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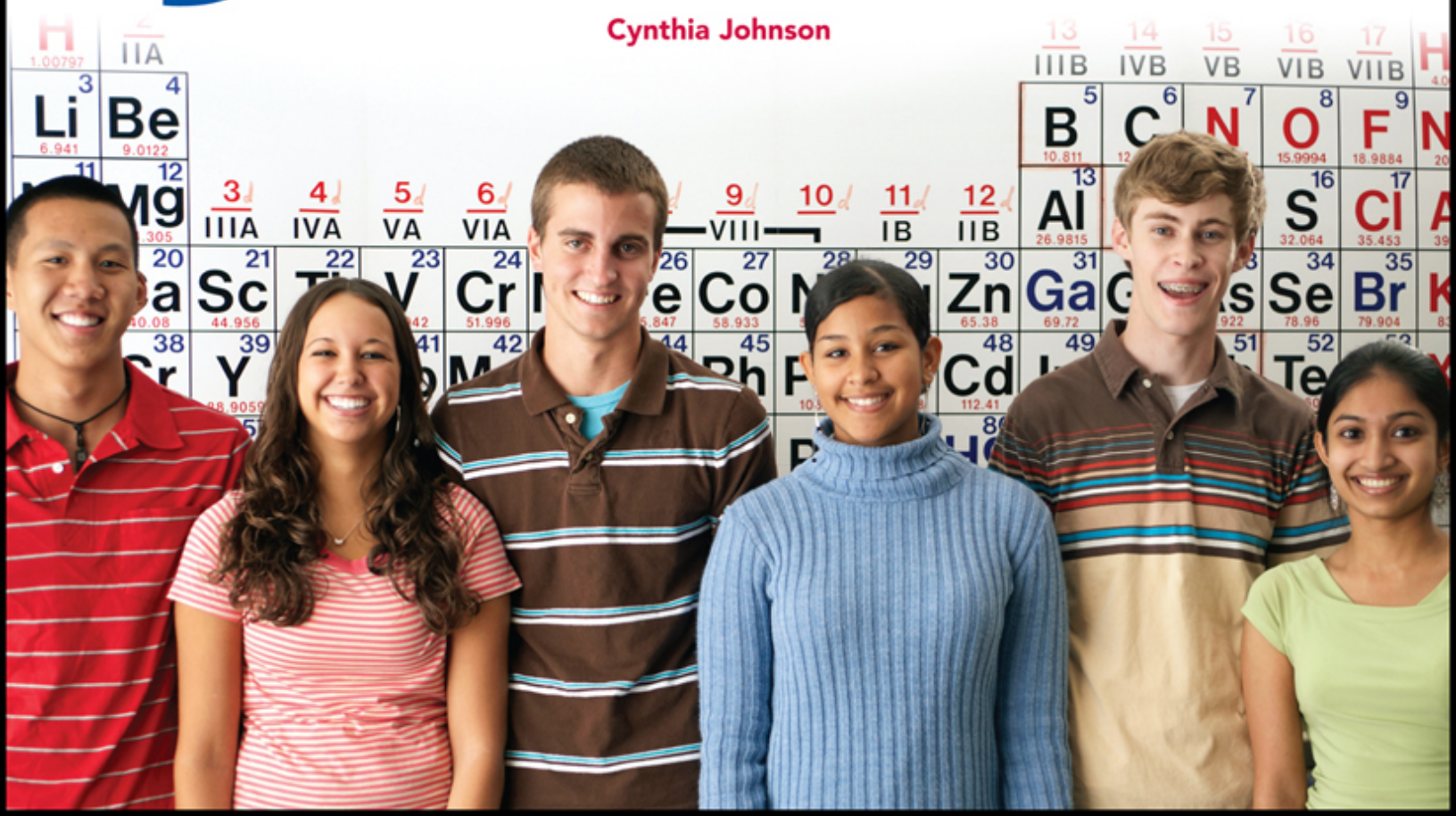
Cynthia Johnson



McGraw Hill's

New York City

SHSAT

Cynthia Johnson

New York Chicago San Francisco Athens London Madrid
Mexico City Milan New Delhi Singapore Sydney Toronto

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PART 1
Getting Started

CHAPTER 1
All About the Specialized High School Admissions Test

Before you know it, you'll be walking the halls of high school. The four years you spend there will undoubtedly be exciting and memorable. Although there are more than 400 high schools in New York City, there are only nine New York City Specialized High Schools. Entrance into these elite, free schools is extremely competitive. In order to be admitted into eight of these programs, students must pass the Specialized High Schools Admissions Test (SHSAT). In fact, this test is the only factor considered in whether or not students are accepted into these schools.

Schools that require students to take the SHSAT are:

- Bronx High School of Science
- Brooklyn Latin
- Brooklyn Technical High School
- High School of Mathematics, Science and Engineering at City College
- High School of American Studies at Lehman College
- Queens High School for the Sciences at York College
- Staten Island Technical High School
- Stuyvesant High School

These schools are run by the New York City Department of Education and were created in order to better meet the needs of gifted students in New York City. Each of the schools has its own unique characteristics. Several offer programs focusing on math and science in addition to the courses that are required by the state. Each of these schools has an outstanding reputation and offers

students a competitive advantage when applying for college. Many college credit and advanced placement (AP) courses are offered. Also, the Specialized High Schools allow students to focus on the fields that interest them.

New York City's Specialized High Schools have been attended by a number of people who have gone on to make extraordinary achievements in many fields. These graduates include Nobel Laureates, politicians, award-winning actors and actresses, musicians, sports figures, and even an astronaut. Students earning the opportunity to graduate from one of these schools are definitely in good company and are taking steps toward a promising future.

Taking the Test

The SHSAT is generally given in late October for eighth grade students, and in the middle of November for ninth grade students who have taken the test in the past and not been accepted into one of the programs. Actual test dates are not released until late in the previous school year. If you plan to take the test, you must register for it through the guidance counselor at your current middle school.

In order to take the SHSAT and be considered for one of the Specialized High Schools, you must live in the Bronx, Brooklyn, Manhattan, Queens, or Staten Island. Because admission into these schools is extremely competitive, being prepared for the SHSAT is a must. Test scores are ranked highest to lowest, with students receiving the highest scores being placed into their first choice school. So, the higher your score, the better your chances of being invited to attend the school you are most interested in. Generally, only about 15 to 20 percent of the students who take the SHSAT are placed into one of the Specialized High Schools each year. That's less than one in five students who take the test. The best way to make sure that your score is near the top of the list is to be prepared.

About 27,000 students took the test in 2009. Of those, fewer than 20 percent were accepted into one of the Specialized High Schools. You will only have one chance to take the test during eighth grade

and possibly one chance during ninth grade. Since there are fewer open spots in tenth grade, the competition for ninth graders taking the test is even tougher. Knowing as much as possible about what to expect and preparing early for the test will help to increase your chances of doing your very best.

What to Expect on the SHSAT

Although the SHSAT consists of verbal and math questions, these questions are different than what you would generally expect to see on a test. Many of the questions require you to use reasoning skills and logic in a way that you usually do not have to in school. This helps determine which students will perform best in a rigorous academic program.

The skills covered on the test are usually part of the eighth grade curriculum. However, since the test is taken early in the school year, they may not have been taught by the time you take the test. Although many of the Specialized High Schools focus on science, there are no science questions on the test. Some of the reading passages may address science topics, but the test will not assess your knowledge of science.

On test day, you will have two and a half hours to complete the SHSAT. The test is divided into two sections, Verbal and Mathematics, but there is no break between the two sessions. You may start with either section and divide your time between the tests in any way you choose. You may move on to the next section whenever you are ready, and you may go back to check your work on the first section after you have completed the second, if time allows.

The following table will give you an idea of how the test is set up:

NUMBER OF QUESTIONS

Verbal: Scrambled Paragraphs	5
Verbal: Logical Reasoning	10
Verbal: Reading Comprehension	30
Mathematics	50

Keep in mind that questions in the scrambled paragraphs section count double.

Verbal Section

There are 45 questions on the Verbal section of the test. Two types of questions measure **verbal reasoning**. The first type includes five "**scrambled paragraph**" questions involving ordering sentences in a logical sequence to form a paragraph. The scrambled paragraph questions are considered to be the hardest on the test, so these are worth twice as many points as any other type of question. The other type of verbal reasoning questions assess **logical reasoning**. There will be 10 of these questions. You will be given information, then expected to figure out a code, form a valid conclusion based on the information, or determine the relative position of objects or people.

The verbal section also assesses **reading comprehension**. There will be five passages with six comprehension questions after each. These 30 items measure how well you are able to understand and interpret information you have read. All of the passages you read in this section will be nonfiction.

Math Section

The Math section of the test includes 50 word problems and computation items assessing your knowledge in four areas of math: arithmetic, algebra, geometry, and probability and statistics.

The ninth grade test includes these skills as well as **trigonometry**. Each question is followed by five answer choices. Other than the scrambled paragraph questions, all items on the SHSAT are multiple choice.

Ranking Schools

On the day of the test, you will rank the Specialized High Schools of your choice on the back of your Admissions Ticket according to which you are most interested in attending. This list must be signed by your parent or guardian. Make sure you know ahead of time which school or schools you would like to apply for, and prioritize your choices. You may choose to select only one, or as many as eight of the schools. You may not change your choices after testing day.

Ranking the schools is extremely important because you will only be assigned to one particular school. Acceptance into the schools is based first on the score you earn on the SHSAT and second on the ranking you have given to the school. Before you rank the schools, there are several things you should take into consideration. Think about:

- Your interests and goals
- Which programs are offered at each school
- The schools' locations
- Commuting options available to each school
- Extracurricular activities
- Sports and other programs available

Make sure you have researched each school's website to make the best choice. Call the school or schools you are interested in and try to arrange to visit them ahead of time.

Be sure to rank as many of the schools as you are interested in attending to increase your chances of being accepted into one of the programs. If you only rank one school and there are no seats available by the time you are placed, you will not be able to attend any of the specialized schools.

Once the test is completed, you will not have the opportunity to change the rankings. Keep in mind that even if you are assigned to a school that was not your first choice, all of these schools are very competitive and offer excellent programs and an outstanding educational experience.

Test Day

On the day of the test, be sure to bring your signed Admission Ticket, with your school choices marked. You will also need several sharpened number 2 pencils, an eraser, and a watch that does not have a calculator. Calculators, cell phones, beepers, and pagers will not be allowed to be brought into the test. NYC public school students will also need to provide their student ID number. In addition to the test booklet and answer sheet, the testing center will provide scratch paper that will be collected after the test.

Test Scores

SHSAT scores are reported as *scaled scores*. These scores are based on how many correct answers you had on the test. Scrambled paragraph questions are each worth two points, and all other questions are worth one point. These points, known as the *raw score*, are combined with the questions' level of difficulty. (There is no penalty for wrong answers.) A formula is used to assign a scaled score based on these factors. The scaled scores for the verbal and math sections are then added together, giving a composite score of up to 800 points.

You are likely to have a different test with different questions than the people sitting near you. There are several versions

of the test. Scaled scores account for these different versions of the test.

You are probably wondering what score you have to earn to get into one of the specialized high schools. Actually, there is not a specific score that guarantees the opportunity to attend one of these schools. Placement is based on how well each student scores on the test.

As you know, student scores are ranked highest to lowest. The student with the highest score is placed into a school first, followed by the student with the second highest score, and so on. Students are placed into their first choice school if there is a spot available. If not, they will be placed into their second choice school. If that school is also full, they will be placed into their third choice, and so on, until all of the spots in all of the schools have been filled. The number of students accepted into each school varies by year. In the fall of 2010, some of the schools accepted approximately 100 freshmen, while others accepted as many as 1,300.

In the February following the test, results will be sent to your current school. Students whose scores earn placement in one of the Specialized High Schools will be offered a seat in a particular school. The offer must be accepted or declined. Students who choose to decline the offer will *not* be offered a chance to attend another of the Specialized High Schools. Once you accept a seat in one of the Specialized High Schools, you must stay at that school for at least one year.

Preparing for the Test

Now that you know what to expect on the SHSAT, you are ready to prepare yourself. Each type of question on the test will be discussed in its own chapter of this book. You will find practice questions, strategies for answering the items, and tips for doing your best on the test. Carefully read through each chapter, become familiar with

the format of the assessment, take the practice tests, and discover your areas of strength, as well as what skills you need to review before test day. Being well prepared will make you more comfortable, increase your confidence, and improve your chances of earning the best possible score on the SHSAT. Now, let's get started!

CHAPTER 2
SHSAT Test-Taking Strategies

Throughout this book, you will learn many skills, definitions, and formulas that you need to know in order to do your best on the SHSAT. You will learn what material will be tested and the types of questions to expect. You are off to a great start in preparing for the test!

In this chapter, we'll talk about a few test-taking strategies that can help you be successful as you prepare for and take the SHSAT. As you read, think about which of these strategies will be most beneficial to you. Everyone is different. Everyone learns differently. And everyone tests differently. Find tips that you think can help you do your best on the SHSAT.

Preparing for the Test

Doing well on a test does not begin on test day. Doing well begins in the weeks and months prior to the test as you study and prepare. This is probably not news to you, since you have already begun to prepare by working through the chapters and practice items in this book. So give yourself a pat on the back, because you are already off to a great start! Now let's talk about a few other things you can do to make sure you are ready for the big day.

Look Ahead

The arithmetic, algebra, geometry, and probability and statistics questions you will find on the SHSAT involve skills that are generally a part of the eighth grade math curriculum, which means that these are all skills you will learn by the end of eighth grade. However, you will be taking the SHSAT toward the beginning of eighth grade, before many of these skills are covered in class. So, what should

you do? Look ahead in your math textbook. Find out what you will learning throughout the year, and determine which skills you are already comfortable with and which you should practice before test day. If you are not yet in eighth grade, borrow a textbook from your school, a friend, or the library. Be sure to look through the glossary as well as the chapters.

Keep in mind that it usually takes a year to learn a whole textbook full of material. You may or may not have that long between now and the time you take the SHSAT. Pace yourself accordingly. There are probably a lot of things you know well already, so a quick review will be sufficient. There may be other things that you will need to spend a little more time learning.

In addition to looking ahead in a textbook, check out the state math standards online. Find out what eighth grade students are expected to know, and make notes of the areas you need to learn more about.

Know the Terms

As you will read repeatedly throughout the practice tests in this book, mathematical terms and formulas will not be provided on the SHSAT. Review these ahead of time. Look at the glossary in your math textbook or find the definitions in this book. Write down terms, definitions, symbols, and formulas on a chart or on flash cards, and then memorize them. Quiz yourself or ask your parents or friends to quiz you.

Remember That Practice Makes Perfect

Practice solving different types of math problems each day. Do some fairly simple questions and some that are more difficult. Do not just practice skills with which you are already comfortable. Try some new things to stretch your knowledge. The more you practice, the more comfortable you will become with the skills.

Read, Read, Read!

Read a variety of materials, such as books, magazines, fiction, and nonfiction. This is a great way to expose yourself to new vocabulary words. If you find unfamiliar words, use context clues to determine

their meaning, then look the words up in a dictionary to be sure you are correct. Even though you will not be expected to read fiction on the SHSAT, reading good fiction is still a terrific way to expand your vocabulary.

Ask Yourself Questions

As you read, ask yourself questions to monitor your own comprehension. Ask yourself what the main idea of a passage is, what facts and details help support the main idea, what the author's purpose might have been for writing the piece, and so on. Do this as you read self-selected materials, as well as when doing required reading for class. Your science and history textbooks offer excellent opportunities for checking your reading comprehension, as well as learning new words and information.

Pretend It Is Test Day

Set aside two and a half hours to do a practice run of the test. Find a quiet place where you will not be disturbed, set the timer, and begin taking one of the practice tests in this book. Not only will you find out how well you know the material, you will also see which parts of the test take the most time for you to complete. Did you finish the verbal questions quickly? Awesome! Did you find that you needed a little bit more time for math? Great information to have! Did you finish the test with enough time left to check your work? Super! All of this information will help you know how to pace yourself on the day of the actual test.

Study Your Mistakes

Everyone makes mistakes. Take the practice tests in this book, and then check your work. If you made a mistake, don't sweat it! This can be an awesome learning tool. Take every opportunity to learn from your mistakes. Reread each question and the correct answer. Notice which types of questions you answered correctly, and which types were incorrect. This will help you find areas in which you need practice. For example, if you missed several algebra questions, you know that a review of algebra skills would be helpful.

Use your tests, quizzes, and assignments in school to learn from your mistakes as well. Pay close attention to the types of skills that trip you up, and spend some time learning how to answer these questions correctly in the future.

Get Some Rest

Make sure you get plenty of sleep the night before the test. It may be tempting to stay up all night to study, but in actuality, it is probably best not to study too much the day before the test. Study, study, study in the weeks and months ahead of time. Then, take it easy the day before the big day. Look over your notes, do a quick review, glance over the vocabulary and formulas, but only spend an hour or so doing this. Then give your brain a break. Take a walk, call a friend, or shoot some hoops.

If you are concerned that you won't be able to sleep if you go to bed early the night before the SHSAT, try getting up extra early the previous morning. That way, you'll be tired before your usual bedtime and have a better chance of catching some extra z's.

Things to Remember on Test Day

You know how to prepare for the test, and you're ready to answer the questions carefully and accurately. Here are a few more things to keep in mind on the day you take the SHSAT.

Before You Leave Home

Making sure you are ready for the test can be as easy as A, B, C!

- **Admission Ticket.** Check your Admission Ticket to find out which test site you have been assigned to and what time you are expected to arrive. Bring the ticket with you to the test. Make sure you have ranked the Specialized High Schools of your choice on the back of the ticket and the list has been signed by your parent or guardian.

- **Breakfast.** Having a nutritious breakfast before the test is important. You need the energy to focus on doing your best, and you don't want a growling stomach to take any of your attention. Even if you don't feel like having a huge meal, at least grab some fruit and trail mix, yogurt and granola, or a smoothie.
- **Clothes.** Wear something that makes you feel good and gives you confidence. You certainly don't want to wear something that is too tight or leaves you feeling too warm or too cold. Consider dressing in layers and bringing a lightweight sweater or jacket. Also, wear a watch if you have one. That way, you will be able to easily keep track of how much time is left during the test.

At the Test Center

Once you get to the test center, there are a few things to keep in mind before you begin taking the test.

- If you get to the test center and do not feel well enough to take the test, be sure to tell the proctor right away. You may not request a makeup test due to illness if you have already begun the test. If you are not feeling well, immediately let the proctor know so your test can be rescheduled. If you start taking the test, no matter how few questions you have answered, you will not be able to reschedule it.
- When instructed to "bubble in" your name, be sure to fill it in as it appears in your school records. Do not use your nickname. For example, if school records indicate that your name is Christopher, fill in *Christopher* on your test, even though your friends and family may call you Chris, CJ, or Junior.
- Make sure you only fill in one answer for each question. If you change an answer, completely erase the first answer before filling in the correct response.

- When filling in answers, make sure the number of the question correlates to the answer on the answer sheet. Be especially careful if you choose to skip any questions. If you skip a question in the test book, make sure you also skip the corresponding answer space on the answer sheet and fill in the answer on the correct line.
- If you start to feel nervous or anxious at any time, take a few seconds to calm down. A couple of deep breaths can help you calm yourself and refocus. Don't worry that this is a waste of time. You'll probably be able to work better if you are feeling calmer.

Tips for Taking the Test

As you know, there are two sections on the SHSAT: Verbal and Mathematics. The Verbal section is first, then the Math section. Let's talk about a few strategies to help you do your best on each of these, as well as some tips that can be used on either part of the test.

Keep Your Eye on the Time

You will have two and a half hours to complete the entire test. The test itself is divided into two parts; however, there will not be a break between the two sections. So, you will need to pace yourself. Since the Verbal section is first, try to spend about half your time, approximately one hour and fifteen minutes, completing these questions. This will leave you an equal amount of time to complete the mathematics questions.

Since there are 45 questions on the Verbal section of the test, you will have an average of a little more than a minute and a half to answer each of these questions. Try to answer the questions as quickly as possible without rushing. If you think that the logical reasoning questions are a breeze, you might try to answer these a little more quickly. That way, if you feel like you could benefit from a little extra time to answer the scrambled paragraph questions, for example, the time will be available for you.

The Math section of the test has a total of 50 problems, which, again, gives you an average of one and a half minutes per question. Try not to spend too much time on any single question. If you complete this section of the test before time is called, you can go back and check your work on either section of the test.

Here are a few other tips for pacing yourself for the test:

- If you feel that the Verbal section of the test will be your strong point, consider allowing yourself only an hour for this section, which will give you additional time on the math.
- If you feel that the Math section will be your strong point, stop working on the verbal questions after half of your time has passed, and move on to math. Work through these items at a faster pace, and then use the extra time to go back to the verbal questions.
- Since there are a total of 95 questions on the SHSAT, and you have 150 minutes to complete the entire test, aim for answering about 10 questions every 15 minutes. That means you will have answered 20 questions in the first half hour, and 40 in the first hour. At this pace, you will complete the test in 135 minutes, leaving an extra 15 minutes to check your work or go back to questions that you may have skipped.

Remember That This Is a Test, Not a Race

Didn't we just discuss the importance of working quickly? Yes. However, working *too* quickly can lead to careless mistakes. Find a pace that is comfortable for you. There is no prize for being the first one finished. There are no bonus points awarded for completing the test in less time. Working too fast and making mistakes could be more detrimental to your score than running out of time to complete the last couple of questions. You definitely need to work quickly, but it is just as important, if not more important, to work carefully. Be aware of the time, but keep your attention on doing your very best.

Use the Time Wisely

With only about a minute and a half to answer each question, you will not be able to spend a lot of time deciding which answer to select. If you come across a question that has you baffled, take your best guess, and then move on. Spending too much time on one tricky question could cause you to run out of time to answer a really simple question later on.

Any time you find a question that you want to take another look at later, be sure to circle the item number in your test book so you can come back to it if time allows. Who knows? You may find that the answer is very clear once you have had some time to think about it.

Read the Questions First

On the reading comprehension questions, read the questions first. This will alert you to what information you should be looking for as you read. Then, completely read the passage; do not try to skim the text to find the information. Read the entire passage completely and carefully, and then select the best answer choice for each question.

Remember, all of the correct answers are based solely on information given in the passage. Even though you may be reading the questions ahead of time, it is important that you do not try to select an answer before reading the text.

Read Everything Carefully

Despite the fact that the clock will be ticking, make sure you take the time to read slowly and carefully enough to catch every fact and detail. Quickly scanning a passage to get the main idea is fine, but then go back and read carefully enough to be able to focus on each and every word.

The importance of reading carefully does not just apply to comprehension passages; this goes for questions and answer choices as well, on both the verbal and math sections of the test. Overlooking a simple word such as *not* or *except* can completely change the meaning of the question and cause you to select the wrong answer. Be careful!

Cover the Negatives

Negative words such as *not* and *except* in a question can cause confusion. When you come across one of these words, try covering it up and reading the question without the negative. Then, see which of the answer choices does not belong. For example:

Which of these is not a planet?

A. Earth
B. Mars
C. Mercury
D. Mexico
E. Venus

Now, that's a pretty simple question, but reread it, eliminating the word *not*: *Which of these is a planet?*

As you can see, Mexico is clearly the answer choice that does not belong. Negative words in a question cause your brain to have to think backward in a way. If you get stuck on a question that contains a negative, try this trick and see if it helps!

Make Your Mark

As you read, underline or circle important information that you will need in order to answer the questions. Consider marking:

- Names
- Dates
- Specific details
- Clues about mood, tone, and theme
- Facts
- Figures
- Numbers
- Percents
- Clue words indicating mathematical operations

This can be helpful when it comes to answering the questions. If the important information has been underlined, you can locate it quickly without having to reread the entire passage each time you need to answer a question.

Try to Answer the Questions Yourself

Try to answer the reading comprehension questions before reading the answer choices. Then, see which answer choice is closest to your response. If none of the choices match your answer, reread the passage and try again.

Make sure your answer is based only on the information in the passage. You may know a lot about a topic, but the test is not intended to measure how much you already knew about a given subject. It is intended to measure how well you comprehended the information in the passage.

Choose the Best Answer

You will see a number of questions that ask you to choose the **best** answer. The best answer is the one that most completely answers the question. More than one choice may be partially correct, or look tempting. Read every answer choice carefully before making a selection. Be sure to choose the **best** answer choice, not the first one that looks good. Answer choice A might look good, but answer choice E could be better!

Think About Each Answer

You may find that the answers to some questions seem completely obvious right away. Be careful to take your time selecting the answer choice, no matter how easy you think the question is. Take the time to read the question and every answer choice carefully, just to be sure the answer you select is truly the best one. It would be a shame to lose points over a careless mistake.

Use Scratch Paper

You will have scratch paper, so make sure to use it. Draw diagrams, charts, tables, pictures, or anything else that will help you solve the

logical reasoning, scrambled paragraphs, or math problems on the test.

You may write in the test book or on scratch paper. However, these will not be scored. Solve problems, draw diagrams, and complete calculations in one of these places, and then mark your final answer on the answer sheet. Make sure there are no extra or stray marks on the answer sheet. Doing work or making marks on the answer sheet could cause answers to be scored incorrectly.

Make a Drawing

Try drawing figures, diagrams, or models to help you solve problems. For example, if you are asked to find the area of a rectangle that is 2 inches longer than a given shape, draw and label the new figure. Being able to visualize the problem can be helpful in solving it.

This strategy can also be helpful in solving the logical reasoning problems. Make a chart, table, or drawing to help you organize the given information and see how the facts fit together.

Check Your Work

Work carefully when solving the math problems. The incorrect answer choices are the result of mistakes that students commonly make. That means, just because your first answer is one of the choices listed, it may not be correct. Work the problems, double-check your work to find any errors, and then read the answer choices.

If you are confident that your answer is correct, but it is not one of the choices, try converting your answer to a different form. For example, if your answer is a fraction, try converting it to a decimal or percent, or try simplifying the fraction. Then, see if your answer is among the choices.

Reread the Question

After selecting the best answer choice, go back and read the question one last time. Make sure that you understood what it was asking and that the answer you chose completely answers the

question. If you determine that your answer does not make sense or is not the best choice for any reason, change your answer and mark your new selection.

Change Things Up

Know which types of questions you do best on, and do these at the start of the test. This can help you build your confidence and get a few correct answers under your belt right away. Just be careful that you do not end up skipping any of the questions by working out of order. Make sure you answer every question during the given time.

Answer Every Question

Make sure you answer every single question, even if it means you have to take your best guess. Your score is calculated by the number of correct answers. On the SHSAT, a question that is left blank is the same as a wrong answer. So, you might as well take a stab at every question, even if you honestly have no idea which is the correct answer. Think back to what you learned about probability; it is impossible to get an answer correct if it is left blank. You are more likely to get it right if you at least fill in something.

If you see that there are only a couple of minutes left, and there is no way you are going to finish all of the questions, just start filling in bubbles. Sounds crazy, right? Wrong! If time is called and the answers are blank, they are wrong. Spend the final minute or so quickly filling in something. That way, at least there is some chance of getting a couple of the answers right. When the competition is as stiff as it is on the SHSAT, every single point helps!

Take Your Best Guess

Obviously, guessing is not the best way to answer any test question, but let’s face it: there are times when guessing is necessary. So, if you’re going to take a guess, make the most of it.

The key to guessing is to first eliminate as many of the incorrect answer choices as possible. This will increase the likelihood of selecting the correct answer. Since there are 5 answer choices for each question, a random guess has a 1/5 probability of being

correct. That's 20 percent. Not great, but better than a 0 percent chance of getting it right if you leave it blank.

If you are able to eliminate one of the incorrect answers, then guess between the remaining four choices, the probability of getting the right answer increases to 1/4, or 25 percent. If you are able to eliminate two incorrect answers, the probability of selecting the correct answer increases to 1/3, or 33 percent. Finally, if you can eliminate three incorrect answers, you will be able to guess between two answer choices: one is correct and one is incorrect. You have increased the probability of selecting the correct answer to 1/2, or 50 percent. That is much better than the 20 percent we started with!

So, how do you eliminate incorrect answer choices?

- Think about what type of answer would make sense, then eliminate answer choices that are not logical. For example, if the question asks about the mean of a set of data, you know that the answer must be between the lowest and highest values in the set. So, you can eliminate any answer choices that do not fit the criteria.
- Estimate the correct answer to math problems. Any choices that are too far off from the estimate can be eliminated.
- Look for an answer choice that sticks out from the rest for some reason. It may be significantly longer or shorter than the other choices. It may be the only negative number listed, or the only decimal number among a set of whole numbers. It might be much higher or lower than the other values.
- Look for answers that are very similar. If there are two answer choices that are nearly the same, other than a few words, consider eliminating the other answer choices and selecting between these two.
- Look for answers that are opposites. Since there is no way for both of the choices to be correct, try to eliminate one of them. Be careful, though, because they may both be incorrect.

- Check for any grammatical hints. This may be especially helpful when answering reading comprehension questions. If an answer choice does not fit grammatically, if the part of speech is incorrect, or if the tense does not match, the answer choice may not be correct, and can be eliminated.
- Watch out for absolutes such as *always*, *never*, and *every*. There are very few things that *always* happen, are *never* true, or occur *every* time. These words could indicate that the answer is not correct, and that it can be eliminated.

Go with Your Gut

Believe it or not, your first instinct regarding the correct answer choice is often correct! If you are confident that you have read the passage carefully or worked the problem accurately, the answer choice that you are leaning toward at the beginning is probably a great choice!

If you go back to check your work at the end of the test and are considering making a change to one of the answers, think carefully. Ask yourself, *Did I make a mistake in the calculations? Did I misunderstand the question or the passage? Is there a valid reason for thinking that this answer choice might not be the best one?* Unless there is a strong reason for changing an answer, it might be a good idea to leave it alone.

Let's review!

In this chapter, you've learned some tips and strategies that will help you do your best on the SHSAT. Let's take a final look at some of the key points for you to keep in mind leading up to the big day, and on test day itself.

- Look ahead in an eighth grade math book to see what concepts will be taught throughout the year. Practice different types of math problems daily to make sure you are comfortable with a wide variety of skills.
- Memorize the math terms, definitions, symbols, and formulas that you will need to use on the test.
- Read a variety of materials in order to expand your vocabulary, and ask yourself questions about the text in order to monitor your own comprehension.
- Taking the practice tests in this book can be helpful for several reasons:
  - You will become familiar with the types of questions that will be on the SHSAT.
  - You will learn which skills you have already mastered and which you would benefit from reviewing.
  - You will find out how to best budget your time on test day.
- Start studying for the test well ahead of time, and then take a break on the day before the test. A quick review on that day should be plenty.
- Getting plenty of sleep, having a good breakfast, and dressing comfortably are important ways to make sure you arrive in tip-top shape for the test!
- Do not start the test if you are at all concerned about whether or not you are feeling well enough to finish. Tell the proctor immediately if you are ill.
- Keep your eye on the time and pace yourself accordingly. It is important to work quickly and carefully, without rushing. Allow yourself more time for the questions or sections that you do not feel are your strength, and move quickly through the areas that are your best skills.
- Try not to spend too much time on any single question. You can always come back later if time allows. Mark your best guess, circle the item number in the test book, and use the time to answer other, possibly easier, questions.

- Read the questions first, then the passages in order to have an idea of what information to look for in the text. Do not, however, actually try to answer the questions before reading the passages. The correct answers are based solely on information in the text, not on what you may already know about the topic.
- Read everything carefully. This includes passages, questions, and answer choices.
- Underline or circle important information in passages, word problems, and questions. This will make it easier to locate the facts needed to select the best answer choice.
- Read every answer choice, then select the one best answer. You may find that two or more of the choices look good, but the correct answer is the one that most completely answers the question. Also, keep in mind that the first answer choice that catches your attention may not be correct. Take the time to carefully consider each and every option.
- Use the scratch paper and your test booklet to complete calculations, draw diagrams, and solve problems. Then, transfer your final answer to the answer sheet.
- Answer every question, even if you have to take your best guess. Any item that is left blank will be marked wrong, so eliminate as many incorrect answer choices as possible, then choose the best option. If you are not sure whether an answer is correct or not, trust your instincts.

A Final Thought

Congratulations! You are taking control of your future and giving yourself the best possible opportunity to do well on the SHSAT by preparing for this test. Doing your best on the SHSAT is an accomplishment you can be proud of. Studying the material, taking the practice tests, and knowing what to expect on the test are beneficial steps in making sure you are well prepared. Now, take a

deep breath, smile, and pat yourself on the back for a job well done!
Best of luck on the SHSAT!

PART 2
SHSAT Diagnostic Test

CHAPTER 3
Diagnostic Test with Answers

On the following page you'll find a form similar to that of the actual SHSAT. Carefully tear it out of the book and use it as you take the diagnostic test for both the Verbal and Mathematics sections.

PART 1 VERBAL

SCRAMBLED PARAGRAPHS

Paragraph 1
- The second sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The third sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The fourth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The fifth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The sixth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ

Paragraph 2
- The second sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The third sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The fourth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The fifth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The sixth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ

Paragraph 3
- The second sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The third sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The fourth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The fifth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The sixth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ

Paragraph 4
- The second sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The third sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The fourth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The fifth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The sixth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ

Paragraph 5
- The second sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The third sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The fourth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The fifth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ
- The sixth sentence is Ⓠ Ⓡ Ⓢ Ⓣ Ⓤ

LOGICAL REASONING

11 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
12 Ⓕ Ⓖ Ⓗ Ⓙ Ⓚ
13 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
14 Ⓕ Ⓖ Ⓗ Ⓙ Ⓚ
15 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
16 Ⓕ Ⓖ Ⓗ Ⓙ Ⓚ
17 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
18 Ⓕ Ⓖ Ⓗ Ⓙ Ⓚ
19 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
20 Ⓕ Ⓖ Ⓗ Ⓙ Ⓚ

READING

21 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
22 Ⓕ Ⓖ Ⓗ Ⓙ Ⓚ
23 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
24 Ⓕ Ⓖ Ⓗ Ⓙ Ⓚ
25 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
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36 Ⓕ Ⓖ Ⓗ Ⓙ Ⓚ
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48 Ⓕ Ⓖ Ⓗ Ⓙ Ⓚ
49 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
50 Ⓕ Ⓖ Ⓗ Ⓙ Ⓚ

PART 2 MATHEMATICS

MATHEMATICS PROBLEMS

51	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	66	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	81	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	96	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ
52	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	67	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	82	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	97	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
53	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	68	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	83	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	98	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ
54	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	69	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	84	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	99	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
55	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	70	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	85	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	100	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ
56	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	71	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	86	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ						
57	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	72	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	87	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
58	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	73	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	88	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ						
59	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	74	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	89	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
60	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	75	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	90	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ						
61	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	76	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	91	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
62	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	77	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	92	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ						
63	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	78	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	93	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						
64	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	79	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	94	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ						
65	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	80	Ⓕ	Ⓖ	Ⓗ	Ⓙ	Ⓚ	95	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						

PART 1
VERBAL
75 Minutes ■ 45 Questions

Scrambled Paragraphs

PARAGRAPHS 1–5

DIRECTIONS: In this section, the first sentence for each paragraph is given; the remaining five sentences are listed in random order. Put the sentences in the order that creates the **best** paragraph. The paragraph must be well organized, logical, and grammatically correct. Each correctly ordered paragraph is worth **double** the value of a question in any other section of the test. Partially correct responses will receive no credit.

Use the blanks to the left of the sentences to keep track of your sentence order. For example, write “2” next to the sentence you think follows the first sentence, write “3” next to the sentence you think follows “2,” and so on. Make changes as necessary. When you have finalized your sentence order, mark your choices on your answer sheet.

Paragraph 1

The building materials that are native to a region influence the type of homes that are built there.

_____ **Q.** Later, when settlers went west to places that had a plentiful supply of clay, they started to build their houses with adobe bricks.

_____ **R.** Other settlers in the East lived in areas that had many stones, so stone houses became the norm, especially during the colonial period in those regions.

_____ **S.** These bricks were made of a mixture of clay and straw that was dried in the sun until the bricks were totally hardened and ready for use.

_____ **T.** Because the climate of these areas was so warm and dry these simple bricks had a long life.

_____ **U.** For example, when this country was first settled there were plentiful forests, so many houses in the East were built of wooden logs.

Paragraph 2

The science of archaeology grew from one person's fascination with the *Iliad*, the exciting story of the Trojan War.

_____ **Q.** However, a German businessman named Heinrich Schliemann decided to search for the site of ancient Troy in Asia Minor and actually found the remains of several cities believed to be Troy, one on top of the other.

_____ **R.** In the nineteenth century, historians speculated that both Homer and the Trojan War he described in the *Iliad* were more myth than reality.

_____ **S.** While Schliemann did much to popularize the science of archaeology, his work is controversial among archaeologists today.

_____ **T.** The Greek poet Homer wrote the *Iliad* more than 2,500 years ago during the period that is known as prehistory.

_____ **U.** He found many relics that he publicized as belonging to ancient Greek heroes of the Trojan War.

Paragraph 3

The human eye is a very sensitive organ since so much of it is exposed to the environment.

_____ **Q.** The eyelid is equally essential, since it shuts over the eye and this protects it from dust or other airborne materials that could irritate.

_____ **R.** Luckily, the facial structure around the eye is designed to protect the exposed parts of the eye from harm or at least keep it to a minimum.

_____ **S.** There are other aspects of the eye that help protect the organ from threats, including the tear glands and the eyelid.

_____ **T.** Tear ducts perform an important role because they provide the fluids that keep the eye moist, which is necessary to the well-being of the organ.

_____ **U.** The brow shields the eye from the top, while the nose and cheekbones guard it at the sides and below.

Paragraph 4

About 80 years ago, a scientist named Alfred Wegener noticed that a map of the world resembled a giant jigsaw puzzle.

_____ **Q.** Experts were skeptical at first, but new facts kept emerging that supported Wegener's theory.

_____ **R.** Wegener argued that long ago, the continents were all one landmass, but something had happened that caused them to break apart and move in different directions.

_____ **S.** He thought the map looked as though the east coasts of North and South America fit into the west coasts of Europe and Africa and theorized that they had been joined at one time.

_____ **T.** If a plate moves, the land above it can shift, which explains how one original landmass became multiple continents over time.

_____ **U.** Scientists now theorize that the continents and the oceans rest on top of giant pieces of Earth's crust, called tectonic plates, that move.

Paragraph 5

Marian Anderson's career represents the triumph of rare talent over poverty and racial prejudice.

_____ **Q.** Marian Anderson soon had a successful career as a soloist in Europe as well as America, even though she often had to face racial prejudice.

_____ **R.** Instead of performing at Constitution Hall, she gave an open-air concert before an audience of 75,000 at the Lincoln Memorial.

_____ **S.** Her greatest achievement, however, came toward the end of her career when she became the first African American woman to perform at the Metropolitan Opera House in New York City.

_____ **T.** Her talent was evident as a child, but her family could not afford to pay for voice lessons, so her church helped her and she began to perform in many different churches.

_____ **U.** Because of her race, she was barred from performing a concert at Constitution Hall in Washington, DC, in 1939.

Logical Reasoning

QUESTIONS 11–20

DIRECTIONS: Read the scenarios below, then choose the **best** answer to each question. You must base your response **only on the information given.**

Read the scenarios with careful attention to the precise meaning of the words used. For example, “The red shirt hanging **between** the yellow and blue shirts” does not necessarily mean “The red shirt is **between and next to** the yellow and blue shirts”; one or more other articles of clothing may separate the red shirt from the yellow shirt or from the blue shirt. This precaution also applies to location words such as **above, below, before, after, ahead of,** and **behind**.

---

11. Five students are standing in a line according to their ages, from oldest to youngest, left to right.

1) Daniela is older than Mark, but younger than Rico.

2) Mark is younger than Bryan, but older than Jeannie.

3) Bryan is younger than Rico.

In which position is Bryan?

A. first
B. second
C. third
D. fourth
E. Cannot be determined from the information given.

12. Crystal will not play soccer if Anita does. Anita will play soccer if Gerry is not late to school.

Based only on the information above, which of the following **must** be true?

F. If Gerry is late to school, Crystal will play soccer.
G. If Anita does not play soccer, Crystal will play soccer.
H. If Gerry is on time for school, Anita will play soccer.
J. Anita will not play soccer if Gerry is late to school.
K. If Gerry plays soccer, Anita will play also.

13. Five letters are pinned next to each other on a wall. They are numbered consecutively from left to right.

1) One letter is an H.
2) The letter C is in the second position.
3) There are exactly three letters between the H and the L.
4) There is exactly one letter to the right of the M.

In which position is the P?

A. second
B. third
C. fourth
D. fifth

E. Cannot be determined from the information given.

14. Most of the people going on a trip to Italy speak Italian. However, some of them are not very fluent.

Based only on the information above, which of the following **must** be true?

F. All of the people who are going on the trip to Italy speak some Italian.

G. Some people going on the trip to Italy speak Italian like a native speaker.

H. Being fluent in Italian is necessary to going on the trip to Italy.

J. Most people going on the trip to Italy are not very fluent in Italian.

K. People who do not speak Italian well can go on a trip to Italy.

Questions 15 and 16 refer to the following information.

In the code below, (1) each letter always represents the same word, (2) each word is represented by only one letter, and (3) in any given sentence, the letters may or may not be presented in the same order as the words.

M	X	R	T	Y	means
"Marie	swam	laps	by	herself."	

X	N	M	Q	R	means
"Laticia	swam	laps	with	friends."	

Q	M	X	R	P	means
"Andy	swam	laps	with	Laticia."	

M	Z	R	X	N	means
"Dante	swam	laps	with	friends."	

15. Which letter represents the word "Dante"?

A. R
B. M
C. X
D. Z
E. Cannot be determined from the information given.

16. Which word is represented by the letter N?

F. herself
G. friends
H. swam
J. Laticia
K. Cannot be determined from the information given.

17. At a fabrics store, there are five different kinds of fabrics piled one on top of the other. The fabrics consist of cotton, linen, silk, corduroy, and wool.

1) The cotton fabric is higher in the stack than the wool is.
2) Exactly two fabrics are between the wool and the silk fabrics.
3) The linen is third from the top.

If the bottom fabric is wool, which fabric is on the top?

A. corduroy
B. cotton
C. silk
D. Either corduroy or cotton, but it is not possible to determine which one.
E. Either corduroy or silk, but it is not possible to determine which one.

18. People who become members of the Sister Cities Club are expected to volunteer 5 hours a month to help with the events the organization holds. Terri volunteers 3 hours for Sister Cities.

Based only on the information above, which of the following **must** be true?

F. Terri is not a member of Sister Cities.
G. If Terri had more free time, she would volunteer more hours to the organization.
H. If all members of the club volunteered only 3 hours a month, the Sister Cities Club could not continue to exist.
J. Terri is a volunteer for other organizations, so she does not have enough time to volunteer the full 5 hours for Sister Cities.
K. If Terri is a member of the Sister Cities Club, she does not do what is expected of her.

19. Four people were riding bikes to the lake. They arrived at the lake one at a time. Their bikes were all different colors. One was red, another yellow, and the other two green and blue.

1) Josh arrived right after Paula.
2) Paula rode the red bike.
3) Chet was not riding the blue bike.
4) The red bike got to the lake before the green bike.
5) Sally, who rode the yellow bike, arrived after Josh.

Based only on the information above, which of the following **must** be true?

A. The blue bike arrived before the green bike.
B. Chet arrived third.
C. Josh rode the green bike.
D. Sally arrived third.
E. The green bike arrived second.

20. If it is raining, the school bus could be late. If it is sunny, the school bus will be on time.

Based only on the information above, which of the following **must** be true?

F. The school bus is on time when it is not raining.
G. The school bus is late when it is cloudy.
H. Rain may cause the school bus to be late.
J. If the bus is late, it must be raining.
K. The school bus is on time more than it is late.

Reading

QUESTIONS 21–50

DIRECTIONS: Read the passages below, then answer the questions that appear after each passage. Select the **best** answer for each question. Reread the passages if necessary. You must base your response **only on information contained in the passage**.

A startling discovery by archaeologists has revolutionized earlier theories about when people first came to the Americas. An excavation at Buttermilk Creek has pushed the date when the first human inhabitants entered the New World back in time some 2,500 years! A huge collection of stone tools such as flint knife blades, chisels, choppers, spears, and other artifacts were found there. The site is a 50 square meter area. It is located 40 miles northwest of Austin, Texas.

These Buttermilk People roamed the plains of North America as early as 15,500 years ago. And they didn't enter the Americas by crossing a land bridge from Siberia, as previously thought. Instead they came by boats crafted from animal skins. Scientists believe this happened at the height of the Ice Age. They think that these people hunted seals along the glaciers and traveled a few miles each day. These early people came with children, babies, and grandparents. They even brought pet dogs. Their first settlements were probably in Washington and Oregon. There is evidence that over centuries they explored all the way to the tip of South America.

For decades scientists had labeled the first inhabitants of the Americas as the Clovis People, also called Paleo-Indians. This was based on the artifacts unearthed near Clovis, New Mexico, in 1936 and 1937. These artifacts dated to 13,000 years ago. At that time these people roamed the Great Plains and the Southwest. Archaeologists must now contend with the new evidence that the Buttermilk People were here much earlier. The evidence of the handcrafted tools and weapons and shreds of flint found alongside the creek is compelling.

The Buttermilk People hunted giant Ice Age mammoths and mastodons, related to present-day elephants, with spears.

Spear points were crafted with flint and made with the bones of the mammoths. The hunting of an animal resulted in meat that would last for many weeks. The meat was also dried to provide food through the harsh winter months. Clothing and shelter were made from the hides.

Probably most surprising was the discovery of chunks of hematite (iron oxide). These were worn down along the sides, a sign that they had been rubbed. They were similar to what school chalk looks like after it is used. When mixed with plant oils, hematite produces red ochre. Archaeologists believe the Buttermilk People painted their spears and decorated their clothes and their bodies. This new discovery has given scientists a new way of looking at early people in the Americas.

21. Which of the following best tells what this passage is about?

A. how scientists are able to date human artifacts that are unearthed
B. the relationship between the Buttermilk People and the Clovis People
C. the discovery that people came to the Americas earlier than thought
D. where the artifacts of the earliest settlers to the New World were found
E. the way in which the first humans to settle in the Americas lived

22. The passage suggests that the Buttermilk People

F. were larger physically than the Clovis People.
G. had difficulty finding food to eat.
H. were able to record their stories through pictures.
J. were better hunters than the Clovis People.
K. made the most of resources available to them.

23. Which of these statements is true about the Buttermilk People's nature?

A. The Buttermilk People had deep religious beliefs.
B. The Buttermilk People were aggressive against others.
C. The Buttermilk People were curious about their world.
D. The Buttermilk People were serious most of the time.
E. The Buttermilk People showed little imagination.

24. Which of the following best describes what is suggested by the statement that the Buttermilk People "even brought pet dogs" (line 24)?

F. The Buttermilk People were affectionate.
G. The Buttermilk People were well educated.
H. The Buttermilk People feared animals.
J. The Buttermilk People felt animals were spirits.
K. The Buttermilk People used animals for protection.

25. What does the use of hematite suggest about the Buttermilk People?

A. They were fearful of enemies.
B. They were friendly to others.
C. They liked colorful objects.
D. They liked to play games.
E. They wanted to look frightening.

26. What was **not** a concern of the Buttermilk People?

F. finding enough food
G. moving from place to place
H. exploring new lands
J. starting a network of trails
K. creating places to live

Along the floor of the Pacific Ocean, more than a mile beneath the surface of the sea, lives a unique creature. Discovered by scientists in 1977 when they developed a submarine capable of going that deep, this animal lives in an environment that is forever dark; no sunlight penetrates the ocean this deep. And the water always remains just a few degrees above freezing.

But how does this creature survive without the life-giving energy of the sun? The animal, called a giant tube worm, is unlike any other. It is able to thrive in such a hostile environment because of the existence of hydrothermal vents or fissures in the surface of the earth that are powered by volcanic activity. Surprisingly, it does not need the energy of the sun, either to keep it warm or to nourish its food supply. The vents release heated vapors from the earth's core that reach temperatures of over 400 degrees Fahrenheit. The vapors contain hydrogen sulfide. Poisonous to man and other animals, the gas is how the giant tube worm receives its nourishment.

Resembling giant tubes of lipstick, the worms have a soft body surrounded by a hard white shell called chitin (pronounced "kite-in"), the same substance that forms the shells of lobsters and crabs. Tube worms have no mouth or digestive system. They get their food from billions of bacteria that live inside their bodies. The bacteria convert the hydrogen sulfide into food for the worms by oxidizing the gas. The bacteria and worms have a symbiotic relationship, which means that both benefit. The worms get nourishment; the bacteria have a place to live. The giant tube worm anchors itself to the ocean floor. On its uppermost part there is a bright red plume, which is filled with hemoglobin. The plume absorbs the chemicals from the seawater that feed the bacteria. The tube worm has few enemies because not many creatures can survive near the vents, but if it is threatened, it can withdraw the plume inside its shell. Sometimes crabs and crustaceans nibble on the plume for nourishment.

These giant tube worms can reach lengths of eight feet. They grow very slowly. Scientists believe that they are among the longest living animals on earth; some are estimated to be over 250 years old. Scientists think tube worms are born with a mouth, which allows them to absorb bacteria on the ocean floor and in the water. As they grow, the mouth disappears. Giant tube worms have no eyes, but they can sense movement and will pull in their plumes to protect themselves. These giant tube worms are closely related to smaller worms that live in shallow water.

27. Which of the following best tells what this passage is about?

A. why giant tube worms develop
B. how bacteria get their nourishment
C. why giant tube worms have a plume
D. how giant tube worms reproduce
E. how giant tube worms survive

28. The author implies that most life on earth

F. has a symbiotic relationship with another animal.
G. lives on land rather than in the ocean.
H. can survive vast changes in temperature.
J. needs the energy of the sun to survive.
K. has the ability to live a very long time.

29. Which of these statements is true about giant tube worms?

A. Giant tube worms are unrelated to any other form of life.
B. Giant tube worms are found in the ocean all over the globe.
C. Giant tube worms are short-lived species that were found by chance.

D. Giant tube worms are aggressive toward enemies who try to eat their plume.

E. Giant tube worms have taught scientists that organisms can exist without direct or indirect access to light.

30. Which best describes the relationship between giant tube worms and lobsters?

F. They both have hard exteriors.

G. They both live off the hot air vents.

H. They have a protective relationship.

J. They both have bacteria inside them.

K. They have a relationship that benefits them both.

31. Which is **not** true of giant tube worms?

A. It is impossible to gain the kind of data needed to fully study them.

B. They are able to live without energy from the sun.

C. They were found when the bottom of the ocean was explored.

D. They do not tolerate extremes in temperature.

E. They cannot exist without the bacteria inside them.

32. What might be inferred about why the mature giant tube worm does not have a mouth?

F. The giant tube worm has no need for a mouth after the bacteria are inside.

G. A mouth would not be useful because of the extreme heat.

H. A mouth would get in the way of the plume.

J. The giant tube worm does not require nourishment.

K. A mouth can cause difficulty so deep in the ocean.

Not all blind people use dogs as guides. A growing number of people who are sightless are turning to miniature horses to ensure their safety and ability to move about. These little animals are equally able to keep a master out of harm's way, and they have certain attributes that may make them more desirable than a Seeing Eye dog.

Miniature horses are no strangers to hard work. Many of their ancestors worked in the coal mines in England and Wales. They have been used for mining in the United States as well. Some of these animals never saw the light of day, living their entire lives underground. The obvious reason for miniature horses working in a mine is their height. They measure 35 inches at their withers. But there were other reasons as well. These little horses are very obedient, easygoing, and able to work long hours. All of these traits make them excellent candidates as guide animals.

And they have other valuable assets. They are hypoallergenic, so people with allergies may prefer them over dogs. They live longer than traditional canine guides, up to 40 years, so people may only have to have one guide animal in their lifetime. They also don't mind spending their downtime out of doors, even though they can be house-trained.

There's another factor to consider as well. Many people are nervous around dogs but feel more comfortable with a small horse. These horses are never aggressive, but they are strong enough to help individuals with physical disabilities, including helping a handler rise from a chair if that is a difficult thing to do. Another good point, say owners, is that dogs are often seen as pets, and it takes some explaining to allow them to act as guides. On the other hand, little horses are recognized quickly as guide animals. And horses are very clean and, unlike dogs, don't get fleas. Plus they are incredibly sweet and willing to take care of their charge.

They are also very adaptable. One sightless person flew with his miniature horse from the farm where she was trained to his home. The horse's name was Cuddles, and she slept beside her master for most of the

flight. They also took a trip to New York City, where locals were impressed by this animal and how she was even able to navigate the subway!

Of course dogs and horses are not the only animals that have served as service animals. Pigs are also trained as guide animals from time to time, and monkeys have proved useful to those with serious physical handicaps. So the saying should probably be changed to this: It's not just a dog that is man's best friend.

33. Which of the following best tells what this passage is about?

A. how different animals are used to guide blind people
B. how miniature horses were used in mines
C. why miniature horses are good for those with allergies
D. how Seeing Eye animals are trained
E. why miniature horses make good guide animals

34. What trait is **not** mentioned as being an asset for miniature horses serving as guide animals?

F. good stamina
G. good memory
H. calm nature
J. able to live outdoors
K. long life

35. According to the passage, which of the following statements about miniature horses is accurate?

A. Miniature horses are able to do as much with a handler as dogs are.
B. Miniature horses are easier to train than dogs are.

C. Miniature horses are less likely to injure their handler than dogs are.

D. Miniature horses are less costly to feed than dogs are.

E. Miniature horses require more veterinary visits than dogs do.

36. The information about miniature horses working in mines was included in order to illustrate

F. how they were misused in the past.

G. why they are obedient.

H. how they behave under stress.

J. why they are no longer used in mining.

K. what hard workers they are.

37. Which of the following is most likely to choose a miniature horse rather than a dog as a service animal?

A. a person who grew up with cats

B. a person who grew up on a farm

C. a person who likes to live in a city

D. someone who feels comfortable around animals

E. someone who is afraid of dogs

38. Why does the author want to change the saying to “It’s not just a dog that is man’s best friend”?

F. to make owners of other animals happy

G. to be more politically correct

H. to show that many animals are helpful

J. to make fun of the saying

K. to insult dog lovers

It was July 20, 1969, and Neil Armstrong took the steps down to the surface of the moon. “That’s one small step for man; one giant leap for mankind,” he said as people worldwide watched the historic moment on television. It was an event that could not be forgotten. Today some people think that the space exploration program should be cut back or eliminated completely. Still others believe this is a self-defeating attitude.

People who favor future space exploration say that it is vital for many reasons. They say it could be key to the future of the human race. It could open unknown doors in medical treatment. It could provide new kinds of ores and other materials. What exploration of outer space will result in ultimately is not something that can be predicted. But proponents say to let space exploration lapse would be a step backward.

Obviously there is a strong-rooted instinct in people to find out about their universe. Humans have always looked up into the stars and wondered what was out there. Some say that space exploration is an essential extension of this urge. It helps provide some of the answers to the questions about our universe. And those in favor of space exploration say it will continue to do so with technology that allows scientists to reach out further into the universe. For thousands of years, brave people from all cultures have risked voyages into unknown wildernesses and across vast seas to find new lands. Intrepid travelers set out to discover uncharted worlds. Polynesian sailors settled far-flung islands of the Pacific starting more than three thousand years ago. Chinese explorers reached what is now Ethiopia by sea in the second century BCE. And Ferdinand Magellan’s expedition, launched in 1519, became the first to sail around the world.

In this spirit, mankind reached out again in 1957 with the launch of *Sputnik* by the Soviet Union. *Sputnik* was the first vessel to travel outside the Earth’s atmosphere. It spurred the United States to develop a space program. This program culminated in the *Apollo* missions to the moon. These missions inspired an entire generation of young people.

55 This exploratory spirit is manifested today by the International Space Station. The ISS conducts intensive environmental research into problems like air quality and climate change. With the population of planet
60 Earth expanding, there is more pressure on the environment and resources. Those who believe in space exploration say the concept of space colonization holds hope for the future of mankind. And they point
65 to what Pulitzer Prize–winning historian William Goetzmann said: "America has indeed been 'exploration's nation,' … that … continually looks forward in the direction of the new." To many it is imperative
70 to continue space exploration for the good of our world and mankind.

39. Which of the following best tells what this passage is about?

A. the many explorers who wanted to chart unknown lands
B. why people are against space exploration increasing
C. the reasons why people think space exploration is important
D. the history of space travel in this country and the Soviet Union
E. the possibility of space colonization

40. The best reason that the author began the passage by describing the landing on the moon was to

F. remind the reader of what happened.
G. impress the reader with its drama.
H. suggest that space exploration is hard to do.
J. show how difficult it was to reach the moon.
K. set the stage for a difference of opinion.

41. Which reason is **not** given for continuing space exploration?

A. improved medical procedures
B. new kinds of materials

C. the possibility of colonization
D. possible ways to contain air pollution
E. a better food source

42. Why are the explorers in the passage called intrepid?

F. They were driven by hunger.
G. They went by ship.
H. They traveled many miles.
J. They found new lands.
K. They were fearless.

43. The author includes information about explorers to demonstrate

A. the extent of traveling that explorers did.
B. his knowledge of the history of exploration.
C. why space exploration is important.
D. how mankind has always wanted to find new lands.
E. how explorers mapped the earth.

44. How will ending the space exploration program most negatively affect mankind's future?

F. People will be unaware of the universe around them.
G. No new frontiers will be explored.
H. Mankind will no longer be curious about its origins.
J. People will no longer want to be rocket engineers.
K. Humans may not be able to find other planets to live on.

Mary Cassatt was a famous and critically acclaimed artist, but it was not easy for her to become one. Mary was born in the mid1800s, a time when young women from well-off families were expected to become educated and then to marry. The education was a polish that would make a woman a charming hostess. But that was not what Mary wanted for herself. At an early age she fell in love with painting. To fulfill her dream, she had to rebel against the wishes of her family, in particular her father.

When Mary was a young girl, her family left the United States to live in Europe. While there, she viewed the work of some of the greatest artists who have ever lived. She even had her first art lesson in Europe. When her family returned home to Pennsylvania, Mary enrolled in the Pennsylvania Academy of Fine Arts in Philadelphia at the age of 17. She was one of a handful of females there. While the other women wanted to learn to paint to round out their education, Mary was a serious student. She soon became frustrated by the slow-paced instruction and vowed to return to Europe once again. Her father did not approve of her pursuing a career as an artist, yet Mary went back to Europe to study painting.

Mary studied painting in Paris. While there she met the famous painter Edgar Degas, and he quickly became her mentor. Soon she joined the Impressionist Movement. This group of artists used bright colors and did not paint in a traditional manner. They painted the “impression” of an object or person, instead of making something look exactly the way it did, they made it look the way it looked to them.

Mary soon started to become well known. Unlike other artists who painted scenes in nature, Mary chose to paint ordinary people doing ordinary things. She often painted women with children. She returned home to be with her family and to meet with other American artists. She left the Impressionist School and went in her own direction. Her paintings were always distinctive. Sometimes her pictures seem to glow with light. At other times they seem very soft and warm. Mary Cassatt gave the world a new way of looking at people and places. She also showed the world that women could become great artists.

45. Which of the following best tells what this passage is about?

A. the city of Paris in the mid-1800s
B. why Mary Cassatt wanted to be an artist
C. the reasons Mary Cassatt's paintings are popular
D. the life and career of Mary Cassatt
E. the founders of the Impressionist Movement

46. How did the experience of living in Europe as a young girl affect Mary Cassatt?

F. She started to show promise as a painter.
G. She became more rebellious.
H. She was exposed to the great masters.
J. She met many artists who helped her develop her art.
K. She learned the techniques of watercolor.

47. Which of the following was **not** something that Mary Cassatt wanted?

A. learning more about painting
B. settling down with her family
C. painting the things she loved
D. meeting many artists
E. breaking with tradition

48. In what way did the Impressionist Movement change the art of the time?

F. Art became much more expensive than it was before.
G. New standards were set about what could be painted.
H. Fewer members of the public collected paintings.

J. More women became well known and respected artists.
K. Artists were freer than before in what and how they could paint.

49. Why do you suppose Mary Cassatt left the Impressionist Movement?

A. She did not want to be labeled.
B. She had an argument with Edgar Degas.
C. She wanted to return to America.
D. She was asked to do so by her family.
E. She wanted to join another artistic group.

50. What can be inferred about Mary Cassatt?

F. She worried about her image in society.
G. She did not trust many people.
H. She would do anything necessary to be famous.
J. She was a talented woman who was selfish.
K. She was a strong woman who knew what she wanted.

PART 2
MATHEMATICS
75 Minutes ▪ 50 Questions

General Instructions

Find the solution to each problem, then choose the best answer from the options given. Mark the letter of your answer choice on the answer sheet. You may use your test booklet or paper provided by the proctor as scrap paper for your calculations. However, DO NOT make calculations or put any stray marks on your answer sheet.

Important Notes

(1) No formulas and definitions of mathematical terms and symbols are provided.

(2) Diagrams are not necessarily drawn to scale. Be careful not to make assumptions about relationships in diagrams.

(3) Diagrams can be assumed to be in one plane unless otherwise stated.

(4) Graphs are drawn to scale. Therefore, you may assume relationships according to appearance. This means lines that look parallel can be assumed to be parallel, angles that look like right angles can be assumed to be right angles, and so forth.

(5) All fractions must be reduced to lowest terms.

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51. What is the greatest common factor of 154 and 196?

A. 2
B. 7
C. 11
D. 14
E. 22

52. Christine, Marylou, and Katy each have a savings account. Marylou has twice as much money in her account as Christine. Katy has three times as much money in her account as Christine. If the total amount in all three accounts is $3,600, how much money does Christine have in her account?

F. $600
G. $720
H. $900

J. $1,200
K. $1,800

53.

In the figure above, points *X* and *Y* are on line segment $\overline{WZ}$.

$$XY = 10 \text{ in.}$$
$$XY : XZ = 1 : 3$$
$$WY : YZ = 5 : 2$$

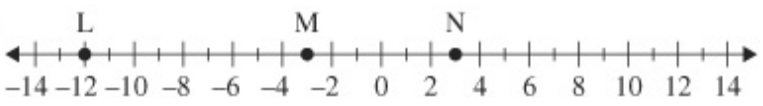
What is the length of $\overline{WX}$?

A. 10 in.
B. 20 in.
C. 30 in.
D. 40 in.
E. 50 in.

54. Express 257.36×10 in scientific notation.

F. 0.25736×10^3
G. 0.25736×10^4
H. 2.5736×10^1
J. 25.736×10^2
K. 2.5736×10^3

55.



On the number line above, M is the midpoint of $\overline{LO}$. Point O is not shown. What is the length of $\overline{NO}$?

A. 3 units
B. 4.5 units
C. 6 units
D. 9 units
E. 10.5 units

56. A hexagon has two sides each of length $3x$ inches. It has three sides each of length $2x$ inches. The sixth side has a length of 15 inches. If the perimeter of the hexagon is 135 inches, what is the value of x?

F. 4
G. 5
H. 10
J. 15
K. 20

57. What is the value of $2|x| - 9|y|$ if $x = -8$ and $y = -3$?

A. –43
B. –11
C. –9
D. 11
E. 43

58. $(\sqrt{81})(\sqrt{25}) =$

F. 5
G. 8
H. 9
J. 40
K. 45

59. It will take Fernando 100 hours to completely thaw a frozen turkey in the refrigerator. If he starts thawing the turkey at 9:00 a.m., at what time will the turkey be completely thawed?

A. 6:00 a.m.
B. 8:00 a.m.
C. 1:00 p.m.
D. 3:00 p.m.
E. 5:00 p.m.

60. *X* is an element of the set {2.8, 3.7, 5.1, 14.0, 20.0} and $\frac{0.3X}{0.25}$ is an integer. What is *X*?

F. 2.8
G. 3.7
H. 5.1
J. 14.0
K. 20.0

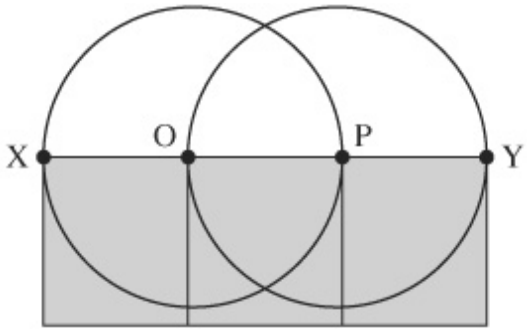
61. How many integers from 1 to 135 have 2 as a factor?

A. 67
B. 68
C. 69
D. 70
E. 71

62. Assume *d* is an odd integer and $1 < d < 11$. What is the mean of all possible values of *d*?

F. 6.0
G. 6.5
H. 12.0
J. 12.5
K. 18.0

63.



The shaded rectangle in the diagram consists of three squares. The rectangle touches the circles at points X and Y. Points O and P are the centers of the two circles. Both of the circles have a diameter of 18 centimeters. What is the area of the rectangle?

A. 27 sq cm
B. 81 sq cm
C. 162 sq cm

D. 243 sq cm
E. 288 sq cm

64. Robyn has 75 feet of copper wire. She needs $\frac{5}{6}$ foot of wire for every electrical circuit she builds. What is the maximum number of electrical circuits Robyn can build?

F. 15
G. 25
H. 90
J. 100
K. 150

65. Ian earns $7.36 per hour. If he worked $2\frac{1}{4}$ hours on Saturday and $2\frac{1}{4}$ hours on Sunday, how much did Ian earn in all?

A. $16.56
B. $18.40
C. $29.44
D. $31.38
E. $33.12

66. Assume that the notation ψ(*o, p, q, r*) means "Multiply *o* and *p*. Subtract *q* from the product, and then raise to the power of *r*." What is the value of ψ(8,2,16,3) + ψ(7,7,39,2)?

F. 18
G. 20

H. 50
J. 100
K. 166

67. In a marathon, 42 out of 60 runners finished the race. What percentage did **not** finish the marathon?

A. 18%
B. 30%
C. 43%
D. 57%
E. 70%

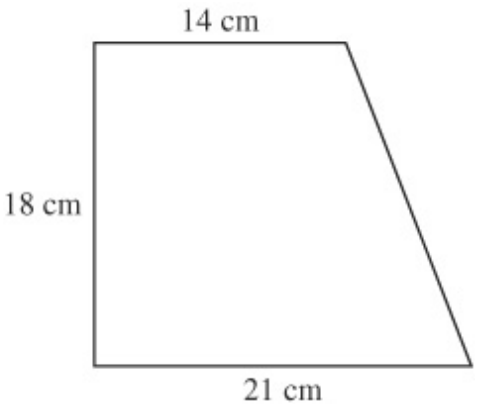
68. The product of 2 consecutive positive integers is 132. What is their sum?

F. 23
G. 28
H. 37
J. 47
K. 66

69. A circle has a circumference of *s* feet and an area of *t* square feet. If $s = 3t$, what is the radius of the circle?

A. $\frac{1}{6}$ ft
B. $\frac{1}{3}$ ft
C. $\frac{2}{3}$ ft
D. 2 ft
E. 6 ft

70.



What is the area of the trapezoid above?

F. 21 sq cm
G. 63 sq cm
H. 126 sq cm
J. 273 sq cm
K. 315 sq cm

71. For what positive value of *x* does $\frac{9}{11} = \frac{x^2}{44}$?

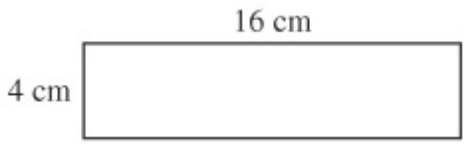
A. 3
B. 6
C. 13
D. 36
E. 53

72. Three consecutive multiples of 3 have a sum of 72. What is the **least** of these numbers?

F. 3

G. 9
H. 21
J. 27
K. 33

73.



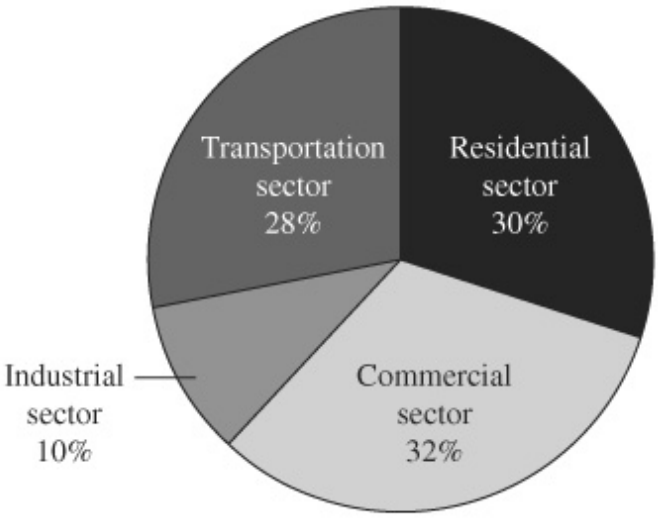
What is the perimeter of a square that has the same area as the rectangle above?

A. 12 cm
B. 20 cm
C. 32 cm
D. 40 cm
E. 64 cm

74. Lilia is 3 inches taller than half of Jeannie's height. If Lilia is 35 inches tall, how tall is Jeannie?

F. 32 inches
G. 64 inches
H. 70 inches
J. 76 inches
K. 105 inches

75. **New York State energy consumption (2008)**



Source: http://www.eia.gov/states/sep_sum/html/pdf/sum_btu_1.pdf

The total energy consumption in New York State in 2008 was about 4,000 trillion btu. About how many trillion btu was consumed by the residential sector?

A. 300
B. 400
C. 1,100
D. 1,200
E. 1,300

76. The following table shows the number of calls received by the customer service department.

Day	1	2	3	4	5	6	7
Number of Calls	119	118	122	120	118	118	121

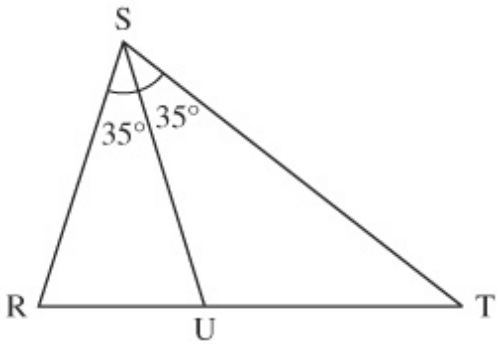
According to the table, what is the median number of calls received by the customer service department in the seven days?

F. 118
G. 119
H. 120
J. 121
K. 122

77. If $x = 9$ and $y = 3$, what is the value of $\frac{12x}{x - y}$?

A. 6
B. 12
C. 18
D. 36
E. 108

78.



Triangles RSU and TSU are similar triangles. What is the measure of ∠SUR?

F. 55°
G. 70°
H. 90°
J. 110°
K. 180°

79. For what value of q is $5(2+q)=3(q+4)$?

A. 1
B. 3
C. 4
D. 11
E. 15

80. Kurt has a jar of marbles. Six of the marbles are blue. The probability of randomly drawing a blue marble is $\frac{1}{5}$. When Kurt adds more blue marbles to the jar, the probability of drawing a blue marble becomes $\frac{1}{4}$. How many blue marbles did Kurt add to the jar?

F. 1
G. 2
H. 3
J. 4
K. 5

81. Julio is driving from New York to Florida. He drives $\frac{1}{5}$ of the total distance on the first day. He drives $\frac{1}{2}$ of the remaining distance

on the second day. What fraction of the total distance does Julio have left to drive?

A. $\frac{2}{5}$

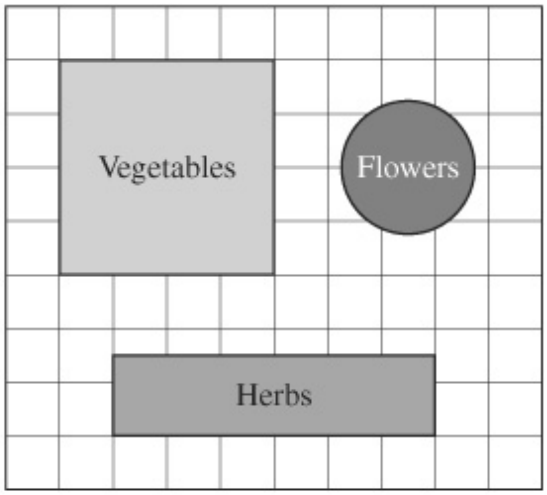
B. $\frac{3}{5}$

C. $\frac{1}{10}$

D. $\frac{3}{10}$

E. $\frac{7}{10}$

82.



Scale: 1 in. = 3 ft

The diagram above shows a plan for a community garden. The plan is drawn on a grid made up of 1-inch squares. About how many square yards will be used to grow herbs?

F. 4.5 sq yd

G. 9.0 sq yd

H. 13.5 sq yd

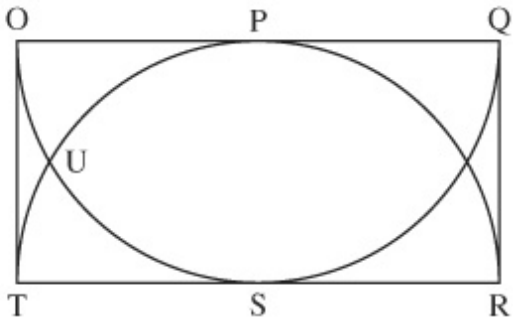
J. 18.0 sq yd

K. 27.0 sq yd

83. Tanisha's website had x visitors on Friday. On Saturday, it had 500 more visitors than it did on Friday. The total number of visitors on Friday and Saturday was twice the number on Sunday. In terms of x, how many visitors did Tanisha's website have on Sunday?

A. $x+250$
B. $2x+250$
C. $2x+1{,}000$
D. $\frac{x+250}{2}$
E. $\frac{x+500}{2}$

84.



In the diagram above, TPR is an arc of a circle centered at S, and OSQ is the arc of a circle centered at P. $\overline{OQ}$ passes through point P, and $\overline{RT}$ passes through point S. The perimeter of triangle PSU (not shown) is 15 centimeters. OQRT is a rectangle, and the length of $\overline{OQ}$ is twice the length of $\overline{OT}$. What is the area of the rectangle?

F. 10 sq cm
G. 20 sq cm
H. 30 sq cm
J. 40 sq cm
K. 50 sq cm

85. Kelly is 3 times the age that Allie was seven years ago. Allie is now 18. How old is Kelly?

A. 11
B. 14
C. 21
D. 25
E. 33

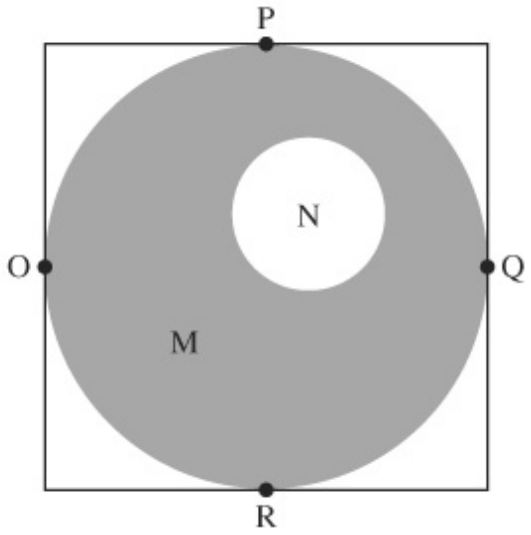
86. In 2000, the population of Utica, NY, was about 60,000. The number of females was 53% of the population. About how many more females than males lived in Utica in 2000?

F. 3,600
G. 4,000
H. 17,000
J. 28,200
K. 31,800

87. Javier poured a total of 128 fluid ounces of lemonade into cups. He poured 11 fluid ounces into all but the last cup. How many fluid ounces did he pour into the last cup?

A. 5
B. 6
C. 7
D. 8
E. 9

88.



Circle M touches the square at points O, P, Q, and R. Circle N lies inside circle M. The area of circle N is 3π sq cm. The area of the shaded portion is 22π sq cm. What is the area of the square?

F. 25 sq cm
G. 50 sq cm
H. 66 sq cm
J. 100 sq cm
K. 121 sq cm

89. There are 25 links in 1 rod and 0.22 yards in 1 link. How many feet are there in 1 rod?

A. 5.5 ft
B. 7.5 ft
C. 16.5 ft
D. 66 ft
E. 75 ft

90. If $x = 5$ and $\frac{4x + 6y}{2y} = 8$, what is the value of *y*?

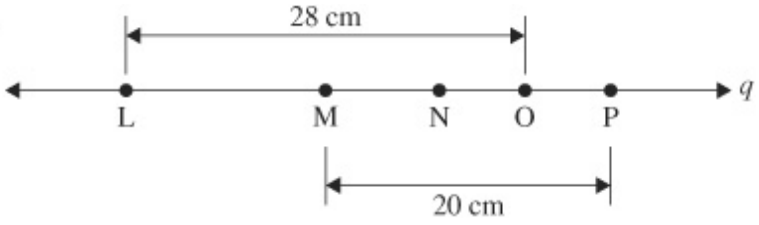
F. 2
G. 4
H. 6
J. 8
K. 10

91. $\frac{r}{30} = \frac{s}{t}$

If $r = 6$ and $s = 9$, what is the value of *t*?

A. 14
B. 20
C. 30
D. 40
E. 45

92.



On line *q*, $\overline{\text{LP}} = 37$ centimeters. If $\text{MN} = 3$ cm, what is the length of $\overline{\text{NO}}$?

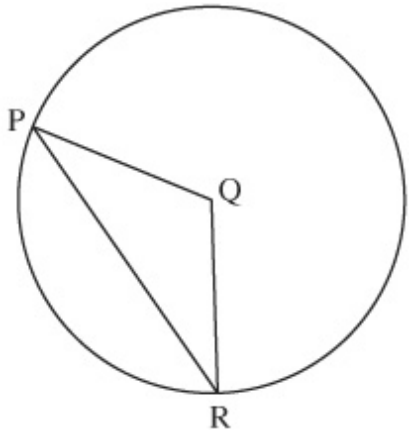
F. 8 cm

G. 9 cm
H. 10 cm
J. 11 cm
K. 12 cm

93. A camera costs x dollars at Store X and y dollars at Store Y. The camera at Store Y costs \$9 less than $\frac{2}{3}$ the cost at Store X. What is the value of y in terms of x?

A. $\frac{2}{3}x + 9$
B. $\frac{2}{3}x - 9$
C. $9 - \frac{2}{3}x$
D. $9x + \frac{2}{3}$
E. $9x - \frac{2}{3}$

94.



Q is the center of the circle. P and R are points on the circle. The perimeter of triangle PQR is 39 centimeters, and PR is 21 centimeters. What is the area of the circle?

F. 18π sq cm

G. 24π sq cm
H. 27π sq cm
J. 81π sq cm
K. 90π sq cm

95. $\left(\frac{3}{4}-\frac{1}{3}\right)\div\frac{5}{8}=$

A. $\frac{2}{3}$
B. $\frac{1}{4}$
C. $\frac{4}{3}$
D. $\frac{4}{15}$
E. $\frac{26}{15}$

96. Each member of the Cobb family packed a suitcase for a trip. Mark's suitcase weighed 51 pounds. Brenda's suitcase weighed 45 pounds. Jonathan's suitcase weighed 31 pounds. How much did Leah's suitcase weigh if the mean weight for the 4 suitcases was 40 pounds?

F. 25 pounds
G. 31 pounds
H. 33 pounds
J. 42 pounds
K. 47 pounds

97. The longest side of a right triangle has endpoints at coordinates (6, 0) and (0,–6). What **could** be the coordinates of the third vertex?

A. (6,6)
B. (–6,–6)
C. (6,–6)
D. (–6,0)
E. (0,6)

98. Nicholas had a jar containing 12 quarters, 3 dimes, 7 nickels, and 14 pennies. He removed 6 quarters, and then randomly drew another coin. What is the probability that the coin was a dime?

F. $\frac{1}{2}$
G. $\frac{1}{4}$
H. $\frac{1}{10}$
J. $\frac{1}{12}$
K. $\frac{1}{15}$

99. Brandy painted $\frac{1}{2}$ of a wall on the first day. On the second day, she painted $\frac{1}{3}$ as much as she did on the first day. What is the ratio of the painted portion of the wall to the unpainted portion?

A. 1:2
B. 1:3
C. 2:1
D. 2:3
E. 3:2

100. If $3x + 1$ is an odd integer, which of the following is also an odd integer?

F. $2x$

G. $5x$

H. $x + 4$

K. $5x + 2$

K. $5x + 2$

THIS IS THE END OF THE TEST. IF TIME REMAINS, YOU MAY CHECK YOUR ANSWERS TO PART 2 AND PART 1. BE SURE THAT THERE ARE NO STRAY MARKS, PARTIALLY FILLED ANSWER CIRCLES, OR INCOMPLETE ERASURES ON YOUR ANSWER SHEET.

Diagnostic Test Answer Key

Paragraph 1

U R Q S T

Paragraph 2

T R Q U S

Paragraph 3

R U S T Q

Paragraph 4

S R Q U T

Paragraph 5

T Q U R S

11. E

12. H

13. B
14. K
15. D
16. G
17. D
18. K
19. A
20. H
21. C
22. K
23. C
24. F
25. C
26. J
27. E
28. J
29. E
30. F
31. D
32. F
33. E
34. G
35. A
36. K
37. E
38. H

39. C
40. G
41. E
42. K
43. D
44. K
45. D
46. H
47. B
48. K
49. A
50. K
51. D
52. F
53. D
54. K
55. A
56. H
57. B
58. K
59. C
60. K
61. A
62. F
63. D
64. H

65. E
66. J
67. B
68. F
69. C
70. K
71. B
72. H
73. C
74. G
75. D
76. G
77. C
78. H
79. A
80. G
81. A
82. G
83. A
84. K
85. E
86. F
87. C
88. J
89. C
90. F

91. E

92. F

93. B

94. J

95. A

96. H

97. C

98. H

99. C

100. J

Explanation of Answers—Verbal Section

Scrambled Paragraphs

Paragraph 1: U R Q S T

Sentence U follows the given sentence because it is an example of the statement about the influence of a region on the materials used to construct a home. R follows next since it is also about "Other settlers in the East." Q follows since it is also about settlers and uses the cue word "Later" at the beginning of the sentence. S explains how the adobe bricks were made, so it comes after Q, and T follows next since it tells what happened after the bricks were dried.

Paragraph 2: T R Q U S

T follows the given sentence that introduces the subject of the *Iliad* because it tells who wrote the epic. R continues to give more information about Homer and the Trojan War and how they were regarded during the nineteenth century. Q is a natural continuation of the subject of how Troy and the Trojan War were doubted and tells what Schliemann did about it. U explains what Schliemann found in the cities that he dug up, so it follows next. S tells how today's

archaeologists view Schliemann's work, so it obviously comes after U.

Paragraph 3: R U S T Q

The given sentence talks about the sensitivity of the eye. R follows the given sentence because it talks about how facial features protect the eye. U follows next since it explains which facial features are protective. The beginning of S, "There are other aspects . . . ," helps to point out other ways that the design is helpful to the eye. T explains how tear ducts, the first item mentioned in S, help the eye, so it comes next. It is followed by Q, which is about the eyelid, which is mentioned second in sentence S. Q also starts with the cue "The eyelid is equally . . . ."

Paragraph 4: S R Q U T

Sentence S come after the given sentence since it explains what the world map looked like to Wegener. R tells what Wegner believed had happened in the past to the continents, so it comes next. Q talks about the reaction of the experts to his theory and would naturally follow R. U refers to what the experts ("scientists") now think is the reason the continents broke apart, and therefore it could not come before Q. T explains what happens when a tectonic plate shifts and acts as a conclusion to the paragraph, so it comes last.

Paragraph 5: T Q U R S

The given sentence tells about Marion Anderson's talent and struggles. The paragraph describes her life, so T is the logical next sentence because it talks about her as a child and how she began to sing. Sequentially Q would follow because it continues to talk about the development of her career. U follows Q because it is an example of the "racial prejudice" that she encountered in her career. R must come next: the word "instead" clues us in that what happened was a result of the information in U. The last sentence, S, talks about the end of Marion Anderson's career and comes last.

Logical Reasoning

11. (E) It can be very helpful to draw a diagram for this kind of problem. Let an initial stand for each name. Write “oldest” on the left-hand side of your scrap paper. Then diagram the first statement.

Oldest: R D M

After reading the second statement, you can put Jeannie into the diagram, but not Bryan. All we know about Bryan is that he goes somewhere to the right of Rico; we don’t know how his age compares with that of Daniela.

Oldest: R D M J

After reading the third statement, we are still not sure about Bryan’s position. We know he is younger than Rico, but he could fit on either side of Daniela. Therefore, Bryan is either second or third, but his exact position “cannot be determined from the information given.”

12. (H) This question contains two conditional statements. It is important that you read each statement carefully. The first sentence may be restated as, “If Anita plays soccer, Crystal will not play soccer.” This is simply a restatement of the two clauses in the first condition. Make sure to observe that the sentence only tells us what will happen *if* Anita plays soccer. *It says nothing about what will happen if Anita does* **not** *play soccer.* Therefore, it is incorrect to conclude that Crystal will play soccer if Anita does not, and Option G can be ruled out. We simply do not know what will happen. Likewise, the second sentence can be rearranged as, “If Gerry is **not** late to school, Anita will play soccer.” But, the sentence does not tell us what will happen if Gerry *is* late for school, so Option J can be ruled out. Option H is correct: if Gerry is on time for school, Anita will play soccer. Option F is incorrect because the two conditions do not address what Crystal will do if Gerry is late to school. Option K can be ruled out because no information is given about Gerry playing soccer.

13. (B) Use a diagram to help solve this question. Write “left” and “right” on your scrap paper and the numbers 1 through 5,

beginning on the left with 1. Then look for specific locations of each letter. Statement two says that the C is in the second position; write “C” above 2. Statement four tells you that there is exactly one letter to the right of the M, so it must be in the fourth position. Statement three says that there are exactly three letters between the H and the L, so the H and the L must be in the first and last positions, even though you cannot tell which one goes in which position. The question asks you in what position the P is. Since you know that positions 1 and 5 are taken and that the C is in the second position and the M in the fourth position, you are left with the third position.

14. (K) Option F is not a valid conclusion because there might be some people who do not speak Italian at all going on the trip. Option G appeals, but there is nothing in the statement about how well some people speak Italian. This could be true, but we cannot assume it is. Option H is a value judgment that is not supported by the information given. While Option J might be true, there is no basis for concluding that “most” people going on the trip are not very fluent in Italian. Option K is a valid conclusion. People who do not speak Italian well can go on a trip to Italy.

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15. and 16. Read the directions carefully. The letters in a sentence may or may not appear in the same order as the words in that sentence. For example, in the first sentence the first letter (M) may or may not represent the first word (Marie). Remember that you don’t need to find out what every letter represents in the code.

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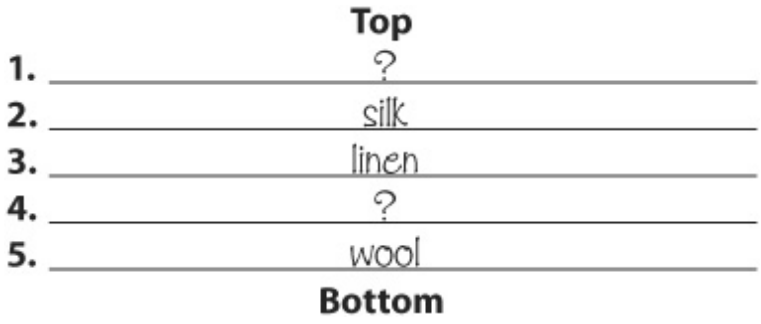
15. (D) “Dante” appears only once in the code, in the fourth sentence. The second and fourth sentences are identical except for the names Laticia and Dante. Thus the letter that is unique in each sentence must represent the name. In sentence 4, the only letter that differs from the letters in sentence 2 is Z. So Z must represent Dante.

16. (G) The letter N appears in the second and fourth sentences, but not in the first or third sentence. The only word that appears in both the second and fourth sentences and not the others is "friends," so Option G is correct.

17. (D) Use a diagram to help answer this question. Write "top" and "bottom" on your scrap paper. Then record what the question says about each fabric. The question says that the bottom fabric is wool, and statement three says that the linen fabric is third from the top. Statement two tells you that exactly two fabrics are between the wool and the silk fabrics, which means the silk fabric is in position 2.



That leaves the corduroy and cotton fabrics for the remaining open positions, which are positions 1 and 4. According to statement one, the cotton fabric is higher than the wool. This condition will be fulfilled whether the cotton fabric is in position 1 or 4, so we cannot determine which position it is in. There is no information at all that can be used to determine the location of the corduroy fabric. Thus, the correct answer is Option D: "Either corduroy or cotton, but it is not possible to determine which one."

18. (K) We need to check each option against the facts to figure out which one is valid. We know nothing about Terri except how many hours she volunteers for Sister Cities. So Option F, although it **might** be true, cannot be verified. Options G and H also **might** be true, but we don't know that from the information that is given. The same is true for Option J. Option K is a valid conclusion, since we are told that members are expected to volunteer more hours than Terri does.

19. (A) This question asks you to match each rider with his or her bike and place the bikes in the order of arriving at the lake. The first thing to do is to look for statements that provide information about the person on each bike. Statement two says that Paula rode the red bike, statement five says that Sally rode the yellow bike, and statement three says that Chet was not riding the blue bike. Therefore the person riding the blue bike must be Josh, which means that Chet was on the green bike. You can put the four bikes and the people riding them in order. Based on statement one, draw a diagram showing what we know of the order in which the bikes arrived at the lake.

First

Paula (red bike)

Josh (blue bike)

Last

Based on statements four and five, Charles (green bike) and Sally (yellow bike) finished behind Josh. Either of the following orders could be correct, based on the information provided.

First	**First**
Paula	Paula
Josh	Josh
Chet	Sally
Sally	Chet
Last	**Last**

In either order, the blue bike, which Josh rode, arrived before the green bike, ridden by Chet. So the correct answer is A.

20. (H) The problem says that the school bus could be late if it is raining, but that it will be on time if it is sunny. We **do not** know that these are the only two circumstances that influence the bus being late or on time. Therefore, you can rule out Option F. Option G is wrong because there is nothing in the statements about what happens when it is cloudy. Option J is wrong because there is no reason to believe that rain is the only reason

the bus might be late. Option K is a false conclusion because there is no way to tell whether it is sunny more times than it is rainy. Option H is correct—rain may cause the bus to be late. It may not, but it could, according to the first statement.

Reading

21. (C) This question asks you to determine the general theme of the passage. Option A is not discussed in the passage. Option B, though both people are mentioned, is not the theme of the passage. Options D and E are both details that are mentioned, but not the focus of the entire passage. The passage describes the discovery that humans came to the Americas earlier than previously thought. Thus, the best answer is Option C.

22. (K) This question asks you to make an inference based on the information in the passage. Option F may be true, but there is not enough information in the passage to draw this conclusion. Option G does not agree with the information in the passage that says the Buttermilk People hunted mastodons and seals. Option H might appeal, but the passage only says that the Buttermilk People used hematite to decorate their clothes, spears, and bodies. It does not mention them using it to tell a story through pictures. Option J could also appeal, since it seems as though the Buttermilk People were good hunters. But this does not mean they were better hunters than the Clovis People. The Buttermilk People seemed to make broad use of the mastodon, using it for food as well as clothing and shelter. Option K is the best answer.

23. (C) The question asks specifically about the Buttermilk People's nature, which is not directly discussed in the passage. In order to answer the question, you must look at the information in the passage and use it to infer the answer. Option A may be true, but there is nothing to support this idea in the passage. Option B could be true or false, but the passage does not discuss this possibility. Option C is supported by the passage, which states, "There is evidence that over centuries they explored all the way to the tip of South America" (lines 26–28). There is no evidence

that Option D is true. Option E seems to be the opposite of what is supported in the passage, which states that the Buttermilk People decorated themselves and their tools. This suggests a good amount of imagination.

24. (F) The phrase “even brought pet dogs” implies a people who cared about animals and, in particular, their own animals. The fact that these early people would even have pets suggests that they were affectionate (Option F). There is no evidence for Option G. The concept of being well educated probably did not exist at that time. There is no support for either Option H or J. The Buttermilk People may have used animals for protection (Option K), but this is not supported by the passage.

25. (C) This is an inference question. It is necessary to look for information that supports one of the options. Neither Option A nor B is supported by the passage. There is no evidence of the Buttermilk People being fearful of enemies or being friendly to others. Option C is supported by the passage. The fact that the Buttermilk People liked to decorate their spears and themselves does suggest that they liked colorful objects. Options D and E are not supported. There is nothing in the passage about these people playing games or wanting to look frightening.

26. (J) To answer this question you have to go through the passage and try to find the one topic that is not discussed. Option F and G are discussed in the paragraph. So is Option H. The topic of where the Buttermilk people settled would suggest that they were concerned with creating places to live. That leaves Option J. There is no mention of them creating a network of trails.

27. (E) This question asks you to determine the general theme of the passage. Option A is not discussed in the passage. Options B and C are mentioned, but they are details, not the theme of the passage. Option D is not discussed in the passage. The passage describes how giant tube worms survive. Thus, the correct answer is Option E.

28. (J) This is an inference question. You need to make an inference based on the information in the passage. Option F is not supported by the passage. While the giant tube worm has a symbiotic relationship with the bacteria inside it, this is not necessarily true of "most life." Option G is not suggested by the passage. This is not a topic of discussion. Option H seems to be refuted by the passage since it seems that giant tube worms are unusual because they can stand vast changes in temperature. Option J is supported by the passage, which states, "Surprisingly, it does not need the energy of the sun," suggesting that most life does. Option K is not supported by the passage either. The giant tube worms do live a very long time, but there is nothing to indicate that is the rule for most life.

29. (E) To answer this question, you need to read the passage carefully to see which option is correct. Option A is a misstatement, since the passage tells the reader that giant tube worms "are directly related" to another form of worm. Option B is not directly refuted in the passage, but the passage suggests that giant tube worms are possibly only found in the Pacific Ocean. Since giants tube worms live until they are 250 years old, they are hardly "short-lived." Option D is contrary to the information in the passage that says that it merely pulls in its plume if it is threatened, hardly an aggressive act. Option E is the correct answer. This can be inferred from the passage because giant tube worms do exist without access to light.

30. (F) To answer this question, you need to reread the section of the passage that talks about lobsters. If you do this, you will find that only Option F is an accurate description of the relationship between giant tube worms and lobsters. Their exteriors are made of the same hard material. Option G might appeal, but there is nothing in the passage to suggest that lobsters live off the hot air vents. There is no mention either of Options H or J. The giant tube worms have a relationship with bacteria that benefits them both, not with lobsters.

31. (D) To answer this question you need to review the passage and find what is true of the giant tube worms. You will also need to use the information to infer the correct answer. Option A does not seem likely because there seems to be a great deal of data on giant tube worms in the passage. Option B is something that is directly stated in the passage, so it can be eliminated. Options C and E are also correct according to the passage. However, Option D does not seem to be correct. There are both very hot and very cold temperatures in the giant tube worms' environment. So Option D is the correct answer.

32. (F) To make an inference, you need to weigh all the information in a passage and come to the most logical conclusion. Options G and H don't seem to be likely. While Option J may appeal, the passage does say the giant tube worm gets nourishment from the bacteria inside it. There is no evidence that Option K is correct. Option F seems to be the only logical reason. Therefore, it is the correct answer.

33. (E) This question asks you to determine what the main idea or general theme of the passage is. Option A is mentioned in the passage, but it is only a detail, not a general theme. Options B and C are also details in the passage, not the main focus of it. Option D is not discussed at all in the passage. The passage describes why miniature horses make good guide animals. Therefore, the correct answer is Option E.

34. (G) Check back through the passage to answer this question. Review the traits that are mentioned. Although the word *stamina* is not used, the passage says that these horses can work long hours, so Option F can be eliminated. Options H, J, and K can also be eliminated because they are mentioned in the passage. So that leaves Option G. There is no mention of the horses' memories.

35. (A) Option A seems to be a logical conclusion from the information in the passage. There is no suggestion that Option B or C is correct. The subject of how much it costs to feed a

miniature horse is not discussed, nor is Option E. Thus Option A is the correct answer.

36. (K) You have to figure out why the author used this information in the passage. Option F might be a possibility, although that particular theme is not addressed in the passage. Option G is off the mark. The horses were obedient, and that is one reason they were used. There is no suggestion that Option H is correct, nor is there any discussion of why they are no longer used in mining, so Option J can be eliminated. Option K is the best answer.

37. (E) You need to use your best judgment to answer this question. Option A does not seem to be a candidate for either a dog or a horse guide. Option B may or may not be a factor in choosing a horse guide, but it is difficult to judge. The same is true of Option C. While Cuddles went to New York City, that does not make horses a preferred guide animal in urban areas. Option D might be likely to choose either a dog or a horse guide. Option E is the most likely answer.

38. (H) To answer this question you have to examine each option carefully. Option F is appealing, but is that really the reason that the author changed the saying? Option G might be possible, but the passage does not seem to be about political correctness. Option H seems to be correct. This seems to be a good reason. Option J and Option K do not seem likely. Option H is the best answer.

39. (C) This question asks you to determine what the main idea or general theme of the passage is. Option A is mentioned in the passage, but it is only a detail, not a general theme. Option B is not really discussed in the passage. Options D and E are only discussed in passing. The passage gives the reasons why people think that space exploration is important. Therefore, the correct answer is Option C.

40. (G) After describing the landing, the passage continues, “It was an event that could not be forgotten” (lines 6–7). Option G best

explains what the author means: the event was dramatic. None of the other options explain why the author used this description.

41. (E) You will need to review the passage to answer this question. Scan through the passage to find what reasons are mentioned for continuing space exploration. Options A, B, and C are definitely mentioned. Option D could be inferred about the research that is being done by the ISS. There is no mention of finding a better food source; hence Option E is the correct answer.

42. (K) A clue to the answer is stated in line 34: "*brave* people" have searched to find new lands. Option K restates this definition. The others do not mean what intrepid means.

43. (D) You will need to analyze the author's intent from what the passage says. It talks about how mankind has always wanted to know about the world. This is the reason for including information about explorers. Option A is not the reason, nor is Option B. This information has no bearing on the importance of space exploration, so Option C is not correct. Option E is only vaguely mentioned. Option D is the answer.

44. (K) While some of the options may occur should space exploration cease, such as Option G and Option J, the most important impact must be Option K. Neither Option F nor Option H would probably be an impact of the loss of the program.

45. (D) This question asks you to determine what the main idea or general theme of the passage is. Option A is mentioned in the passage, but it is only a detail, not a general theme. Options B and C are details in the passage, not the main focus of it. Option E is discussed, but is not the main theme. The passage describes how Mary Cassatt happened to become a unique artist. Thus, the correct answer is Option D.

46. (H) Use the information in the passage to find the answer to this question. While the passage says that Mary took her first art

class in Europe, it does not say she showed promise, so Option F can be eliminated. There is no reason to believe she became more rebellious, although it appears she always had a mind of her own, so Option G is not correct. Option H is something that the passage supports. Therefore it is the correct answer. Options J and K may or may not be true, but not according to the passage.

47. (B) A careful review of the passage will help answer this question. Option A is something that Mary wanted to do, as are Options C and E. Option D probably happened because she was in Paris. That leaves Option B. Mary did not seem to want to settle down. She wanted to become an artist.

48. (K) The passage talks about how the Impressionists used bright colors and did not paint things exactly the way they looked. The option that fits best with this information is Option K. Artists were freer to do what they wanted. There is no evidence for Option F or H. Again, the Impressionists were more about breaking rules than setting standards, so Option G can be eliminated. Perhaps Option J is true, but that is not discussed in the passage.

49. (A) You have to use the information and your own prior knowledge to answer this question. Option A is appealing. Mary did not seem to want to be part of any organization. There is no evidence that Option B is correct. While she did return to America, she did not necessarily have to leave the Impressionist Movement to do so. There is no suggestion that Mary was asked by her family to leave the movement, so Option D can be eliminated. There is nothing in the passage about her wanting to join another group, so Option E cannot be correct. The best answer is Option A.

50. (K) You need to use the information in the passage to make an inference about Mary Cassatt. Options F and G do not seem to fit with her personality. Mary did not seem to be worried about her image, nor did she seem not to trust people. While Mary wanted to be a famous artist, it doesn't seem as though she

would do anything to achieve this goal. Option J doesn't seem to fit because Mary did unselfish things in her life like coming back to America to be with her family. Option K seems to fit her personality best. She was a strong woman who knew what she wanted.

Explanation of Answers—Mathematics Section

51. (D) Factor each number into its prime factors:

$$154 = 2 \cdot 7 \cdot 11$$

$$196 = 2 \cdot 2 \cdot 7 \cdot 7$$

The common prime factors are 2 and 7:

$$154 = \boxed{2} \cdot \boxed{7} \cdot 11$$

$$196 = \boxed{2} \cdot 2 \cdot \boxed{7} \cdot 7$$

The greatest common factor (GCF) is the product of the common prime factors:

$$\text{GCF} = 2 \cdot 7 = 14$$

52. (F) Let x represent the amount of money that Christine has in her account. Marylou has twice as much as Christine, so Marylou has $2x$ in her account. Katy has three times as much as Christine, so Katy has $3x$ in her account. Together, they have $x + 2x + 3x = \$3{,}600$. Solve for x:

$$x + 2x + 3x = \$3{,}600$$

$$6x = \$3{,}600$$

$$x = \$600$$

$\$3{,}600$, so

53. (D) The ratio of XY: XZ is 1:3 and $\text{XY} = 10$ in. Set up a proportion to find XZ:

$$\frac{XY}{XZ}=\frac{1}{3}$$
$$\frac{10}{XZ}=\frac{1}{3}$$
$$10\cdot 3=XZ$$
$$30=XZ$$

Solve for YZ:

$$YZ=XZ-XY$$
$$=30-10$$
$$=20$$

Set up another proportion to find WY:

$$\frac{WY}{YZ}=\frac{5}{2}$$
$$\frac{WY}{20}=\frac{5}{2}$$
$$WY\cdot 2=5\cdot 20$$
$$WY=50$$

Solve for WX:

$$WX=WY-XY$$
$$=50-10$$
$$=40$$

54. (K) A number in scientific notation is written as $a\times 10^{b}$, where $1\le a<10$ and *b* is a whole number. Options H and K are the only two options written in scientific notation:

Option H:

$2.5736\times10^{1}\rightarrow1\le2.5736<10$ and $b=1$

Option K:

$2.5736\times10^{3}\rightarrow1\le2.5736<10$ and $b=3$

However, only option K is numerically equivalent to 257.36×10:

$257.36\times10=2573.6$

$2.5736\times10^{3}=2573.6$

55. (A) The formula for the midpoint between two numbers *a* and *b* on a number line is $\frac{a+b}{2}$. Use the formula to find the location of point O:

$$\text{midpoint}=\frac{a+b}{2}$$

$$\text{M}=\frac{\text{L}+\text{O}}{2}$$

$$-3=\frac{-12+\text{O}}{2}$$

$$-6=-12+\text{O}$$

$$6=\text{O}$$

The formula for the distance between two points is $b-a$. Use the formula to find the length of $\overline{\text{NO}}$:

$$\text{distance}=b-a$$

$$=\text{O}-\text{N}$$

$$=6-3$$

$$=3$$

56. (H) The sum of all six sides of the hexagon is 135, so $2(3x)+3(2x)+15=135$. Solve for *x*:

$$\begin{aligned} 2(3x)+3(2x)+15&=135\\ 6x+6x+15&=135\\ 12x&=120\\ x&=10 \end{aligned}$$

57. (B) Substitute the values of *x* and *y* into the expression:

$$\begin{aligned} 2|x|-9|y| &= 2|-8|-9|-3|\\ &= 2(8)-9(3)\\ &= 16-27\\ &= -11 \end{aligned}$$

58. (K) $(\sqrt{81})(\sqrt{25})=(9)(5)=45$

59. (C) Since there are 24 hours in a day, it will be 9:00 a.m. after 24 hours, 48 hours, and 96 hours. Therefore, 100 hours after Fernando starts thawing the turkey will be 9:00 a.m. plus 4 more hours, or 1:00 p.m.

60. (K) First, simplify the fraction:

$$\frac{0.3X}{0.25}=\frac{30X}{25}=\frac{6X}{5}$$

For $\frac{6X}{5}$ to be an integer, the product of 6 and *X* must end in a 0 or 5. For example:

if $6X=30$, then $\frac{6X}{5}=6$

if $6X = 32$, then $\frac{6X}{5} = 6.4$

if $6X = 35$, then $\frac{6X}{5} = 7$

The only number in the set for which 6*X* will end in a 0 or 5 is 20.0. Therefore $X = 20.0$.

61. (A) All even integers have 2 as a factor. There are five even integers from 1 to 10, five even integers from 11 to 20, and so on:

1–10	2, 4, 6, 8, 10
11–20	12, 14, 16, 18, 20
. . .	. . .
121–130	122, 124, 126, 128, 130
131–135	132, 134

There are 13 consecutive groups of 10 integers. Each of these groups has five even integers, so the number of even integers from 1 to 130 is $13 \times 5 = 65$. There are two more even integers from 131 to 135. Therefore, the total number of even integers is $65 + 2 = 67$.

62. (F) The odd integers between 1 and 11 are 3, 5, 7, and 9. The mean of these integers is

$$\frac{3+5+7+9}{4} = 6.$$

63. (D) Each circle touches the center of the other circle, so the length of the rectangle is equal to the length of the diameter of

one circle plus the length of the radius of the other circle: $18+9=27$ cm.

The rectangle is made up of three squares, so the side length of each square is $27\div 3=9$ cm.

The width of the rectangle is equal to the side length of a square, so the area of the rectangle is $27\times 9=243$ sq cm.

64. (H) Divide to find the number of $\frac{5}{6}$ foot pieces of wire Robyn can cut from 75 feet:

$$75\div\frac{5}{6}=75\times\frac{6}{5}=90.$$

65. (E) The total number of hours Ian worked is $2\frac{1}{4}\times 2=4.5$. Therefore, the total amount that he earned is $\$7.36\times 4.5=\$33.12$.

66. (J) Translate $\Psi(o, p, q, r,)$ into an algebraic expression:

$$\Psi(o,p,q,r)=[(o\times p)-q]^r$$

Substitute the values of *o*, *p*, *q*, and *r* into the expression and solve:

$$\Psi(8,2,16,3)=[(8\times 2)-16)]^3=(16-16)^3=0^3=0$$

$$\Psi(7,7,39,2)=[(7\times 7)-39]^2=(49-39)^2=10^2=100$$

$$\Psi(8,2,16,3)+\Psi(7,7,39,2)=0+100=100$$

67. (B) The number of runners who did not finish the marathon is $60-42=18$.

The fraction of runners who did not finish the marathon is $\frac{18}{60}$.

Reduce the fraction to lowest terms and then multiply by 100 to find the percentage who did not finish the marathon:

$$\frac{18}{60}=\frac{3}{10}\times 100=30\%$$

68. (F) Through quick estimation, we can conclude that the two consecutive positive integers are 11 and 12: $11\times 12=132$. The sum of 11 and 12 is $11+12=23$.

69. (C) The formula for the circumference of a circle is $C=2\pi r$. The formula for the area of a circle is $A=2\pi r$. Substitute the formulas into the equation $s=3t$ to solve for the radius of the circle:

$$s=3t$$
$$2\pi r=3(\pi r^2)$$
$$\frac{2}{3}\pi r=\pi r^2$$
$$\frac{2}{3}r=r^2$$
$$\frac{2}{3}=r$$

70. (K) The formula for the area of a trapezoid is $A=\frac{1}{2}\times \text{ height } (\text{base}_1+\text{ base}_2)$, where base_1 and base_2 are the lengths of the parallel sides of the trapezoid. Substitute the given values into the equation and solve:

$$A = \frac{1}{2} \times \text{ height } (\text{base}_1 + \text{ base}_2)$$
$$= \frac{1}{2} \times 18(14 + 21)$$
$$= 9(35)$$
$$= 315$$

71. (B) Cross multiply and then solve for *x*:

$$\frac{9}{11} = \frac{x^2}{44}$$
$$11x^2 = 9(44)$$
$$x^2 = \frac{9(44)}{11}$$
$$= 9(4)$$
$$= 36$$
$$x = 6 \text{ or } -6$$

72. (H) Let *x* represent the smallest of the three numbers. In terms of *x*, the next two consecutive multiples of 3 are $x + 3$ and $x + 6$. The sum of the three numbers is 72, so

$x + (x + 3) + (x + 6) = 72$. Solve for x:

$$x + (x + 3) + (x + 6) = 72$$
$$3x + 9 = 72$$
$$3x = 63$$
$$x = 21$$

73. (C) The area of the rectangle is $16 \times 4 = 64$ sq cm. Therefore, the area of the square is 64 sq cm, and the side length of the square is $\sqrt{64} = 8$ cm. The perimeter of the square is $8 \times 4 = 32$ cm.

74. (G) Let *j* represent Jeannie's height in inches:

$$\frac{1}{2}j+3=35$$

$$\frac{1}{2}j=32$$

$$j=64$$

75. (D) The residential sector consumed 30% of the total energy:

$$4{,}000\times 30\% = 4{,}000\times 0.30 = 1{,}200$$

76. (G) The median is the middle value in a list of numbers. To find the median, list all the numbers in order from least to greatest. The number in the middle is 119.

$$118,\ 118,\ 118,\ \boxed{119},\ 120,\ 121,\ 122$$

77. (C) Substitute the values of *x* and *y* into the expression:

$$\frac{12x}{x-y}=\frac{12(9)}{9-3}=\frac{108}{6}=18$$

78. (H) Corresponding angles of similar triangles have the same measure. Since triangles RSU and TSU are similar, angles R and T have the same measure.

Let *x* represent the measure of angle R and angle T. The sum of the angles of triangle RST is 180°, so $35+35+x+x=180$. Solve for *x*:

$$35+35+x+x=180$$
$$70+2x=180$$
$$2x=110$$
$$x=55$$

Let *y* represent the measure of ∠SUR. The sum of the angles of triangle RSU is 180°, so $55+35+y=180$. Solve for *y*:

$$55+35+y=180$$
$$90+y=180$$
$$y=90$$

79. (A)

$$5(2+q)=3(q+4)$$
$$10+5q=3q+12$$
$$2q=2$$
$$q=1$$

80. (G) Let *x* represent the original number of marbles in the jar. There are 6 blue marbles, and the probability of randomly drawing one of them is $\frac{1}{5}$. Set up a proportion to solve for *x*:

$$\frac{6}{x}=\frac{1}{5}$$
$$x=30$$

Let *y* represent the number of blue marbles Kurt added to the jar. When Kurt adds the blue marbles, the probability becomes $\frac{1}{4}$. Set up another proportion to solve for *y*:

$$\frac{6+y}{30+y}=\frac{1}{4}$$
$$4(6+y)=30+y$$
$$24+4y=30+y$$
$$3y=6$$
$$y=2$$

81. (A) Julio drives $\frac{1}{5}$ of the distance on the first day, so the fraction of the total distance that remains after the first day is $1-\frac{1}{5}=\frac{4}{5}$.

The fraction of the total distance that Julio drives on the second day is $\frac{1}{2}\times\frac{4}{5}=\frac{2}{5}$.

After two days, the fraction of the total distance that Julio has driven is $\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$.

Therefore, the fraction of the total distance that Julio has left to drive is $1-\frac{3}{5}=\frac{2}{5}$.

82. (G) The drawing of the herb garden has a width of about 1.5 inches and a length of about 6 inches. The scale is 1 in. = 3 feet, so the width of the actual herb garden is $1.5\times 3=4.5$ feet and the length is $6\times 3=18$ feet.

One yard is equal to 3 feet, so the width is $4.5\div 3=1.5$ yd and the length is $18\div 3=6$ yd. Therefore, the area of the herb garden is $1.5\times 6=9$ sq yd.

83. (A) If *x* represents the number of visitors on Friday, then the number of visitors on Saturday is expressed as $x + 500$. The total number of visitors on Friday and Saturday is $x + x + 500 = 2x + 500$.

The total number of visitors on Friday and Saturday is twice the number on Sunday, so the number of visitors on Sunday is expressed as $\frac{2x + 500}{2} = x + 250$.

84. (K) The sides of triangle PSU are the radii of circle P and circle S: $\overline{SU}$ is a radius of circle S, $\overline{PU}$ is a radius of circle P, and $\overline{PS}$ is a radius of both circles. Since both circles share $\overline{PS}$, their radii are all the same length. This means that triangle PSU is an equilateral triangle, and the length of the radii of both circles is $15 \div 3 = 5$ cm.

$\overline{OQ}$ is the diameter of the circle P and a side of rectangle OQRT. Since the radius is 5 cm, OQ is $5 \times 2 = 10$ cm. The length of OQ is twice the length of $\overline{OT}$, so OT is $10 \div 2 = 5$ cm. Therefore, the area of the rectangle is $10 \times 5 = 50$ sq cm.

85. (E) Allie is now 18 years old. Her age seven years ago was $18 - 7 = 11$. Therefore, Kelly's age is $3 \times 11 = 33$.

86. (F) The number of females was $60{,}000 \times 0.53 = 31{,}800$. The percentage of males in the population was $100\% - 53\% = 47\%$, so the number of males was $60{,}000 \times 0.47 = 28{,}200$.

Subtract to find how many more females than males lived in Utica in 2000:

$31{,}800 - 28{,}200 = 3{,}600$

87. (C) Divide 128 by 11 to find the number of cups Javier filled with 11 fluid ounces of lemonade. The remainder represents the amount he poured into the last cup:

$$128 \div 11 = 11\text{R}7$$

88. (J) The area of circle M is the area of the shaded portion plus the area of circle N: $22\pi + 3\pi = 25\pi$. The formula for the area of a circle is $A = \pi r^2$. Solve for the radius:

$$\pi r^2 = 25\pi$$
$$r^2 = 25$$
$$r = 5$$

The radius of circle M is 5 cm, so its diameter is 10 cm. The diameter of circle M is equal to the side length of the square, so the area of the square is $10 \times 10 = 100 \text{ sq cm}$.

89. (C) There are 3 feet in one yard and 0.22 yard in 1 link, so the number of feet in one link is $0.22 \text{ yd} \times 3 = 0.66$.

Since $1 \text{ rod} = 25$ links and $1 \text{ link} = 0.66$ ft, the number of feet in 1 rod is $25 \times 0.66 = 16.5$.

90. (F) Substitute the value of *x* into the expression and solve for *y*:

$$\frac{4(5)+6y}{2y}=8$$
$$\frac{20+6y}{2y}=8$$
$$20+6y=16y$$
$$20=10y$$
$$2=y$$

91. (E) Substitute the values of *r* and *s* into the equation and solve for *t*:

$$\frac{r}{30}=\frac{s}{t}$$
$$\frac{6}{30}=\frac{9}{t}$$
$$6t=270$$
$$t=45$$

92. (F) Subtract to find the missing lengths:

$$\begin{aligned}\text{OP}&=\text{LP}-\text{LO}\\&=37-28\\&=9\text{ cm}\\\text{MO}&=\text{MP}-\text{OP}\\&=20-9\\&=11\text{ cm}\\\text{NO}&=\text{MO}-\text{MN}\\&=11-3\\&=8\text{ cm}\end{aligned}$$

93. (B) Translate the verbal description into an algebraic equation:

If the cost of the camera at Store Y is \$9 less than $\frac{2}{3}$ the cost at Store X, then $y = \frac{2}{3}x - 9$.

94. (J) $\overline{PQ}$ and $\overline{QR}$ are two sides of triangle PQR. They are also the radii of the circle, so they have the same length. The perimeter of the triangle is 39 cm and PR is 21 cm, so because the sum of the three sides of the triangle equals its perimeter, you can solve for the radius, which is equal to PQ and QR:

$$\begin{aligned} 21 + r + r &= 39 \\ 2r &= 39 - 21 \\ 2r &= 18 \\ r &= 9 \end{aligned}$$

The radii PQ and QR are each 9 cm. Therefore, the circumference of the circle is $C = 9^2\pi = 81\pi \text{ sq cm}$.

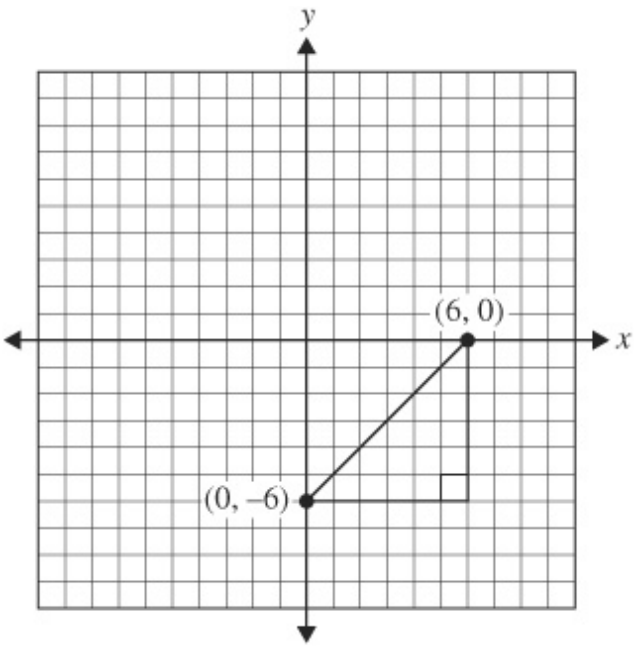
95. (A)

$$\begin{aligned} \left(\frac{3}{4} - \frac{1}{3}\right) \div \frac{5}{8} &= \left(\frac{9}{12} - \frac{4}{12}\right) \div \frac{5}{8} \\ &= \frac{5}{12} \div \frac{5}{8} \\ &= \frac{5}{12} \times \frac{8}{5} \\ &= \frac{40}{60} \\ &= \frac{2}{3} \end{aligned}$$

96. (H) Let *x* represent the weight of Leah's suitcase, and solve for *x* using the formula for the mean.

$$\frac{51+45+31+x}{4}=40$$
$$\frac{127+x}{4}=40$$
$$127+x=160$$
$$x=33$$

97. (C) Look at the coordinate plane below. There are two possible coordinates for the third vertex, (0, 0) and (6, -6). Of the two, only (6, -6) is offered as an answer choice.



98. (H) There were 36 coins in the jar, and then Nicholas removed 6 quarters. This leaves 30 coins in the jar. Three of those coins are dimes, so the probability of randomly drawing a dime is $\frac{3}{30}=\frac{1}{10}$.

99. (C) Brandy painted $\frac{1}{2}$ of the wall on the first day. On the second day, she painted $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$ of the wall. The fraction of the wall that

is painted is $\frac{1}{2}+\frac{1}{6}=\frac{2}{3}$. The fraction of the wall that is unpainted is $1-\frac{2}{3}=\frac{1}{3}$. The ratio of painted to unpainted is $\frac{\frac{2}{3}}{\frac{1}{3}}$ or 2:1.

100. (J) If $3x+1$ is an odd integer, then 3*x* must be an even integer (because one less than an odd integer must be even), and *x* must also be an even integer (or 3*x* could not be even). For example:

$3(2)+1=7$

$3(3)+1=10$

$3(4)+1=13$

$3(5)+1=16$

You can see that $3x+1$ is only odd when an even integer is substituted for *x*. Substitute an even integer into each of the answer options. Only $2x+7$ produces an odd integer.

PART 3
SHSAT Topic Reviews

CHAPTER 4
Scrambled Paragraphs

As you know, some of the questions on the Verbal section of the SHSAT involve scrambled paragraphs. The purpose of these questions is to find out how well you are able to organize information. These paragraphs are often thought to be the most difficult part of the test, but don't let that make you nervous. Once you understand exactly what the questions require and learn a few key things to look for, you will be better prepared to tackle these items.

There will be five scrambled paragraphs, each containing six sentences. The first sentence will be in the correct order. You will then rearrange the remaining five sentences into the order in which the author originally placed them. There will only be one correct way to arrange the sentences to form a complete, correct, well-organized paragraph.

One thing to remember is that scrambled paragraphs are worth twice as many points as any other question on the test. Although you must arrange five sentences, you must have all of them in the right order to get any points. When the test is scored, the scrambled paragraphs will either be completely right or completely wrong. There is no partial credit for getting a few of the sentences in the right order. In other words, if three of the sentences are arranged correctly but two are misplaced, you will not earn any points. That's why it is crucial that you place all of the sentences in the most logical order.

The good news is that there are clues to help you correctly arrange the information. Words and phrases within the sentences will give hints about what ideas and information should come next.

Getting Started

What should you do first with scrambled paragraphs? As with so many other situations, the first thing you should do is read. Start with the first sentence. As you know, the main idea of a paragraph is often found in the first sentence. Since this is also the only sentence in the scrambled paragraph that you know is already in the correct order, you'll at least have an idea of what the finished paragraph will be about.

Now that you know the topic of the paragraph, read the rest of the sentences. Even though the sentences are not in the correct order, you will know what information is present. This will give you a more complete idea of what the final, corrected paragraph will include.

As you read the sentences, look for clues such as **transition words**, **grammatical clues**, **sequential ideas,** and **time order** that will help you determine the order in which the sentences belong. These clues may be very subtle, so pay close attention. We'll talk more about each of these types of clues later in this chapter.

Now, reread the first sentence and choose which sentence should follow it. Write a number 2 on the blank line next to this sentence so you know that it should be the second sentence in the paragraph. Continue numbering the sentences until you have determined the placement of each.

Once you have numbered all of the sentences, read them in order. Make sure that the paragraph flows smoothly and that the ideas are in a logical order. If any part of the paragraph doesn't make sense or doesn't sound right, try rearranging the ideas. The finished paragraph should be grammatically correct, the ideas should connect logically, and the final sentence should wrap things up.

After reading the scrambled paragraph sentences in order, if you feel that the paragraph leaves things unfinished, go back and check your work. The correct paragraph should leave you feeling that the ideas are complete. Make sure you have finished with a sentence that creates a conclusion to the piece and doesn't leave you hanging.

So, the basic steps in how to approach the scrambled paragraphs are:

1. Read the first sentence to get the main idea of the paragraph.
2. Read the remaining sentences.
3. Look for clues to help determine the order of the sentences.
4. Choose which sentence is second in the paragraph.
5. Write the number of the sentences in the blanks.
6. Read the paragraph and make any changes.

Now, let's talk about those all-important scrambled paragraph clues!

Looking for Clues

The scrambled paragraphs contain a total of six sentences. Six sentences is a relatively short amount of text in which to convey a complete explanation, thought, or account. That means the author would have chosen his or her words carefully and built the ideas upon each other in a very precise way. The ideas must be presented in a logical sequence for the paragraph to make sense. This works out well for you! There will only be a single correct order in which the sentences can be arranged and make perfect sense. Your job is to use the clues provided to determine that sequence of ideas.

Transitional Words and Phrases

You probably already know the importance of using transition words when you write. These words and phrases help to move the story along, introduce contrasting ideas, and alert readers to additional information.

These transition words can also help you determine the correct sequence of ideas in the scrambled paragraphs. Notice the use of the transition word *however* in the following sentences.

Most students taking the SHSAT are currently in the eighth grade.

However, some ninth grade students also take the test each year.

The word *however* introduces a contrasting idea. It links the information in the first and second sentences. It also gives a clue about the correct order of the sentences. Notice the difference when the sentences are arranged incorrectly:

However, some ninth grade students also take the test each year.

Most students taking the SHSAT are currently in the eighth grade.

In this order, the sentences are not logical. *However* offers a clue that the information that follows is different from the previous information.

Look at the following sentences. Use the transition words to help you arrange the sentences in the correct order. As with the scrambled paragraphs, the first sentence is given.

To determine which students will be invited to attend the specialized high schools, SHSAT scores will be listed from highest to lowest.

_____ Because of this, students with the highest scores are more likely to be placed into their first choice school.

_____ Students with lower scores, on the other hand, will be less likely to be placed into one of their highest ranked schools.

_____ Once this happens, students with the highest scores will be assigned to a specialized high school first.

What transition words did you notice? *Because of*, *on the other hand*, and *once* can be clues to help you arrange the sentences. Let's start by finding sentence 2.

Look at the sentence that begins with "Because of …." Ask yourself, *Because of what?* Are students more likely to be placed into their first choice school *because* the scores were listed from highest to lowest? Not exactly. So this is probably not sentence 2.

Now read the sentence "Students with lower scores …. "The phrase *on the other hand* tells us that the information contrasts with what has already been stated. Does this sentence contrast with scores being listed from highest to lowest? No. So this is not sentence 2 either.

Be sure to check the entire sentence for clues. The word or phrase you need could be in the beginning, middle, or end of the sentence.

Read the sentence that begins "Once this happens …." This phrase tells us that once a previously mentioned event occurs, the actions in this sentence can take place. Are students with high scores able to be assigned to schools once scores are ordered from highest to lowest? Yes! Does this sentence make sense following the first sentence? Yes! So we have found sentence 2.

(1) To determine which students will be invited to attend the specialized high schools, SHSAT scores will be listed from highest to lowest. (2) Once this happens, students with the highest scores will be assigned to a specialized high school first.

Let's look at the remaining sentences to find which is sentence 3. Look at the sentence that begins "Because of this …." Does the event in this sentence happen *because of* the previous sentence? Yes! Students with high scores are more likely to be placed into their first choice school because these students are assigned to schools first.

Now, read the final sentence, "Students with lower scores …." This sentence introduces an idea that contrasts with previous information. Does the information about students with lower scores contrast with sentence 3? It does! So we've found the final sentence. Now read the completed paragraph to make sure the order of the sentences is logical.

(1) To determine which students will be invited to attend the specialized high schools, SHSAT scores will be listed from highest to lowest. (2) Once this happens, students with the highest scores will be assigned to a specialized high school first. (3) Because of this, students with the highest scores are more likely to be placed into their first choice school. (4) Students with lower scores, on the other hand, will be less likely to be placed into one of their highest ranked schools.

The paragraph makes sense, and the ideas flow in a logical order. The transitions in these examples helped to correctly arrange the sentences. Transitions that could be helpful in solving the scrambled paragraphs on the SHSAT include those in the following list.

At any rate	In addition
Alternately	In that case
Although	Likewise
Another	Moreover
Because of	Nevertheless
Conversely	Once
Despite this	Similarly
Due to	Specifically
Furthermore	Still
Hence	Thus
However	

As you come across other transitions, add them to the list. Keep your eyes open for these helpful clues.

Grammatical Clues

Another type of clue that can be helpful in arranging the scrambled paragraphs on the SHSAT are grammatical clues. Often the placement of words themselves offer hints about the order in which the sentences should be arranged.

> *The Lincoln Memorial* is located in Washington, DC.
>
> *The monument* was opened in 1922 as a tribute to our 16th president.

These two sentences are already in the correct order, but did you notice any grammatical clues that could have helped you if these sentences were part of a scrambled paragraph? Look at the italicized words. The second sentence includes the words *the monument*. These obviously refer to a specific monument that has already been mentioned. You could use this clue to figure out that the sentence including *the monument* would need to follow a sentence that tells which monument is being discussed.

Look for clues that will help you correctly arrange the next few sentences.

The 36 columns included in the Lincoln Memorial represent the number of states in the Union at the time of President Lincoln's death.

_____ Lincoln gave this famous speech in 1863 at the dedication of the Soldiers' National Cemetery.

_____ However, these are not the only historical representations included in the monument.

_____ The words of the Gettysburg Address are inscribed into part of the impressive memorial.

The sentence "Lincoln gave…" includes a grammatical clue. The words *this famous speech* tell that this sentence needs to follow a sentence in which the speech is discussed. And since the first sentence does not talk about a speech, we know that this is not sentence 2.

The sentence that begins with "However, these…" refers to historical representations that have already been mentioned. It also tells that other representations will be discussed in a coming sentence. This could be sentence 2 since the first sentence mentions the columns as historical representations.

The sentence "The words of…" could make sense as sentence 2. But then the other sentences would not fit into place logically. Since all of the sentences must work together to make a cohesive paragraph, this sentence cannot be second. What we do know is that *the Gettysburg Address* is included here. So this must precede the sentence that includes the words *this famous speech*.

Keeping all of these clues in mind, determine the correct order of the scrambled sentences. Remember to find sentence 2, number the sentences, and read them in order to be sure they make sense.

Hopefully, you determined that the paragraph should be arranged in the following order.

(1) The 36 columns included in the Lincoln Memorial represent the number of states in the Union at the time of President Lincoln's death. (2) However, these are not the only historical representations included in the monument. (3) The words of the Gettysburg Address are inscribed into part of the impressive memorial. (4) Lincoln gave this famous speech in 1863 at the dedication of the Soldiers' National Cemetery.

Other words that can serve as helpful hints when arranging the scrambled paragraphs are pronouns. As you know, pronouns take the place of nouns. The nouns to which the pronouns refer are called *antecedents*. Common pronouns include those in the list below.

he	she
her	that
hers	their
herself	theirs
him	them
himself	themselves
his	there
it	these
its	they
itself	this
me	us
our	we
ours	

Pronouns often follow the nouns they are replacing.

In 1860, Abraham Lincoln was nominated for president. He was reelected in 1864.

Look at the pronoun *he*. It replaces *Abraham Lincoln*. Abraham Lincoln, the antecedent, is introduced in the first sentence, and the pronoun, *he*, is used in the following sentence. When you find a pronoun in one of the scrambled paragraphs, identify who or what

the pronoun refers to. It is likely that the sentence with the pronoun belongs after the sentence containing the antecedent.

Before becoming president, Lincoln worked as a clerk in a general store, which was often a community meeting place.

_____ They found him to be honest, intelligent, and likeable.

_____ It was there that he met and impressed many of the people in his town.

_____ As a result, Lincoln easily won the majority of the votes in his town when he ran for the legislature.

Look at the sentence beginning with “They found…” The pronoun *they* is a clue about the order of the sentences. Whom does the pronoun *they* refer to? The people of his town. Since a pronoun usually follows the antecedent, this sentence should probably follow the sentence that includes *the people in his town*.

Look at the sentence that begins “It was there…” The pronoun *there* also offers a clue. Where is *there*? It is the general store mentioned in the first sentence. So it is likely that this could be sentence 2.

Finally, look at the sentence “As a result…” Ask yourself, *As a result of what?* This sentence offers a transition clue rather than a pronoun. He won votes as a result of being honest, intelligent, and likeable. So this sentence should come after the one discussing the characteristics that helped him win votes.

Now, number the sentences in the correct order. Use the pronouns and transitions to help you. Then read to make sure the paragraph makes sense and is arranged logically.

Are your sentences in the same order as the paragraph below? If so, great!

(1) Before becoming president, Lincoln worked as a clerk in a general store, which was often a community meeting place. (2) It was there that he met and impressed many of the people in his town. (3) They found him to be honest, intelligent, and likeable. (4) As a result, Lincoln easily won the majority of the votes in his town when he ran for the legislature.

Sometimes substitute words are used in a way that is similar to the way we use pronouns. These words take the place of or help to explain a word that has been previously used.

The *duck-billed platypus* is native to Australia. Although it is a mammal, this *unusual-looking creature* lays eggs.

In this example, the words *unusual-looking creature* are used as a substitute for the name of the animal, *duck-billed platypus*. As with pronouns, the substitute words are likely to follow the actual name of the noun being discussed.

The platypus has a bill like a duck, a tail like a beaver, fur like an otter, and is approximately the size of a cat.

_____ Since it has no teeth, the platypus also takes gravel into its bill to help mash its nutritious find.

_____ Living in burrows beside freshwater streams and rivers, the carnivore hunts underwater for shellfish, insects, and larvae.

Substitute words can help you determine the correct order of these sentences. In the sentence beginning with “Since it has no teeth …,” the words *nutritious find* refer to the food that the platypus hunts. So this sentence probably follows a list of what the animal eats.

Another clue in that same sentence is the word *also*. It suggests that the gravel is taken into its bill in addition to something that has

already been mentioned.

In the sentence “Living in burrows …,” the substitute words *the carnivore* refers to the platypus, suggesting that this sentence follows one in which the animal is actually named.

Keep in mind that a sentence can offer more than one type of clue. Look carefully for any information that could be helpful in organizing the sentences in scrambled paragraphs!

As you have probably determined, the corrected paragraph should be in the order shown here.

> (1) The platypus has a bill like a duck, a tail like a beaver, fur like an otter, and is approximately the size of a cat. (2) Living in burrows beside freshwater streams and rivers, the carnivore hunts underwater for shellfish, insects, and larvae. (3) Since it has no teeth, the platypus also takes gravel into its bill to help mash its nutritious find.

Sequence of Ideas

Another clue that can help you organize the sentences in scrambled paragraphs is the sequence of the ideas themselves. Some sentences must follow others in order to make sense. Let’s check out a few hints that suggest a sequence of ideas.

Descriptions or Explanations

A sentence may describe or further explain ideas mentioned in a previous sentence.

> Weather is one significant cause of erosion.
>
> As rain falls, soil and rocks are washed down mountains or streams.

In this example, the second sentence explains an idea that was mentioned in the first sentence. It tells *how* weather causes erosion. Often a sentence that discusses a topic is followed by another sentence that explains *how* or *why* something occurs.

Now read the sentence below. Decide how it fits with the sentences about erosion.

> Another way erosion occurs is when the ocean's waves crash against the shore, rocks, or even cliffs.

The words *another way* tell that this sentence is further describing a previously mentioned topic. This sentence would follow the information about erosion and rain.

Additional Information

Often, one sentence introduces a topic or idea. Then the next sentence refers to or tells more about that idea. Words such as *another way* or *a better idea* suggest that a sentence is adding to something that has already been mentioned.

> (1) Centuries ago, it was widely believed that the world was flat.
>
> (2) But Christopher Columbus had a better idea.
>
> (3) He believed that the world was round, and he had a plan to prove this theory.
>
> (4) He would sail west in order to reach the east.

These sentences about Columbus are in the correct order. Let's see how these ideas link together. Notice that the first sentence introduces a popular belief of long ago. The phrase *a better idea* in sentence 2 refers to the previously mentioned belief. The pronoun *he* in sentence 3 refers to the explorer mentioned in sentence 2. Also, the phrase *He believed that the world was round* offers additional information about the *better idea* mentioned in the second sentence.

Finally, sentence 4 offers additional information about the plan mentioned in sentence 3.

References to Earlier Sentences

Sometimes a sentence will refer to events that were mentioned in a previous sentence. This can help you determine the order in which the sentences should be placed.

> Although many of us use a cell phone for daily communication, the first cell phone call was made in 1973 by the device's inventor, Dr. Martin Cooper.
>
> The introduction of this portable phone came less than 100 years after the first telephone was invented by Alexander Graham Bell.

The second sentence refers to the invention of the cell phone mentioned in the first sentence. The words *this portable phone* point out that the phone has already been discussed and offer a clue that this sentence must follow a sentence that discusses the phone. Also, notice that the paragraph mentions the cell phone before discussing the first telephone that Bell invented a century before. Don't let dates fool you. Paragraphs do not always present events in the same order that they actually took place.

Be careful! Not all paragraphs are written in chronological order. Sometimes a sentence may look back to an event that occurred earlier in history.

Read the following sentences. Pay attention to how they refer to ideas mentioned in earlier sentences. Then, number them in the correct order.

The first telephone was created by Alexander Graham Bell in 1878.

_____ By 1884, long-distance calls could be made between New York City and Boston.

_____ Within twelve years of Bell's invention, the first telephone exchange was established.

_____ This new communication tool was originally called an "electrical speech machine."

Let's look at the sentence that begins "This new communication tool …." These words refer to the first telephone mentioned in the first sentence. This could be sentence 2.

Now look at the sentence beginning "Within twelve years of Bell's invention …." This also refers to the first telephone, or "electrical speech machine," mentioned in previous sentences, but the end of the sentence discusses the first telephone exchange being established. In order for the paragraph to make sense, this information should follow the idea that the new communication tool was originally known by a different name. This sentence should be number 3.

Finally, read the sentence that begins "By 1884 …." It tells about long-distance calls. This would require use of a telephone exchange, as mentioned in sentence 3.

The correctly organized paragraph should be arranged as shown here:

(1) The first telephone was created by Alexander Graham Bell in 1878. (2) This new communication tool was originally called an "electrical speech machine." (3) Within twelve years of Bell's invention, the first telephone exchange was established.

(4) By 1884, longdistance calls could be made between New York City and Boston.

Comparison or Contrast of Ideas

Comparing and contrasting ideas is another way to determine the sequence of sentences. When similarities or differences are discussed, the sentences are likely to refer to ideas that have already been mentioned.

The earliest microwave oven stood more than five and a half feet tall and weighed in excess of 750 pounds.

A more effective design was produced, making the appliances more practical as well as more cost effective for consumers.

The phrase *A more effective design* indicates that the ideas in the sentence are a comparison to previously mentioned information. This could offer a clue about the correct order of the sentences.

Key words that often indicate comparison and contrast include:

Although	On the contrary
As well as	On the other hand
Both	Same
But	Similarly
Contrary to	The reverse
Even though	The same as
Have in common	Too
However	Unlike
In the same way	Whereas
Instead	Yet
Like	

Examples

Often, paragraphs will describe an object or explain an idea, then follow this information with an example. When you find a sentence

containing an example in the scrambled paragraphs, it is likely that the example belongs *after* the introduction of the idea.

Many painters in the mid- to late 1800s used an artistic style known as Impressionism.

One of the first of these was Monet, whose painting *Impression, soleil levant* lent its name to the style.

Here, the first sentence introduces the idea of Impressionist painters. The second sentence names Monet as an example of these artists. Notice that reversing the order of the sentences would not make sense. The example belongs after the mention of the topic.

Look at the sentences below. Identify the topic and example(s), then arrange the sentences in the correct order.

At some point, many families consider adding the companionship and love of an animal to their home.

_____ Another example is the collie, which is known to be calm and gentle.

_____ The golden retriever is one such breed, since it is smart and typically good with children.

_____ There are certain breeds of dogs that tend to make great family pets.

The first sentence introduces the idea of having pets. The second talks about a specific type of pets: great family dogs. Then, golden retrievers and collies are given as examples of such breeds. Since the sentence about the Collie starts with the words *Another example*, we could determine that a different example, the golden retriever, was given first.

Notice that the word *example* is included in one of the sentences about breeds of dogs but not the other. Sometimes you will find the words *one example* or *for example*, which will make it relatively easy to identify the fact that an example is being offered. Other times, you will have to read the sentence and figure out for yourself that it is giving an example of the topic.

The correct paragraph should be arranged as shown here.

> (1) At some point, many families consider adding the companionship and love of an animal to their home. (2) There are certain breeds of dogs that tend to make great family pets. (3) The golden retriever is one such breed, since it is smart and typically good with children. (4) Another example is the collie, which is known to be calm and gentle.

Continuing an Idea

Some sentences continue a story or idea mentioned in a previous sentence. Take a look at the sentences we read earlier about Monet. As you remember, the first sentence introduced the topic of Impressionist painters, and the second discussed Monet as an example of these artists. Here, the third sentence continues the previous ideas.

> Many painters in the mid- to late 1800s used an artistic style known as Impressionism.
>
> One of the first of these was Monet, whose painting *Impression, soleil levant* lent its name to the style.
>
> He later explained that this 1872 oil painting showed his impression of the sunrise over a French harbor, rather than an actual view of the harbor.

Notice that the third sentence continues the idea of the painting giving an impression. Other clues about the order of the sentences are included here as well. The pronoun *he* refers to the previously

mentioned Monet, and *this 1872 oil painting* refers to the previously named painting.

Sentences may include more than one clue about the order in which they should be arranged. Use as much information as you can find to help determine the proper sequence of ideas.

Summarizing Previous Sentences

As you know, a summary wraps up the important ideas in a text. A sentence that wraps things up would come after the ideas that are being summarized. It could also give a feeling of finality to the paragraph.

> Impressionism began with Monet's pivotal painting more than a century ago, and the style continues to be popular among collectors today.

This example briefly summarizes the information already mentioned about Impressionism and leaves us with a feeling that the discussion of the topic is complete. A sentence such as this would follow all of the other information on the topic.

Time Order

As we have talked about in this chapter, the ideas and information in a passage follow some sort of sequence. Often, this sequence is time order. Biographies and accounts of historical events are examples of the types of texts that are likely to be written chronologically.

> English scientist Humphry Davy invented the first electric light in 1800 using carbon and an electric arc.
>
> More than half a century later, in 1860, Sir Joseph Wilson Swan improved on Davy's invention using a filament of carbon

paper; however, the light quickly went out.

Near the end of the nineteenth century, it was Thomas Alva Edison who introduced a long-lasting electric light by placing a carbon filament in a bulb that did not contain oxygen.

Over time, continued developments and improvements have been made to the work of these men to give us the convenience of modern electric lights that we use today.

These sentences are arranged according to time order. Notice the dates 1800 and 1860 in these sentences. Also notice the phrases that suggest time, such as *more than half a century later*, *near the end of the nineteenth century*, *modern*, and *today*.

Not all chronological information will include dates. Other words and phrases are commonly used to suggest time order. Look for clues such as the following transitions in order to correctly organize sentences:

After	Formerly
After that	Immediately
Afterward	In the meantime
At last	Last
Before	Meanwhile
Begin with	Next
Concurrently	Previously
Earlier	Simultaneously
Eventually	Soon
Finally	Suddenly
First	Then

Think about what you've learned about time order, and arrange the following sentences into the correct sequence. Remember to look at dates, as well as other words or phrases suggesting time order.

Spanish explorer Hernando de Soto was the first European to explore the area we now know as Florida.

_____ His next expedition was to Peru, where he helped to conquer the Incas.

_____ He was born around the year 1500 in an area near Portugal.

_____ For the following three years, he searched for precious metals and fought the natives in this area until his death in 1542.

_____ In 1524, he went on an expedition to Nicaragua, and he lived in that part of South America for some time.

_____ Finally, the most notable of his expeditions came in 1539, when he landed along Florida's west coast.

What clues helped you organize the sentences properly? You probably noticed several dates, including 1500, 1524, 1539, and 1542. You also probably noticed the words *first*, *next*, *finally*, and *for the following three years*.

The only sentence that did not include a date was the one that began *His next expedition was to Peru*. Although no exact date was given, the word *next* tells that this followed a previous expedition. Also, since a different sentence tells us "*Finally, the most notable of his expeditions came in 1539*," you could probably figure out that the trip to Peru was not his last.

The correct order of the scrambled paragraph is shown below. See how well you did!

(1) Spanish explorer Hernando de Soto was the first European to explore the area we now know as Florida. (2) He was born around the year 1500 in an area near Portugal. (3) In 1524, he went on an expedition to Nicaragua, and he lived in

that part of South America for some time. (4) His next expedition was to Peru, where he helped to conquer the Incas. (5) Finally, the most notable of his expeditions came in 1539, when he landed along Florida's west coast. (6) For the next three years, he searched for precious metals and fought the natives in this area until his death in 1542.

Now that you've practiced using dates to arrange sentences in a scrambled paragraph, there is one other important fact to note: dates may set a chronological order, going either *forward or backward*. Consider the sentences below.

During the Renaissance, people began to demand that books be printed in languages other than Latin.

As a result of Gutenberg's printing press, books were able to be printed at a lower cost, making them accessible to more people, many of whom did not speak Latin.

Prior to this, books were extremely expensive since the words and illustrations had to be copied by hand.

Before the books could be copied, animal skins had to be cleaned and turned into parchment for scribes to write on.

Despite the labor involved in preparing parchment, it replaced papyrus, which had been a commonly used writing material hundreds of years earlier.

Notice that the paragraph starts by discussing the fact that during the Renaissance, people wanted books to be printed in their own languages. The following sentence talks about the printing press that had been invented prior to these demands. The sentence after that explains how books were copied before Gutenberg's invention. The next sentence tells what had to be done before the books could be copied. And the final sentence tells about the material used hundreds of years earlier than parchment. These chronological events are discussed in reverse order, starting with the most recent occurrence and moving backward through time.

Sequence or Steps

Think about a time you have read directions for how to do something. Perhaps you were following a recipe or following steps to complete a project. In either case, making sure you complete each step in the proper sequence is imperative.

Carefully measure each board *first*, marking the exact length with a pencil.

Next, use a saw to cut each of the boards to the proper length.

Often, when the sequence of information is important, you will find words that help you understand the correct order of the steps. Some of these clue words are similar to those used to show time order. These words can be helpful in determining the correct arrangement of the sentences in a scrambled paragraph. A list of commonly used sequence words is shown below.

After	Next
Before	Once that is completed
Finally	Second
First	Then
Following	Third
Last	To begin with

Using the clues given, arrange the following sentences in the correct order.

Making parchment for writing was a long and tedious process.

_____ After they were cleaned, the skins were stretched, and then they were scraped to produce an even thickness.

_____ The final step in preparing the skins involved whitening them with chalk.

_____ Animal skins had to first be washed thoroughly using a quicklime.

_____ Next, the scraped skins were rubbed with pumice to smooth them into a thin writing material.

Let's see how well you did! The first sentence introduces the topic of making parchment. The sentence beginning "After they were cleaned…" tells you that this step happens after something else. We know that this is not sentence 2 and that it must come after the sentence explaining the cleaning process.

The next sentence includes the words *The final step*, which are a strong clue that this sentence is most likely at the end of the paragraph.

The sentence that begins "Animal skins…" includes the word *first*, so we know that this is the first step in the parchment-making process. It should follow the introductory sentence and come before all of the other steps. So we have found sentence 2.

The sentence that begins "Next, the scraped skins…" must come after the sentence that explains how the skins are scraped.

Take a look at the correct paragraph. The sequence words are italicized to show how these work together to show the steps in the process.

(1) Making the parchment was a long and tedious process. (2) Animal skins had to *first* be washed thoroughly using a quicklime. (3) *After* they were cleaned, the skins were stretched,

and then they were scraped to produce an even thickness. (4) *Next*, the scraped skins were rubbed with pumice to smooth them into a thin writing material. (5) The *final* step in preparing the skins involved whitening them with chalk.

A paragraph may include only a few of the sequence or step clue words. You may not find them in every sentence, but rather sprinkled throughout the paragraph. These clues are often used in addition to other types of clues. Be sure to use all of the clues available and determine how they work together to form the completed text.

Just Plain Logic

Wouldn't it be great if every single sentence offered an obvious clue about exactly where it belonged in a scrambled paragraph? Let's face it: that's not likely to happen. There will definitely be times where you have to figure out the correct arrangement of sentences using plain old everyday logic.

But that really won't be as tough as it may sound. Some ideas simply make more sense in one place than another. For example,

- A description would need to follow the introduction of what is being described.
- A biography that focuses on someone's career may end with a sentence about his or her retirement.
- A paragraph listing the steps of a process is likely to end with the final step, or the completion of the task.

As you read the sentences in a scrambled paragraph, think about where the sentences make the most sense. Let's try it.

The invention of the printing press had a tremendous impact on the spread of knowledge at that time.

_____ Businessmen and bankers, as well as the university students, began learning arithmetic.

_____ Bookshops and printing businesses became plentiful in towns where universities were located.

_____ As printing was simpler and less costly, books became more easily accessible, making learning much easier.

The first sentence tells that the paragraph will be about the spread of knowledge as a result of the printing press. It would make sense for the next sentence to explain how this invention affected the spread of knowledge. The sentence including the words *books became more easily accessible* explains this idea. It should be sentence 2.

Look at the sentence about the bookshops and printing businesses. It is logical that this would follow sentence 2 because it explains how the books became more accessible, especially to students.

Now read the sentence about businessmen and bankers. It explains that these people, in addition to the university students, were learning more. This sentence should follow the sentence first mentioning the university.

The final paragraph should be arranged as shown here:

> (1) The invention of the printing press had a tremendous impact on the spread of knowledge at that time. (2) As printing was simpler and less costly, books became more easily accessible, making learning much easier. (3) Bookshops and printing businesses became plentiful in towns where universities were located. (4) Businessmen and bankers, as well as the university students, began learning arithmetic.

How'd you do? Using logic, without many other grammatical or word clues, is definitely possible. That's something to keep in mind, even when clues are given. Make sure that the order of the sentences makes sense.

You Try It!

So far, you've practiced arranging a few sentences at a time using clues such as transitions, grammatical clues, sequence of ideas, time order, and logic. Now, let's put it all together. These paragraphs have six sentences, just like the scrambled paragraphs on the SHSAT. Look for all of the clues that you've learned in this chapter to help you.

Example 1

Choose the order of the sentences that will create the best paragraph. Use the blanks to keep track of your sentence order.

Composer Ludwig van Beethoven was different from other musicians in a number of unique ways.

_____ However, these characteristics of the great composer were less significant than the fact that deafness eventually prevented Beethoven from hearing his own amazing compositions, which the rest of the world continues to appreciate today.

_____ These titles were a way to help listeners connect to the feelings of the piece by picturing the images he had in mind when writing the notes.

_____ Born in 1770 in what is now known as the Classical period, his compositions created what became known as the Romantic period, in which passion and emotion impacted the music.

_____ Adding singers to his symphonies was another way in which Beethoven was unlike other composers, who had never included the human voice with the instruments.

_____ One way he brought this emotion to his masterpieces was to name each of the movements in the symphonies he wrote.

Let's see how well you did! The first sentence introduces the topic of Beethoven's unique characteristics. Look at the sentence that begins with *However*. This transition lets us know that the ideas in the sentence will contrast in some way with a previously mentioned idea. Since no ideas have been mentioned yet, we know that this is not sentence 2. Also, the phrase *these characteristics … were less significant than …* is a clue that this sentence must follow other characteristics of Beethoven.

The words *These titles* at the beginning of the next sentence refer to previously mentioned titles. So this sentence must follow one that discusses the titles Beethoven gave to the movements in his symphonies.

Born in 1770 indicates that the sentence will discuss an event at the beginning of Beethoven's life. Logically, this sentence should be at the beginning of the discussion. This is sentence 2.

Look at the sentence that states, "Adding singers to his symphonies was another way in which Beethoven was unlike other

composers.” The words *another way* are a clue that at least one way he was different must be mentioned in previous sentences.

The sentence beginning “One way…” includes the words *this emotion*, which tells that it follows an earlier sentence that discusses emotion. Also, *one way*, as opposed to *another way*, could indicate that this sentence introduces the first of the list of Beethoven’s unique characteristics.

Now, compare your paragraph to the corrected text shown below:

> (1) Composer Ludwig van Beethoven was different from other musicians in a number of unique ways. (2) Born in 1770 in what is now known as the Classical period, his compositions created what became known as the Romantic period, in which passion and emotion impacted the music. (3) One way he brought this emotion to his masterpieces was to name each of the movements in the symphonies he wrote. (4) These titles were a way to help listeners connect to the feelings of the piece by picturing the images he had in mind when writing the notes. (5) Adding singers to his symphonies was another way in which Beethoven was unlike other composers, who had never included the human voice with the instruments. (6) However, these characteristics of the great composer were less significant than the fact that deafness eventually prevented Beethoven from hearing his own amazing compositions, which the rest of the world continues to appreciate today.

If you didn’t quite get the paragraph arranged correctly, look at the differences between your order and the completed paragraph. Find which sentences are out of order. Why did you choose that placement for the sentence? What clues did you overlook? See if you can determine why the correct order is more logical, and use this information to help you with the next scrambled paragraph.

Example 2

Let’s try another one. Choose the order of the sentences that will create the best paragraph. Use the blanks to keep track of your sentence order.

Kangaroos are marsupials, meaning that the mothers carry their babies in a pouch on the front of their bodies.

_____ One of the ways the mother is able keep the baby safe is by contracting the muscles of her pouch to pull it tighter against her body as she hops.

_____ The tiny joey crawls up his mother’s belly and into her pouch, where it stays for several months.

_____ Once the joey is finally old enough to leave the pouch, it will return to this safety when feeling threatened until the time it is about eight months old.

_____ During that time, the joey is protected and kept warm while it continues to grow and develop inside the pouch.

_____ When a baby kangaroo, called a joey, is born, it is about the size of a jelly bean, is blind, and has no hair.

The first sentence indicates that the paragraph will be about mother kangaroos and their babies. The sentence that begins “One of the ways…” suggests that the idea of keeping the baby safe has already been discussed. This sentence is an example of ways the baby is kept safe. As you know, examples usually follow the introduction of an idea, such as the fact that the mother keeps the

baby safe in its pouch. At this point, we don't know exactly where the sentence belongs in the paragraph, but it is probably not sentence 2.

Moving on to the sentence "The tiny joey …," we can figure out that a previous sentence must have introduced the word *joey*. This sentence does not offer any explanation about what a joey is but explains its actions as if a joey has been mentioned before. So this sentence will probably follow a sentence that tells about a joey.

The sentence "Once the joey is finally old enough…" discusses things that happen several months after it is born. This information belongs after its birth and early months are addressed. Also, the word *finally* indicates that this sentence belongs late in the paragraph.

Look at the sentence that begins "During that time …." It refers to a time that has already been mentioned. This is an example of a sentence referring to a previous sentence, and such clues can be very useful. We need to find out during which time the joey is protected and kept warm and place this sentence after that time is introduced.

The sentence "When a baby kangaroo…" gives information about the animal's birth. Logically, a discussion of its birth would belong toward the beginning of the paragraph. Also, since this sentence defines the word *joey*, we can determine that this is likely the first mention of the word. So we have found sentence 2.

Now let's check your work. Compare the arrangement of your paragraph to the paragraph below.

> (1) Kangaroos are marsupials, meaning that the mothers carry their babies in a pouch on the front of their bodies. (2) When a baby kangaroo, called a *joey*, is born, it is about the size of a jelly bean, is blind, and has no hair. (3) The tiny joey crawls up his mother's belly and into her pouch, where it stays for several months. (4) During that time, the joey is protected and kept warm while it continues to grow and develop inside the pouch. (5) One of the ways the mother is able keep the baby safe is by contracting the muscles of her pouch to pull it tighter against her body as she hops. (6) Once the joey is finally old

enough to leave the pouch, it will return to this safety when feeling threatened until the time it is about eight months old.

Example 3

Now try this. Determine the order of the sentences that will create the most logically organized paragraph. Use the blanks to keep track of your sentence order.

In June 1933, the first drive-in movie theater was opened in Camden, New Jersey.

_____ The entrepreneur wanted to find a comfortable way for people to enjoy watching movies.

_____ First known as Park-in Theaters, these family-friendly entertainment venues were the result of a concept by Richard Hollingshead, a young man who enjoyed both cars and movies.

_____ He patented the idea in May 1933, and at the height of their popularity about three decades later, approximately 5,000 drive-in theaters existed throughout the country.

_____ To develop the possibility for her and others to watch movies from the comfort of an automobile, Hollingshead tried various ways to project movies and provide sound in his own driveway.

_____ This goal was a result of the fact that his own mother found traditional indoor theaters to be uncomfortable.

Were you able to find clues that were helpful in correctly arranging the sentences? Let's see what information is given to solve this scrambled paragraph about drive-in theaters.

The sentence that begins with "The entrepreneur…" is obviously referring to a person that has already been named. This sentence would need to follow one that introduces the businessperson being discussed.

Look at the next sentence. The phrase *these family-friendly entertainment venues* includes substitute words to replace the phrase *drive-in movie theaters*. This indicates that the theaters were previously mentioned. Another clue is the phrase *First known as Park-in Theaters*. This is describing an idea that has already been discussed. And since it tells what the theaters were originally called, it makes sense that this fact would be toward the beginning of the paragraph. This is sentence 2.

The sentence "He patented the idea…" includes the pronoun *he*, indicating that this sentence should follow one that includes the antecedent *Richard Hollingshead*. It also discusses events that occurred *three decades later.* Chronologically, this sentence belongs after any that discuss the invention and early years of the theaters.

The sentence "To develop the possibility…" includes the pronoun *her*, so it must follow the mention of a female antecedent.

The words *This goal* at the beginning of the next sentence refer to the goal of finding a comfortable way for people to watch movies, so the sentence must come after the goal is mentioned. Also, *his own*

mother is the only reference to a female in the paragraph. That shows that this sentence must come before the one that contains the pronoun *her*.

Take a look at the correctly arranged paragraph. Compare the order of these sentences to the order in which you placed them.

> (1) In June 1933, the first drive-in movie theater was opened in Camden, New Jersey. (2) First known as Park-in Theaters, these family-friendly entertainment venues were the result of a concept by Richard Hollingshead, a young man who enjoyed both cars and movies. (3) The entrepreneur wanted to find a comfortable way for people to enjoy watching movies. (4) This goal was a result of the fact that his own mother found traditional indoor theaters to be uncomfortable. (5) To develop the possibility for her and others to watch movies from the comfort of an automobile, Hollingshead tried various ways to project movies and provide sound in his own driveway. (6) He patented the idea in May 1933, and at the height of their popularity about three decades later, approximately 5,000 drive-in theaters existed throughout the country.

Let's Review!

In this chapter, you've learned several strategies for tackling the scrambled paragraphs on the SHSAT. Let's take a final look at some of the ideas to keep in mind for this section of the test:

- Read all of the sentences in the paragraph, then identify which should be sentence 2. Use clues to help you number the remaining sentences accordingly.
- Transitional words and phrases can help you figure out the order of the sentences.

- There are several types of grammatical clues that can be helpful in organizing the scrambled paragraphs. Look for:
  - Words that refer to previously mentioned ideas
  - Pronouns
  - Substitute words and phrases
- The sequence of ideas in the paragraph can offer clues about the correct order of the sentences. Look for:
  - Descriptions or explanations
  - Additional information
  - References to earlier sentences
  - Comparison or contrast of ideas
  - Examples
  - Continuation of an idea
  - Summaries of previous sentences
- The information in some paragraphs is arranged chronologically, going either forward or backward in time. Watch out for time order words that offer clues about the correct sequence of the sentences.
- Some paragraphs include ideas that must follow a particular sequence, such as steps for completing a project. Clue words are often used to show the proper arrangement of the ideas.
- Using logic is an important way of determining the correct sequence of the sentences in scrambled paragraphs. No matter what other clues are offered, make sure that the final paragraph is arranged in a way that makes sense.

CHAPTER 5
Logical Reasoning

Ten of the questions on the verbal section of the SHSAT assess your logical reasoning skills. These questions will ask you to do one of four things. You might need to:

- Figure out a code
- Draw a valid conclusion
- Identify a correct assumption
- Determine the relative positions of people or objects

Think of the logical reasoning questions as puzzles you must solve. The trick here is to base your answers only on information you are given and not to form conclusions that are not based on the facts presented.

There are a few helpful things to keep in mind as you approach the logical reasoning questions.

- It is imperative that you read the information and directions very carefully.
- Do not make any assumptions that are not completely based on the information you are given.
- Drawing a diagram or making a chart can be helpful tools in organizing the information you are given.
- Underline key facts in the statements.
- Read the questions first so you know what information you are being asked to find.

Now, let's take a closer look at each of the types of logical reasoning questions you will encounter on the SHSAT.

Figuring Out a Code

Some of the logical reasoning questions on the SHSAT will require you to figure out a code like the one below.

T	Q	L	M	R	means
"Ella	likes	English	class	best."	

In this example, each of the letters T, Q, L, M, and R are code for one of the words in the sentence "Ella likes English class best." Given just this one sentence, it would be nearly impossible to determine which word is represented by each of the letters. However, you will not be given just a single sentence. You will be given a set of sentences that, when considered together, will allow you to decode the word or words needed to answer the question.

Let's see what we can figure out about the original sentence when it is presented along with other information.

T	Q	L	M	R	means
"Ella	likes	English	class	best."	

Q	S	M	R	F	means
"Franklin	likes	math	class	best."	

These two sentences contain several common words. Now we can start to figure out the code. The letters Q, M, and R are part of the code for both sentences. The words *likes*, *class*, and *best* are also part of both sentences. So we can determine that Q, M, and R each stand for one of these common words. Given the information we have, we don't know which letter represents each word, but we have a start. Let's look at the rest of the words and letters associated with our sample sentences.

T	Q	**L**	M	R	means
"Ella	likes	*English*	class	best."	
Q	**S**	M	R	**F**	means
"Franklin	likes	*math*	class	best."	

In the first sentence, we have not yet figured out which words are represented by T and L. We also have not associated any code letters with *Ella* or *English*. So we can determine that T and L stand for either *Ella* or *English*.

The same reasoning can be used with the second sentence. The letters S and F must stand for either *Franklin* or *math*.

Suppose we are trying to find which letter represents the name *Ella*. We have determined that either T or L is the correct letter. But we would need more information to choose which of these letters represents *Ella*.

T	Q	L	M	R	means
"Ella	likes	English	class	best."	
Q	S	M	R	F	means
"Franklin	likes	math	class	best."	
R	A	Y	L	X	means
"Ella	does	not	like	history."	

Let's think about what we already know first, then apply this to the code in the third sentence, which includes *Ella*. We already figured out that *Ella* is represented by either T or L. Since L is repeated in the code for the third sentence, and T is not, we can determine that L represents *Ella*.

This third sentence can also help us decode one of the other words. The word *like* is in the third sentence, and it was also in sentences 1 and 2. We already figured out that *like* is represented by either Q, M, or R. Since the only one of these letters that is repeated in the third sentence is R, then R must represent the word *like*.

This still leaves several words that we have not figured out the code for. That doesn't matter. You do not need to figure out every word in every sentence. In fact, that won't even be possible sometimes. You are only required to decode the word or words that the questions ask about.

Don't worry about figuring out every single word in the code. See what word or words you have to decode, and focus on those.

Let's try another one. The directions given before this example are similar to those you might find on the SHSAT.

> In the code below, (1) each letter always represents the same word, (2) each word is represented by only one letter, and (3) in any given sentence, the letters may or may not be presented in the same order as the words.

E	O	P	B	K	means
"Thursday	was	a	sunny	day."	
P	T	K	B	U	means
"Saturday	was	a	rainy	day."	
Z	U	K	J	W	means
"It	was	not	hot	Saturday."	

Which letter represents the word "Saturday"? There are quite a few words and letters in this code. The only one we need to solve is the letter that represents the word *Saturday*.

Let's start by figuring out what words and letters are common among the sentences. The first two sentences both contain the words *was*, *a*, and *day*. The codes for these two sentences also both contain the letters P, B, and K. That tells us that *was*, *a*, and *day* are represented by either P, B, or K.

Now, let's focus on the second sentence, which contains the word *Saturday*, since this is the word we need to find the code letter for. We can start by ruling out what letters are incorrect. The code letters for the sentence "Saturday was a rainy day" are P, T, K, B, and U. We already determined that P, B, and K represent the words *was*, *a*, or *day*. That means that *Saturday* and *rainy* are represented by one of the two remaining letters in the code for the second sentence, T or U.

Take a look at the third sentence and its code. This sentence also includes the word *Saturday*, so we can now look for repeated code letters. We know that either T or U represents *Saturday*. And since T is not part of the code for this sentence and U is, U must be the letter that represents the word *Saturday*. Whew!

So the steps we followed to break the code were to:

1. Determine which word or words you must decode.

2. Find common words and letters among the sentences.

3. Use these to rule out any incorrect letters.

4. Look for repeated code letters among the sentences.

As with any test item, reading the directions extremely carefully for the coded questions on the SHSAT is vital. The directions for these items will *not* always be the same.

Take another look at the third direction for the code we just solved: "in any given sentence, the letters may or may not be presented in the same order as the words." That means that the first letter in the code could possibly represent the first word in the sentence.

However, there will be times when the directions state that the letters are not in the same position as a word. You will likely find a direction line that states, "in any given sentence, the position of a letter is **never** the same as that of the word it represents." This means that the first letter of the code *cannot* represent the first word in the sentence. This can be extremely helpful information. The position of a letter can help you rule out whether or not it represents a given word.

Read the directions carefully. Sometimes the letters in a code may be in the same order as the words, sometimes they are not. Underline key words and phrases in the directions to help you remember exactly what is stated.

Let's work through another example. Carefully read the directions before you begin. The directions given are similar to those you may find on the SHSAT.

> In the code below, (1) each letter always represents the same word, (2) each word is represented by only one letter, and (3) in any given sentence, the position of a letter is **never** the same as that of the word it represents.
>
> D M Y V A means
> "Terrell played soccer with Marco."
>
> E Y M F D means
> "Terrell played chess with Amelia."
>
> L V D E M means
> "Amelia played tennis with Marco."

1. What word is represented by the letter D?

2. What letter represents the word "tennis"?

For this code, we need to solve two things: We need to find the word that is represented by the letter D and the letter that represents the word *tennis*.

First, find which words and letters are common among the sentences. Each sentence contains the words *played* and *with*, and each code contains the letters D and M. So D either represents *played* or *with*.

We know from the directions that no letter is in the same position as the word it represents. Let's use this information to help us find which word is represented by D. In each sentence, *played* and *with* are in the second and fourth positions, respectively. Now, look at the first code and sentence.

D	**M**	Y	V	A	means
"Terrell	**played**	soccer	**with**	Marco."	

Here, M and *played* are both in the second position. So we know that M cannot represent *played*. By process of elimination, M must represent the word *with*, and D must represent the word *played*.

Next, we need to find which letter represents the word *tennis*. Let's take another look at the sentence containing the word.

L	V	D	E	M	means
"Amelia	played	tennis	with	Marco."	

We've already established that D represents the word *played* and M represents *with*. That leaves the letters L, V, and E. So L, V, and E must represent the remaining words in the sentence: *Amelia*, *tennis*, or *Marco*.

This is the only sentence that includes the word *tennis*. However, the words *Amelia* and *Marco* are used in other sentences. So the letters representing *Amelia* and *Marco* will be found in other codes. The letter representing *tennis* will not.

Look at the first sentence again. Does it contain L, V, or E?

D	M	Y	**V**	A	means
"Terrell	played	soccer	with	Marco."	

Yes! It contains V. We know that V must represent *Amelia*, *tennis*, or *Marco*. Since *Marco* is the only one of these words in the sentence, V must represent *Marco*.

Take a look at the second sentence. Does it contain L or E?

E	Y	M	F	D	means
"Terrell	played	chess	with	Amelia."	

It does! It contains E. We know that E must represent either *Amelia* or *tennis*. Since *Amelia* is the only one of these words in the sentence, E must represent *Amelia*.

Now, take a look at the third sentence again.

L	V	D	E	M	means
"Amelia	played	tennis	with	Marco."	

We know which words are represented by V, D, E, and M. That only leaves L, so we know that L represents the word *tennis*.

Now you try it! Use what you have learned to decode these items. The only difference is that this is a multiple choice question, just like the questions on the SHSAT. Read the directions, decode the words, and choose the correct answer choice.

In the code below, (1) each letter always represents the same word, (2) each word is represented by only one letter, and (3) in any given sentence, the position of a letter is **never** the same as that of the word it represents.

R	X	F	S	Q	means
"They	ate	steak	for	dinner."	

F	R	Q	T	S	means
"They	ate	chicken	for	dinner."	

T	Y	Q	S	F	means
"They	ate	chicken	for	lunch."	

1. Which letter represents the word "chicken"?

A. X
B. Q
C. S
D. T
E. Y

2. Which word is represented by the letter S?

A. they
B. ate
C. for
D. Either “they” or “for, “but it is not possible to determine which one.
E. Either “they” or “ate,” but it is not possible to determine which one.

One strategy for answering multiple choice questions is to see which answers could possibly be correct and eliminate those that are not. When figuring out a code, you can use this approach to make sure the answer you have chosen is the right one. It’s a great way to check your work and make sure you have decoded the sentences correctly.

Let’s see how well you did! Look at the answer choices for question 1, and figure whether or not each could be the correct answer. The first choice, X, only appears in the first sentence, which does not include the word *chicken*. The letters Q and S also appear in the first sentence, but *chicken* does not. The letter T is included in both sentences that include the word *chicken*. However, the letter Y is not part of the second sentence, and the word *chicken* is. So, the letter T represents the word *chicken*.

Now let’s check your work for the second question. The letter S is included in every sentence. The words *they*, *ate*, and *for* are also included in every sentence. The directions state that the position of a letter is **never** the same as that of the word it represents. In the third sentence, S and *for* both hold the fourth position. This means that S cannot represent the word *for*. That means S must represent either *they* or *ate*.

These words hold the first and second positions, respectively, of every sentence. The letter S is never in the first or second position,

so neither of these words can be ruled out. That means the final answer choice is correct. In the code, the letter S represents either *they* or *ate,* but it is not possible to determine which one.

Drawing Valid Conclusions

As you know, a valid conclusion is a reasonable decision or opinion that is made based on facts. For example, suppose the school cafeteria always serves pizza on Fridays. If you walk into the cafeteria on a Friday morning and see the cook tossing pizza dough in the air, you can conclude that he is making a pizza. While it is possible that he is using the dough to make a calzone or some other dish that requires a similar crust, this would not be a reasonable conclusion since there are no facts to suggest that he would make something other than pizza on this day. You might also assume that he is making pepperoni and sausage pizza. However, without any evidence, this conclusion would not be valid.

Suppose you notice that the menu board in the cafeteria tells that today's lunch is vegetable pizza. You hear the cook mention that the spinach and mushrooms look fresh today. A valid conclusion could be that he is putting the spinach and mushrooms on the vegetable pizzas.

The key to drawing a valid conclusion is to base your decisions on the facts that are given. If the stated information does not completely support the conclusion, it is not valid. Consider the facts shown below.

> Spot has four legs.
>
> Spot has fur and a tail.
>
> Spot stays on the floor in my room at night.

What conclusion can be drawn based on these facts? Well, we know that something that has four legs, fur, and a tail is probably an

animal. And the fact that it stays on the floor at night supports this fact. It would be reasonable to conclude that Spot is an animal.

Can we conclude that Spot is a dog? Well, *Spot* is a common name for dogs, but there is no evidence to support this. Spot could be a cat, or a gerbil, or a number of other four-legged creatures.

Can we conclude that Spot is someone's pet? Think carefully! Since Spot stays on the floor at night, we do know that Spot is inside the house, and probably not a stray or a wild animal. However, the facts do not state that Spot is a *living* creature. Spot could be furry stuffed animal, like a teddy bear. So, as tempting as it may be to believe that Spot is a pet, this would not be a valid conclusion.

When looking for a valid conclusion, think of yourself as a detective. Your job is to look at all of the facts and evidence, then base your decision only on the information given.

On the SHSAT, questions requiring you to draw a valid conclusion will be multiple choice. You will read the facts, read each answer choice, then select the one answer choice that offers the *best* conclusion. To answer these questions, follow the steps shown below.

1. Carefully read the information that is given.
2. Identify all of the facts in the statements.
3. Check each of the answer choices against the facts given in the item.
4. Decide which answer choice is a valid conclusion based only on the facts.

Let's take a look at an example that is similar to the items you will find on the SHSAT.

All of Karina's pets were adopted from the pound.

Her favorite pet is her dog, Rascal.

Which of the following statements is a valid conclusion from the statements above?

A. All of Karina's pets are dogs.
B. The pound has both dogs and cats.
C. Rascal was adopted from the pound.
D. The first pet Karina adopted was Rascal.
E. Most of the animals at the pound are dogs.

First, determine exactly what facts were given.

- All of her pets were adopted from the pound.
- Her favorite pet is Rascal.
- Rascal is a dog.

Let's take a look at each of the statements and determine whether each is a valid conclusion based solely on the information given.

A. All of Karina's pets are dogs.

We do know that at least one of her pets, Rascal, is a dog. But, no information is given regarding any of Karina's other pets. They may all be dogs; however, she may have other animals as well. Since we don't know for sure, this statement is not a valid conclusion.

B. The pound has both dogs and cats.

Often, many types of animals can be found at the pound, including both dogs and cats. In this case, though, we have to select a valid conclusion based only on the facts that were stated. Since there was no mention of cats, we cannot conclude that there are cats at the pound.

D. The first pet Karina adopted was Rascal.

The statements do tell that Karina adopted Rascal, but no information is given about when she did this. He is her favorite, and he may have been her first pet. He also could have been the pet she adopted most recently. There is no way to know based on the information given. Since we do not know for sure when Rascal was adopted, this is not a valid conclusion.

E. Most of the animals at the pound are dogs.

Since Rascal is a dog, we can conclude that at least *some* of the animals at the pound are dogs. But there is no indication that *most* of the animals at the pound are dogs.

C. Rascal was adopted from the pound.

Think about the facts: all of Karina's pets were adopted from the pound. Rascal is one of her pets. Therefore we can conclude that Rascal was adopted from the pound.

Now you try it. Read the information, then select the answer choice that is a valid conclusion, based only on the information given.

> Everyone at the dance was an eighth grade student, with the exception of Mrs. O'Malley. Marissa was at the dance.

Which of the following statements is a valid conclusion from the statements above?

A. Mrs. O'Malley was a chaperone at the dance.
B. Mrs. O'Malley was at a school dance.
C. Marissa is Mrs. O'Malley's daughter.
D. Marissa is in the eighth grade.
E. Marissa is 13 years old.

Did you determine that the only valid conclusion is answer choice D? If so, great job! Let's review why this is the correct answer. First,

look at the facts given.

- Everyone at the dance except Mrs. O'Malley was an eighth grade student.
- Mrs. O'Malley was the only one at the dance that was not an eighth grade student.
- Marissa was at the dance.

Next, check each answer choice against these facts.

A. Mrs. O'Malley was a chaperone at the dance.

As you know, adults attending an eighth grade dance are usually chaperones. It is likely that this was Mrs. O'Malley's role at the dance. However, the facts do not state why she was there. She could have been the DJ at the event, or the caterer, or maybe she was a parent who came to pick up her child. There is no way to know since this information is not given. So, while it may be true that she was the chaperone, this is not a valid conclusion based on the facts.

An answer choice may be true, but if it cannot be determined from the statements included in the item, it is not the correct choice.

B. Mrs. O'Malley was at a school dance.

You may have thought that this one was a little tricky. That facts state that Mrs. O'Malley was at a dance that was attended by eighth grade students, however, the information does not state that it was a school dance. It could have been a birthday party. It could have been a dance at summer camp. But, no matter what type of dance it was, this answer choice is incorrect because the facts do not state that it was a school dance.

C. Marissa is Mrs. O'Malley's daughter.

Think about the facts. Marissa was at the dance. Mrs. O'Malley was at the dance. We have no way to know what the relationship is between these two people. So this answer choice is incorrect because there is no statement to suggest that these individuals are related in any way.

E. Marissa is 13 years old.

As you know, many eighth grade students are 13 years old, but this is not a requirement, nor is any age stated in the facts. Although there is the possibility that an answer choice might be true, it is not correct unless it can be concluded according to the facts. So we cannot conclude that this is Marissa's age.

Now that we've compared each of the incorrect answers to the facts given, let's review why answer choice D is correct.

D. Marissa is in the eighth grade.

Marissa was at the dance. Everyone at the dance was an eighth grade student, except for Mrs. O'Malley. So if Marissa was at the dance, and she is not Mrs. O'Malley, she must be an eighth grade student.

Identifying Correct Assumptions

An *assumption* is something that is believed to be true. On the SHSAT, you will be asked to look at evidence, then choose the statement that must be true, based on the information given. The way you approach these questions will be quite similar to the way you chose a valid conclusion.

Key things to remember when identifying correct assumptions include the ideas below.

- It is imperative that you read all of the information carefully.
- Do not assume anything is true unless it is explicitly stated.

- The correct answer must satisfy all of the conditions given.
- The answer must be based only on the information given.

Let's try it! Read the information, then select the statement containing a correct assumption.

If a question asks what *must* be true, do not choose an answer that *might* be true.

There will be a pep rally next Friday after school if the football team makes it to the playoffs. The team will be in the playoffs if they win the game tonight.

Based only on the information above, which of the following **must** be true?

A. There will be a pep rally tonight before the football game.
B. If there is a pep rally next Friday, the team will win the playoffs.
C. If the team does not win tonight, the pep rally will be canceled.
D. There will not be a pep rally tonight since the team is not in the playoffs.
E. If the team wins tonight, there will be a pep rally next Friday.

In this type of question, the only information on which you can base your selection is exactly what has been stated. Think about what facts have been given.

- If the team wins tonight, they will be in the playoffs.
- If the team makes it to the playoffs, there will be a pep rally next Friday.

Sometimes it is helpful to rearrange the statements without changing their meaning. This can make it easier to see the relationship between the conditions. Just be careful that the facts are not altered when you rearrange the information.

Now, we have to determine whether each option *must* be true based only on this information. Notice that this situation includes two conditional statements. Items on the SHSAT may contain more than one conditional statement. The correct answer must satisfy *all* statements given.

A. There will be a pep rally tonight before the football game.

Nothing is ever stated about a pep rally tonight. The only pep rally that is mentioned is one that may take place next Friday. While there may actually be a pep rally tonight also, the question asks you to identify which answer choice *must* be true, not which *might* be true.

B. If there is a pep rally next Friday, the team will win the playoffs.

The facts state that if the team is *in* the playoffs, there will be a pep rally. The facts do not state that the team will *win* the playoffs as a result of the pep rally.

C. If the team does not win tonight, the pep rally will be canceled.

This statement assumes that the pep rally has already been scheduled, and that it will be canceled if the team loses tonight. Remember, you must base your answer only on the information given. Do not assume anything that does not rely on the stated facts. We cannot assume that the pep rally has been scheduled nor that it will be canceled based on the outcome of tonight's game. No information was given regarding the pep rally being canceled.

D. There will not be a pep rally tonight since the team is not in the playoffs.

Again, we cannot assume anything that is not stated. There was no mention of whether or not a pep rally would take place tonight, so we cannot make any assumptions in this regard.

E. If the team wins tonight, there will be a pep rally next Friday.

Remember the facts:

- If the team wins tonight, they will be in the playoffs.
- If the team makes it to the playoffs, there will be a pep rally next Friday.

So, a win tonight means a trip to the playoffs and a pep rally next Friday. This answer choice meets all of the conditional statements given and is a correct assumption based on the facts.

Determining Relative Position

The fourth type of logical reasoning questions you will find on the verbal section of the SHSAT involve determining relative position. In these questions, you will be given a set of facts about a group of people or items and asked to use these facts to figure out the order in which people or items are arranged. For example, you might need to find which student is the tallest, which runner placed third in a race, which color house on Pine Street is third from the corner. Again, think of these items as a puzzle.

There are several things that you should keep in mind when trying to answer these questions.

- **Know what to you need to solve.** As you work, pay close attention to what information you are being asked to find. You may or may not be able to determine the relative position of every

person or object mentioned. Don't worry about that. You will be given enough information to be able to figure out what you need to know to correctly answer the questions. Look at the questions to determine who or what you need to know the position of, and focus on figuring out that information.

At times, there will not be enough information given to figure out the exact positions of the objects. However, you will be given enough information to be able to answer the question.

- **Use a visual.** It can be helpful to draw a chart or diagram to determine the correct order. For example, when organizing students' names from shortest to tallest, you might draw five blank lines, then label the first *shortest* and the last *tallest*. As you work, you can fill in names and see how the heights relate to one another. The same concept can work for arranging race horses from first to last place, or placing plants on a series of shelves from top to bottom.
- **Watch for specifics.** Look for information that is definite, such as "Jordan is the tallest," "Kayla is shorter than Maddie," or "Devon is not the shortest." Facts such as these give you a strong starting point around which you can arrange the rest of the information.
- **Find the facts.** Record what information is given about each person or object. Remember, your answer must be based on exactly what facts were stated in the question.
- **Read carefully.** Read the facts exactly as they are stated. If the item tells that Jordan is between Kayla and Maddie, it does not mean that he is right next to either of them. It also does not mean that he is an equal distance from each of them. Keep this in mind with words such as *above*, *below*, *before*, *after*, *ahead of*, *behind*, *to the right of*, and *to the left of*.

Take a look at the following example.

There are five shelves in Mr. Franklin's refrigerator, with the shelves numbered consecutively from top to bottom. He keeps a different type of food on each shelf.

1) Vegetables are on one of the shelves.

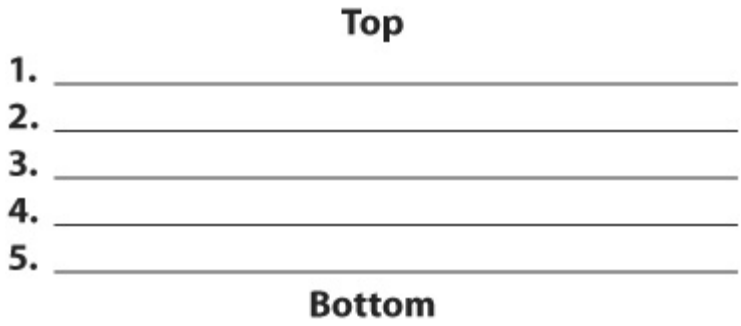
2) Juice is on the top shelf.

3) There are exactly two shelves between the meat and the fruit.

4) Yogurt is directly above the meat.

On which shelf are the vegetables?

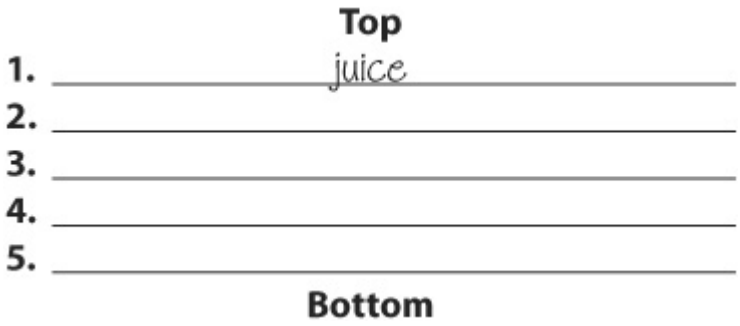
The first step in determining relative position is knowing what you need to solve. In this example, we need to find which shelf the vegetables are on. In order to do this, let's make a diagram of the shelves.



As we find facts about the locations of the items in the refrigerator, we can add them to the diagram.

After carefully reading the question, we know that there are five types of food: fruit, juice, meat, vegetables, and yogurt. Each food is on a separate shelf.

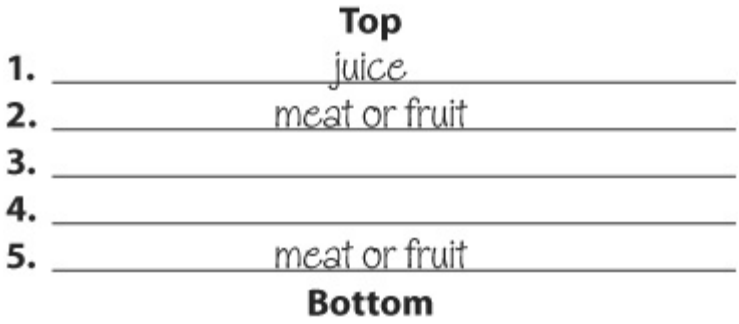
Now, look for any specific information that gives the exact locations of the foods. Since the second statement tells that juice is on the top shelf, we can write this on line 1 of the diagram.



Next, look for other facts that are given, such as the third statement.

▶ There are exactly two shelves between the meat and the fruit.

For there to be two shelves between the meat and the fruit, these must be placed on either shelves 1 and 4, or shelves 2 and 5. Since we already know that juice is on shelf 1, we can determine that meat and fruit must be on shelves 2 or 5. What we do not know, however, is which is on shelf 2 and which is on shelf 5. So we'll mark them on the diagram in both locations until we learn more.



Now, move on to the fourth fact.

▶ Yogurt is directly above the meat.

This fact is very important in determining the locations of the remaining items. The word *directly* lets us know that the yogurt is on the shelf immediately above the meat. If meat were on the second shelf, yogurt would have to be on the top shelf. But we already know that juice is on the top shelf, which means yogurt must be in a different location, so meat cannot be on the second shelf. Fruit must

be on the second shelf, and meat must be on the bottom shelf. If meat is on the bottom shelf, yogurt will be on the fourth shelf.

Top

1. juice
2. ~~meat~~ or fruit
3.
4. yogurt
5. meat ~~or fruit~~

Bottom

Think about the question we are trying to answer.

On which shelf are the vegetables?

Notice that no facts are given about the location of the vegetables. The only way to determine the shelf on which they are located is the process of elimination. We found which items belong on each of the other shelves, and only the third shelf remains empty. That means vegetables belong on the third shelf.

Top

1. juice
2. fruit
3. vegetables
4. yogurt
5. meat

Bottom

Keep in mind that the position you are looking for may not be mentioned. It is possible that you will need to use the process of elimination to find the correct answer.

Quickly go back and double-check the arrangement against the facts stated.

1. Vegetables are on one of the shelves.
2. Juice is on the top shelf.
3. There are exactly two shelves between the meat and the fruit.
4. Yogurt is directly above the meat.

The arrangement does include vegetables on one of the shelves. Juice is on the top shelf. Exactly two shelves separate the meat and the fruit. Yogurt is directly above the meat. Perfect!

Let's try another one.

The first four acts in the school talent show showcased students playing instruments. The instruments played were the cello, harp, piano, and trumpet.

1) Kaitlyn performed after Malcolm.

2) Trevor played the harp.

3) The student playing the piano was first to perform.

4) Sofia played right before Kaitlyn, but she did not perform first.

5) Trevor performed after the trumpet and cello.

Based on the information, which of the following **must** be true?

A. Kaitlyn played the cello.
B. Trevor was the last to perform.
C. Malcolm performed after Sofia.
D. Sofia played the trumpet.
E. Kaitlyn performed right after the piano.

In this example, there are two relationships that must be determined. First, you must find the order in which the students performed, and second, you must also find which instrument each student played. In this case, you could make a chart with separate columns for all of this information.

	Student	Instrument
First		
Second		
Third		
Fourth		

The specific information given includes:

- Trevor played the harp.
- The student playing piano was first to perform.
- What these specific facts also tell is that since Trevor played the harp, he did not perform first. Now, look at the other facts.
- Kaitlyn performed after Malcolm.

This means that Kaitlyn was not first, and she did not play the piano. This also means that Malcolm was not last. Since this statement does not say Kaitlyn performed immediately after Malcolm, it is possible that another student performed between them.

- Sofia played right before Kaitlyn, but she did not perform first.

If Sofia did not perform first, she does not play the piano either. So we know that Trevor, Kaitlyn, and Sofia do not play the piano. That only leaves Malcolm; he plays the piano, which means he performed first.

Also, if Sofia played right before Kaitlyn, but she was not first, she was either second or third. Kaitlyn would have to have performed third or fourth. If Sofia was second, Kaitlyn was third, and Trevor would have to be fourth. If Sofia was third, Kaitlyn was fourth, and the only spot left for Trevor would be second.

	Student	Instrument
First	Malcolm	piano
Second	Sofia or Trevor	
Third	Sofia or Kaitlyn	
Fourth	Kaitlyn or Trevor	

Now, take a look at the final statement given.

▶ Trevor performed after the trumpet and cello.

As we already determined, Trevor had to be second or fourth. If he was second, he could not have performed after the trumpet and cello, which means he had to have performed fourth.

If Trevor was fourth, Sofia had to have been second and Kaitlyn third. And, since the second statement given told that Trevor played the harp, that means that Sofia and Kaitlyn played either the trumpet or the cello. However, we have not been given enough information to determine which instrument each girl played.

	Student	Instrument
First	Malcolm	piano
Second	Sofia	trumpet or cello
Third	Kaitlyn	trumpet or cello
Fourth	Trevor	harp

Based on the facts given, only one of the answer choices *must* be true.

B. Trevor was the last to perform.

While Kaitlyn *might* have played the cello and Sofia *might* have played the trumpet, we do not have enough information state that either answer choice A or D *must* be true. And, since Malcolm performed first, answer choice C cannot be true because he did not perform after Sofia. Likewise, since Kaitlyn performed right after Sofia, who played either trumpet or cello, she did not perform right after Malcolm played the piano.

Use initials, symbols, or codes to save time. For example, draw a triangle rather than writing out the word *triangle* when determining the relative position of the shape. Write *J* rather than *Josephine* when ordering names to find her relative position.

As you saw with this example, there may be more than one arrangement that fits the given conditions.

	Student	Instrument
First	Malcolm	piano
Second	Sofia	trumpet
Third	Kaitlyn	cello
Fourth	Trevor	harp

	Student	Instrument
First	Malcolm	piano
Second	Sofia	cello
Third	Kaitlyn	trumpet
Fourth	Trevor	harp

Either of these arrangements could have been correct, based on the information given in the question. There may be times on the SHSAT that this happens. But don't worry. You may not need to know the positions of every person or object in order to answer the question.

You will always have enough information to determine the correct arrangements of the people or items in question. Just be careful not to say that something *must* be true if there is more than one possible answer.

You Try It!

In this chapter, you've learned how to solve the types of logical reasoning questions you will have to answer on the SHSAT. Use what you've learned to figure out the code, draw a valid conclusion, identify the correct assumption, and determine the relative positions.

Examples 1 and 2

Refer to the following information: In the code below, (1) each letter always represents the same word, (2) each word is represented by only one letter, and (3) in any given sentence, the letters may or may not be presented in the same order as the words.

C	J	V	G	R	means
"Joshua	gave	books	to	Abigail."	
U	V	K	R	C	means
"Natalia	gave	books	to	Phil."	
R	J	M	V	C	means
"Abigail	gave	books	to	Colleen."	
V	R	U	C	G	means
"Phil	gave	books	to	Joshua."	

1. Which letter represents "Joshua"?

A. J
B. G
C. R
D. U
E. K

2. Which word is represented by the letter M?

A. Abigail
B. gave
C. books
D. to
E. Colleen

Let's check your work. For question 1, we know that Joshua must be represented by one of the words in the first code, since his name is in the first sentence. So, either C, V, G, J, or R must represent Joshua. Since V, C, and R appear in every sentence, these letters must represent either *gave*, *books*, or *to*. So, based only on the code for the first sentence, Joshua must be represented by either G or J.

Now, take a look at the last sentence, which is the only other sentence to include *Joshua*. The letter G is in the code for this sentence, but the letter J is not, so G must represent *Joshua*.

Using the process of elimination, let's rule out all of the other answer choices for question 1 to check your work. We can rule out J because the code for the final sentence does not include J, even though the sentence does include *Joshua*. We can also rule out R because it is included in every code, even those that have sentences without *Joshua* in them. For this reason, R must represent one of the words that appear in every sentence: *gave*, *books*, or *to*. We can also rule out U and K because *Joshua* is in the first sentence, and neither of these letters is. So that leaves G, which is the letter that represents *Joshua*.

For question 2, you probably recognized that M only appears in the sentence "Abigail gave books to Colleen." That means the letter cannot represent any of these words that are also included in any of the other sentences. So, we can rule out *Abigail*, *gave*, *books*, and *to*, since all of these words are in at least one other sentence. That only leaves *Colleen*, so M represents this word.

Example 3

Read the information, then select the answer choice that is a valid conclusion, based only on the information given.

> Bobcats are carnivorous animals. Animals that are carnivorous typically eat only meat. A farmer lives near a forest known to be home to bobcats and deer. The farmer sees animal tracks in his garden and notices that something is eating the squash he planted.

Which of the following statements is a valid conclusion from the statements above?

A. Deer are eating the squash.

B. Bobcats are eating the squash.

C. Deer are not eating the squash.

D. Bobcats are not eating the squash.

E. Bobcats and deer have been in the garden.

Let's see how well you did! While several of the answer choices are possible, there is only one that is a valid conclusion based on the facts that have been given. According to the definition given, carnivores eat meat. Bobcats are carnivores, therefore, they eat meat, not vegetation. So, we can conclude that bobcats are not eating the farmer's squash.

As you know, deer do eat vegetation. But the conclusion must be based only on the facts stated. It is possible that deer are eating the farmer's squash, but it is also possible that rabbits or another animal are the culprits. So we cannot conclude that deer are, or that deer are not, eating the vegetables.

Since we have already determined that bobcats eat meat, it would not be reasonable to conclude that they are eating the squash. It is possible, however, that bobcats and deer have been in the garden since both are known to live in the area, and the farmer has seen animal tracks in the garden. It is also possible that a different type of animal is leaving the footprints, so we cannot conclude that any certain type of animal has been in the garden.

Example 4

Now try this question:

Amanda and Raven both want to play the lead character in the school play.

1) One girl is in seventh grade, and one is in eighth grade.

2) Amanda is in the drama club, but not the chorus.

3) One girl was the lead in last year's play.

4) The seventh grade student is in the chorus, but not the drama club.

5) The eighth grade student was the lead in last year's play.

Based only on the information above, which of the following **must** be true?

A. Raven is not in the seventh grade.
B. Raven is in the drama club.
C. Amanda is in the seventh grade.
D. Amanda had the lead in last year's play.
E. Raven is not in the chorus.

How did you do? Let's look at the facts so we can select the statement that meets all of the conditions given. Reorganizing the facts can be helpful, so we'll group information in a way that makes it easier to recognize the correct assumption. Look at the second and fourth facts.

- Amanda is in the drama club, but not the chorus.
- The seventh grade student is in the chorus, but not the drama club.

By looking at these statements together, we know that since Amanda is in the drama club, she is not the seventh grade student. That means she is in the eighth grade and Raven is in the seventh grade. This means that answer choice C is not true. And if Raven is the seventh grade student, she is in the chorus, but not the drama club. This means answer choices A, B, and E must not be true.

Now take a look at the fifth fact that was given.

- The eighth grade student was the lead in last year's play.

We already determined that Amanda is in the eighth grade. So we can use this information to verify the fact that answer choice D must

be true. It is a correct assumption, based on the facts given.

Examples 5 and 6

Refer to the following information.

A florist arranged four vases side by side in the window of her shop. Each vase held either pink, purple, red, or yellow flowers.

1) The first vase held flowers that were yellow.
2) Exactly two vases were between the orchids and the roses.
3) The carnations were not pink or purple.
4) The purple flowers were in the third vase.
5) The tulips were next to the orchids and the carnations.

In which position were the orchids?

A. first
B. second
C. third
D. fourth
E. Cannot be determined from the information given.

Based on the information, in which vase are the red flowers?

A. first
B. second
C. third
D. fourth
E. Cannot be determined from the information given.

Let's see how well you did! In these questions, you need to find the position of the orchids, and the position of the red flowers. Do not assume that since the two questions ask about red flowers and orchids they are referring to different vases of flowers. It is possible that the orchids *are* the red flowers. Remember, always use the facts to solve the problems. Never assume anything on the logical reasoning questions!

Now, look for specifics to fill in the chart. Consider the first and fourth facts:

- The first vase held flowers that were yellow.
- The purple flowers were in the third vase.

This means that the second and fourth vases contained either pink or red flowers.

Flower	Color
First	yellow
Second	pink or red
Third	purple
Fourth	pink or red

Next, consider the second fact:

- Exactly two vases were between the orchids and the roses.

In order for there to be two vases between these flowers, the orchids and roses must either be first or fourth.

	Flower	Color
First	orchids or roses	yellow
Second		pink or red
Third		purple
Fourth	orchids or roses	pink or red

Consider the third fact:

- The carnations were not pink or purple.

Since the third vase contains purple flowers, the carnations are not third. We also know that the carnations cannot be first or fourth, since these positions are occupied by either orchids or roses. That leaves only the second position for the carnations. And since the vase in the second position must have pink or red flowers, and the carnations are not pink, they must be red. This also means that if the second vase contains red flowers, the fourth vase must have pink flowers.

	Flower	Color
First	orchids or roses	yellow
Second	carnations	~~pink or~~ red
Third		purple
Fourth	orchids or roses	pink ~~or red~~

Next consider the fifth fact:

- The tulips were next to the orchids and the carnations.

In order for the tulips to be next to these other flowers, they must have been third. This also means that the orchids must have been fourth. And if the orchids are fourth, the roses have to be first. So the correct arrangement of the vases is shown below.

	Flower	Color
First	roses	yellow
Second	carnations	red
Third	tulips	purple
Fourth	orchids	pink

So, to answer the questions, the orchids were in the fourth vase, and the red flowers were in the second.

Let's Review!

In this chapter, you've learned what to look for and what steps to take to answer the logical reasoning questions on the SHSAT. Let's take another quick look at some of the ideas to keep in mind for choosing the best answer to these questions.

- The following steps can be helpful in figuring out a code.
  1. Find out which word(s) or letter(s) you need to decode.
  2. Look for common words and letters among the sentences.
  3. Use these to rule out any incorrect letters.
  4. Look for code letters that are repeated among the sentences.
- Once you have figured out the code, use the process of elimination to make sure you have selected the correct multiple choice answer.
- When selecting a valid conclusion, keep the following steps in mind:
  1. Attentively read the information that is given.
  2. Identify all of the facts, information, and definitions given.
  3. Check each of the answer choices against the facts in the statements.
  4. Determine which answer choice is a valid conclusion based solely on the facts.

- Remember that even though an answer choice may be true, it is not a valid conclusion unless information to support it has been given.
- When selecting a correct assumption, choose the statement that *must* be true based only on the facts given.
- Correct assumptions must meet all of the conditional statements.
- Things to remember when identifying the correct assumption are
  - Read all of the information carefully.
  - Do not assume anything is true unless it is explicitly stated.
  - Choose an answer that satisfies all of the conditions given.
  - Select a correct answer based only on the information given.
- When determining relative positions, first find out what you need to solve.
- Creating a visual, such as a diagram, table, or chart, can be helpful in determining the relative positions of people or objects. Begin by filling in any specific information given, then add facts that were stated in the question.
- Regardless of the type of logical reasoning question you are solving, always read every word carefully, and check your answer against the facts given.

CHAPTER 6
Reading Comprehension

As you know, the verbal section of the SHSAT contains a total of 45 questions. Five of the questions are scrambled paragraphs, and ten are logical reasoning. The remaining 30 questions assess your reading comprehension skills. In other words, these questions test how well you understand the information you read.

On the SHSAT, you will be given five nonfiction passages to read. The passages will be between 350 and 450 words each. The topics of the passages may include:

- Biographies
- Historical events
- Science
- Art
- Human interest stories
- Discussions sharing a point of view

As with any reading test, the key is to read the information carefully. After reading each passage, you will be given six multiple choice questions. The first question will always ask you to identify the main idea of the passage. The other questions can be answered based on the facts, details, and information given in the passage, or on inferences that can be drawn based on this information.

You may find that more than one of the answer choices looks correct. Your task is to choose the one best answer, based solely on the information you have read. You may need to combine ideas found in different parts of the text to come up with the best answer to a question.

You also must be careful not to base your answer choice on information you previously knew about the topic. For example, you

might be given a passage to read about the Statue of Liberty. Perhaps you have written research papers on this remarkable monument and have even visited on several occasions. You might even consider yourself an expert on the topic. However, none of the information you have learned elsewhere is to be used to answer the questions on the SHSAT. The one best answer is based only on the facts and details that are included in the passage.

Make sure you separate information you already know about a topic from the ideas you read in a passage. Questions on the SHSAT must be answered based only on the information you are given in the passage.

Most likely, you have already learned a number of reading comprehension skills in the past, so much of what you read in this chapter may be review. This section of the test could be a great place for you to earn points that will increase your overall score on the test. Just make sure you read each word carefully, know exactly what each question is asking, and select the best answer for the questions.

With that in mind, let's go over some of the skills you will need to correctly answer the reading comprehension questions on the SHSAT.

Main Idea and Supporting Details

Every passage you read, whether fiction or nonfiction, has a **main idea**. The main idea, sometimes referred to as the *big idea*, tells what the passage is mostly about. It is the most important information, key concept, or main message that the author wants you to gain by reading the text.

For example, if you were to read a passage about Mount Rushmore, the main idea might be:

> Mount Rushmore is a national memorial that features the faces of several presidents carved into the side of a mountain.

This would be the key piece of information that the author wants you to learn by reading the text. His or her main reason for writing the passage is to tell readers that Mount Rushmore is a national memorial that shows the faces of some of our leaders carved into the side of a mountain.

The main idea may be the author's opinion about the subject matter or a general idea. Be careful not to confuse the main idea of a passage with the **topic**. The topic, or subject, is a single word or short phrase that tells what the passage is about. In the example above, the topic is *Mount Rushmore*. The main idea, on the other hand, is a complete sentence that tells the most important information about the topic. Often, the main idea is introduced in the **topic sentence**. This sentence is usually near the beginning of the text and tells what the passage will be about.

Identifying the topic first can be a helpful way to determine the main idea of a passage. To find the topic, ask yourself, *What or who is the subject of the passage?* Then, to find the main idea, ask, *What is the most important thing the author wants me to know about this topic?*

In a passage that includes several paragraphs, each paragraph will have its own main idea. Often, the main idea of a paragraph is found in the first sentence; however, it can be anywhere in the paragraph. These paragraphs work together to explain and support the main idea of the entire passage as a whole. The main idea of the entire passage will tell the central message that is being explained by all of the paragraphs together. In this case, you will have to read

the full passage, then determine the most important idea that the author wanted to relay.

As you read, you will find **supporting details** that explain, prove, or give more information about the main idea. They help to make the main idea stronger and clearer so that the reader is better able to fully understand its importance. Types of details often used to support the main idea are:

- Examples
- Reasons
- Definitions
- Facts
- Descriptions

In the passage discussed earlier about Mount Rushmore, supporting details could include statements such as those shown below.

> This monument is located in the Black Hills of South Dakota.
>
> The cost of creating the memorial was nearly $1 million, the majority of which was provided by the U.S. government.
>
> The rock carving features 60-foot-high likenesses of Presidents Washington, Jefferson, Roosevelt, and Lincoln.

Each of these details helps readers more fully understand the main idea that *Mount Rushmore is a national memorial that features the faces of several presidents carved into the side of a mountain.* Notice that the supporting details tell more about the main idea, such as *where* the monument is located, *how* much it cost to build, *who* was responsible for funding the project, and *which* presidents are represented. This information helps to explain Mount Rushmore.

Supporting details often answer the questions:

- Who?

- What?
- Where?
- How?
- When?
- Why?

Being able to recognize the main idea and identify supporting details will help you better understand what you read. You will know what message the author is trying to share and be able to organize the information in the passage in a way that helps you understand the topic.

Read the following paragraph. As you read, look for the main idea and supporting details.

> While his name may be unfamiliar to many, Gutzon Borglum was an artist who was responsible for a number of works that have become parts of American history. Born around 1867, Borglum spent his teens and twenties studying art. After painting a portrait of a wealthy general, he was introduced to Theodore Roosevelt. Several years later, the artist created a bust of Abraham Lincoln, which Roosevelt displayed in the White House, giving Borglum a reputation around the country. As a result, he was commissioned to carve his first mountain sculpture, a bust of Robert E. Lee on Georgia's Stone Mountain. Although he was not the artist to complete this project, he did develop techniques that would be useful in his following, and most famous, carving. After leaving what he felt was a project that would only be supported by people in a limited area of the country, Borglum went on to create a masterpiece that he felt would gain national support: carving the faces of four United States presidents into Mount Rushmore.

First, let's find the topic. Ask:

What or who was the subject of the paragraph?

Since all of the information is about the life and work of artist Gutzon Borglum, he is the topic of the paragraph.

Now that we know the topic, let's identify the main idea. Ask, *What was the passage about?* Remember, the main idea is a complete sentence. The key concept that the author wanted us to know is that *Gutzon Borglum was an artist whose works are an important part of American history*. This sentence tells the big idea of the whole paragraph. All of the other information you read gives information, facts, details, and examples that explain the main idea.

Finally, let's find the supporting details in the paragraph that help to build a complete understanding of the main idea. Ask, *What information did the author include that helped to explain the main idea?*

The main idea is that Borglum's work is an important part of American history. Details that explain this include:

- He created a bust of Abraham Lincoln, which Roosevelt displayed in the White House.
- He worked on a carving of Robert E. Lee in the side of Georgia's Stone Mountain.
- He carved the faces of four United States presidents into Mount Rushmore.

As you probably noticed, the main idea is a small part of the passage. The majority of the passage is made up of supporting details. As you read, think about how each supporting detail tells more about the main idea. You will probably find that some supporting details are more informative and helpful than others.

Some questions on the SHSAT may ask you to determine which statement best tells what the passage is about, but will not actually include the words *main idea*. In this case, you are still looking for the answer choice that tells the general point the author was trying to make throughout the passage. Select the answer choice that

most closely states the main idea, since the main idea tells what the passage is about.

Fact and Opinion

As you already know, **facts** are true statements that can be proven. **Opinions** are someone's personal views or thoughts, which may or may not be able to be verified.

The average rainfall in June is 6.8 inches.

There was too much rain in July.

This was the wettest summer on record.

All of these statements tell about summertime rain; however, only two of the statements can be verified. The first and third statements could be proven using rainfall data from weather reports. Therefore, these statements are facts. The second statement is an opinion, because we cannot prove that there was too much rain. A statement that says that there was too much, too little, or just the right amount of rain expresses a personal viewpoint. Not everyone would agree that there was too much rain. Some people might have enjoyed the wet weather and believe that July had the perfect amount of rain.

As you read, it is important to be able to distinguish between facts and opinions. Since all of the passages on the SHSAT are nonfiction, the majority of the information included will be facts. But it is still possible that an opinion will occasionally appear in the text, especially when the purpose of the passage is to share a point of view.

As you know, facts can be used as supporting details. While an opinion may agree with the main idea of a passage, it will not be the strongest statement supporting the main idea.

As you read the following paragraph, look for facts and opinions. Notice which statements best support the main idea of the text.

Not only was Wolfgang Amadeus Mozart a renowned composer, he was also an accomplished musician who mastered a number of instruments. Mozart's father, Leopold, was a composer and violinist. By the time he was three, the younger Mozart had learned to play the piano. Within a short time, he had also mastered the harpsichord, organ, and violin. By the time he was six years old, he performed concerts for European royalty on these instruments, which his father had taught him to play. Later in life, he wrote a number of piano concertos, which allowed him to be recognized as both the composer and soloist when performing. Many of these concertos featured the piano and wind instruments of the orchestra working together to create some of his most beautiful compositions.

Which fact from the passage best supports the main idea?

A. Wolfgang Amadeus Mozart, composer and musician
B. Mozart's father, Leopold, was a composer and violinist.
C. Within a short time, he had also mastered the harpsichord, organ, and violin.
D. Not only was Wolfgang Amadeus Mozart a renowned composer, he was also an accomplished musician who mastered a number of instruments.
E. Many of these concertos featured the piano and wind instruments of the orchestra working together to create some of his most beautiful compositions.

Let's look at each of the answer choices and see how well you did identifying the fact that best supports the main idea. You probably recognized that answer choice A is the topic rather than a supporting fact.

Now, take a look at answer choice B. This is a fact. It can be verified by checking the information in a biography or a book about musical history. However, even though this fact is in the passage, it does not support the main idea. It is definitely interesting, and it offers insight into Mozart's family, but it does not help to explain the main idea of the passage.

Answer choice D is the main idea. This piece of information is the key concept that the author wanted to share when he or she wrote the paragraph. In this question, we are looking for the fact that best supports this statement.

Answer choice E is an opinion. This statement shares the author's views of the concertos. Not everyone would agree that these are the most beautiful of Mozart's compositions. This information does help to support the main idea in that it offers insight into the composer's talent as an accomplished musician and pianist. However, we are looking for the *fact* that best supports the main idea.

Let's go back to answer choice C. This statement is a fact that could also be verified through a quick look at a biography or musical history book. This statement also supports the main idea. So, as you probably recognized, this is the correct answer to the question.

Certain words may offer clues that a statement contains an opinion. Look for words such as *better*, *best*, *worst*, *beautiful*, *awful*, *easiest*, *too difficult*, *too much*, *terrific*, and *should*. Often, these indicate a personal viewpoint, rather than a verifiable fact.

Inferences

Suppose you come home an hour past your curfew. Mom is standing in the doorway with her eyebrows furrowed, arms folded across her chest, foot quickly tapping on the ground. Without her ever saying a

word, you can read between the lines to figure out that she's angry. You have just made an **inference**.

There are times when an author expects you to read between the lines as well. He or she will make suggestions, give information, or offer clues about something but never directly state what it is that he or she wants you to know. Readers have to make an inference, or read between the lines to discover information that is implied or suggested in the passage.

When making an inference, be sure to reflect on:

- Clues and hints in the text
- Details and information given
- What the author does and does not say
- The word choice of the author
- Your own prior knowledge and observations

An inference is not a random, wild guess that is made out of the blue. It is a logical conclusion based on information that is suggested or implied. Take a look at the following example.

> Rebecca put down her pompons and nervously climbed to the top of the human pyramid, trying to put aside her fear of performing in front of a stadium full of football fans.

You've probably figured out that Rebecca is a cheerleader. The following facts are all clues that would lead you to this inference:

- She has pompons.
- Rebecca is climbing to the top of a human pyramid.
- She is performing in a stadium full of football fans.

Let's look at another example:

> Every Friday night, the football stadium is packed with high school students anxiously watching their aggressive peers toss

> the ridiculously shaped ball to each other while running from a herd of oversized, helmet-wearing thugs who want nothing more than to pummel them into the ground.

What does the author's word choice in this sentence suggest? Words and phrases such as *aggressive peers, ridiculously shaped ball, helmet-wearing thugs*, and *pummel them into the ground* suggest that the author is not exactly a football fan. Often, an author's word choice can be used to make inferences about his or her feelings on the topic. Based on this sentence, it would be logical to infer that the author does not like football and probably does not attend these games.

Sometimes, what an author does *not* say can offer information as well. Read the paragraph below, then choose the best inference.

> Four exciting touchdowns clinched the win for the Northside Mustangs last night! Number 24, freshman Christopher Young, intercepted a pass by the Tigers, and the crowd was on their feet! Young barreled more than 40 yards down the field, dodging and outmaneuvering his opponents, to score the team's first points of the night. Undoubtedly, cheers could be heard for miles! In the second half of the game, Darnell Thompson carried the ball into the end zone, adding another touchdown to the Mustangs' score. Jonathan Atkins also scored two touchdowns in the final quarter.

Which inference could be made, based on the passage?

A. The author is a fan of Christopher Young.
B. Most of the players on the team are freshmen.
C. The passage was written by the coach of the Mustangs.
D. Darnell Thompson is the most valuable player on the team.
E. The author feels that Jonathan Atkins is the most valuable player.

You were probably able to infer that the author is a fan of Christopher Young. The author included information such as Young's jersey number and class, as well as details about the touchdown he made, but did not mention this information about any of the other players. By reading between the lines, you could logically conclude that the author thinks highly of Young.

Let's consider each of the other answer choices and determine why these would not be the best inference. Although Young is a freshman, there is no evidence to indicate that any of the other players are in the same class, so answer choice B is not correct. And while the paragraph may have been written by the coach, this is not a logical inference, since there is no evidence to imply who wrote the passage. Therefore, answer choice C is not correct. Darnell Thompson did make a touchdown; however, nothing suggests that he is more valuable than any of the other players, so answer choice D is not the best inference. And while the fact that he made two touchdowns in the final quarter might suggest that Jonathan Atkins is a valuable member of his team, the author did nothing more than mention him in the paragraph. Since the author did not say much about Atkins, we cannot infer that he or she feels that this is the most valuable player, despite the number of touchdowns he scored in the game.

So, for whatever reason, the author appears to believe that Christopher Young and the touchdown he made were more important than the other players and the points they scored for the team.

Drawing Conclusions

In many ways, **drawing conclusions** is similar to making inferences. To draw a conclusion, you must look at the information given and make a reasonable guess about what is happening.

Suppose you walk past a hospital. A happy young man is carrying a fistful of pink balloons that read "Congratulations." In his other hand, he has a gift bag with a pink teddy bear sticking out of it and a

bouquet of flowers. Beside him, a nurse is pushing a smiling young woman in a wheelchair. The woman has a hospital band on her arm and is gently cradling a tiny bundle wrapped in a pink blanket. Using the information you have gained through observation, as well as what you already know, you can logically conclude that this couple has just welcomed a baby girl into their lives.

> His use of light and shadow causes the images to come to life on the canvas. The uniqueness of the brushstrokes gives each piece a distinctive quality that is only found in his artistic style.

Let's look at the facts and details mentioned in the paragraph:

- Use of light and shadow
- Images come to life on the canvas
- Brushstrokes
- His artistic style

By looking at the information given, and using what you already know, you were probably able to conclude that the author is referring to an artist. And since the artist uses a canvas and brushstrokes, it would be logical to in turn conclude that the artist being discussed in this instance is a painter.

Now it's your turn. As you read the following sentences, look for information that offers clues about the passage. Combine these clues with your own prior knowledge and draw a logical conclusion about the text.

> The structure of Russia's Kizimen has been compared to Mt. St. Helens in the United States. It is approximately 12,000 years old and nearly 8,000 feet high, and it is made of layers of ash, lava, and rocks from earlier eruptions. Although ash emissions have been recently recorded, the only explosive eruption in recorded historical time occurred in 1928.

Which statement best describes Kizimen?

A. It is a volcano.
B. It is a mountain.
C. It is nearly 100 years old.
D. It erupts on a regular basis.
E. It is older than Mt. St. Helens.

Let's take a look at the clues in the paragraph that offer information about Kizimen.

- It has been compared to Mt. St. Helens.
- It is made of ash, lava, and rocks from earlier eruptions.
- There have been recent ash emissions.
- There was an explosive eruption in 1928.

Now, let's think about the prior knowledge you probably already have related to this topic.

- Mt. St. Helens is a volcano.
- Ash, lava, and rocks come from volcanoes.
- Volcanoes erupt.

Based on the information given as well as your own prior knowledge, it is likely that you were able to conclude that Kizimen is a volcano. You may have figured that out without too much thought. In fact, when you read the question and answer choices, you might have even thought that answer choice A was actually stated in the paragraph. Notice that the word *volcano* is not used in the passage at all. If you knew that Kizimen is a volcano, it is because you drew that conclusion. Good for you!

Context Clues

The purpose of reading is to gain understanding of the passage. In order to understand the passage, you have to understand the meanings of the words. Occasionally, when you are reading, you will come across a word that is unfamiliar. Sometimes you can grab a dictionary to find the definition. But this is not always convenient, or even possible. For example, if you find a word you do not know on the SHSAT, you will not have access to a dictionary, so you'll have to find another way to determine the word's meaning.

Often authors will include **context clues**, which are hints that can help you determine the meanings of words. Types of clues that can be helpful in figuring out meaning include

- Synonyms
- Antonyms
- Definitions
- Explanations
- Examples

As you know, **synonyms** are words that have the same or similar meanings, such as *small* and *tiny*.

> Following the merger of the companies, Miss Winchester is hoping to retain, or keep, her current position.

In this sentence, the synonym *keep* is included to explain the meaning of the less familiar word, *retain*. By including this context clue, the author is helping readers to understand the text.

Antonyms, or opposites, are another type of context clue that can help readers determine meaning. Consider this example:

> All of the community members' concerns, whether petty or important, were addressed during the city council meeting.

In this case, the words *petty* and *important* are stated as being words with different meanings. The sentence tells that the concerns were either *petty* or *important*, so these words must mean different things. We can determine that *petty* must be the opposite of *important*. So *petty* means “something that is unimportant, insignificant, or trivial.”

An author may include a **definition** that states the meaning of a word that he or she feels could be unfamiliar to readers:

> Lola’s exuberant reaction to the election results proved that she was full of enthusiasm about being the new class president.

In this example, the definition of *exuberant* is found later in the same sentence when the author explains that Lola was full of enthusiasm. A reader who was unfamiliar with the word would be able to tell what type of reaction Lola had to the news. Similarly, an **explanation** may also be included to clarify a word’s meaning:

> When he attends college, Samuel plans to take classes in horticulture to learn about the science of growing plants.

Reading the beginning of this sentence, you might wonder what Samuel will study in horticulture classes. Then the second half of the sentence explains that he will learn about the science of growing plants in these classes. This explanation tells readers the meaning of *horticulture*.

Examples are also used to clarify the meaning of words that readers might find unfamiliar:

> The types of organisms, such as plants and animals, that are found in a habitat vary depending on the environment.

In this sentence, plants and animals are given as examples of types of organisms. From this, readers can figure out that organisms are living things.

Often, context clues are found within the same sentence as the word they are helping to explain. Other times, they may be found in the surrounding sentences. As you read, be sure to look at information in sentences before and after the unfamiliar word in order to determine what the word means. Take a look at the following sentence.

> It generally takes three business days for a requisition to be processed. Department managers must approve all official written requests for needed materials prior to the supplies being ordered.

What is the meaning of the word *requisition*? By reading the first sentence, we can tell that it is something that must be processed. Context clues in the second explain that this is an official written request for needed materials.

Now you try it. Read the sentences below, then use context clues to help you answer the question.

Some may find the work of the researcher to be provincial. However, experts in the field disagree with this claim and state that his latest journal is not at all unsophisticated and simple. He uses scientific facts as well as his own investigations and experiments to explain which outcomes are plausible and which are unachievable.

Which is the meaning of *provincial*?

A. expert
B. investigations
C. researcher
D. simple
E. sophisticated

Did you choose answer choice D? Great! The second sentence uses the definition “unsophisticated and simple” to explain the meaning of *provincial*. Pay close attention when you read! The paragraph says

that some find the researcher to be *provincial*. Then, it states that experts disagree; his work is *not* unsophisticated and simple, as others claim. In other words, some think it is unsophisticated and simple. These are the people who said it is provincial. Be careful not to overlook words such as *however* and *disagree* that point out an alternate point of view.

Let's try another one. Answer the following question, based on the same paragraph.

Which has the same or nearly the same meaning as *plausible*?

A. impossible
B. outcome
C. reasonable
D. result
E. unachievable

This question takes a little more thinking. You have to use context clues to determine the meaning of *plausible*, then figure out which of the answer choices has a similar meaning. First, let's find the meaning of *plausible*. The paragraph states that the researcher explains "which outcomes are plausible and which are unachievable." This sentence includes antonyms to explain the two types of outcomes: plausible and unachievable. So, *plausible* is the opposite of *unachievable*. *Plausible* means achievable, or possible.

Which of the answer choices could explain an outcome that is achievable or possible? *Impossible* and *unachievable* are opposites of *plausible*. *Plausible* is an adjective, so its synonym will also be an adjective. *Outcome* is a noun from the paragraph, and it does not have the same meaning. *Result* is a noun that means the same as *outcome*. An outcome that is plausible would be reasonable. *Reasonable* is also an adjective, and it has a meaning that is similar to that of *plausible*. So the correct answer is C.

Multiple-Meaning Words

As mentioned earlier, in order to be able to understand what you read, you must know the meaning of the words. As you are aware, some words have more than one meaning. For example, think about the meaning of the word *breach*. You might have thought that *breach* means “failure to obey or preserve something.” Or you may have thought that a *breach* is “a gap or hole.” Maybe you thought that the meaning of *breach* is “the leap a whale makes out of the water.” Actually, all of these are correct definitions. But in order to comprehend text that includes multiple-meaning words, you must know which definition the author has in mind. Think about how the word *breach* is used in the following sentence:

> Dr. Thompson’s discussion of Kylie’s condition was a *breach* of confidentiality.

The only way to know which meaning is correct is to consider how the word is used. In this example, the doctor’s discussion did not create a hole, nor did any large mammals leap out of the water, so these are not the correct definitions. The discussion did, however, fail to preserve the confidentiality between a doctor and his patient.

Read the following sentence. Look at how *faculty* is used in the sentence to determine which of the word’s meanings is correct.

> The principal expected Mrs. Ramirez, the Spanish teacher, to attend a meeting after school with all of the other members of the school’s *faculty*.

In this sentence, the principal expects a teacher to attend a meeting. The school’s *faculty* will be at this meeting. Using clues in the sentence, we can determine that in this case, the correct meaning of *faculty* is “the teaching staff at a school.”

Now, consider the following sentence containing the word *faculty*.

> One reason Ling does so well on tests is that she has a great faculty for memorizing facts quickly.

In this sentence, *faculty* does not refer to a teaching staff. The meaning of the word in this case is “a capacity or ability.”

As you read, make sure to look at how the words are used, and pay attention to context clues in order to determine which definition of a multiple-meaning word is being used. Assuming the incorrect definition of a word can hinder you from fully comprehending the passage.

Read the sentence below. Look at how the italicized word is used to help you determine which meaning is correct.

> Following the storm, a contractor was asked to give a *quote* for repairing the damage to Mr. Patel’s roof.

Which is the correct meaning of *quote* in this sentence?

A. to copy someone’s exact words
B. reuse of art or written material
C. to refer to something as support or proof
D. an estimate of the cost for providing a service
E. written marks used to indicate someone else’s words

The sentence tells that the contractor gave a quote for repairing the roof. Although all of the answer choices give definitions for *quote*, only answer choice D makes sense in the context of the sentence. He would have given an estimate of the cost for making the repairs.

Remember, the part of speech can be helpful in determining which of a word’s meanings is being used. In the sample question, *quote* is a noun, so the correct meaning must also be a noun. Since answer choices A and C give definitions of verbs, these are not correct.

Summarizing

Think about the last research paper or book report you wrote. In order to complete the project, you probably read information in books or online, then restated the most important points using your own words. What you did was **summarize**.

Being able to summarize what you have read is one way to demonstrate how well you understood the text. To summarize, you have to focus on the main idea and most important supporting details, then state or explain them in your own words. On the SHSAT, you may be asked to choose a statement that best summarizes the passage you have read. The answer choices will probably all include facts from the passage. Some of the choices might even summarize a particular paragraph. However, the best summary will focus on the most important information from the entire passage.

Read the following paragraph. As you read, look for the main idea and most important details. Then, select the statement that best summarizes the passage.

> Hurricanes are large storms that swirl around a center, or eye. These storms have maximum sustained winds of at least 74 miles per hour. At this speed, a hurricane is classified as a Category 1, the slowest, weakest type of hurricane. The strongest of the storms are classified at Category 5 and have winds exceeding 155 miles per hour.

These powerful weather systems form over the warm waters of the ocean, which provide the energy needed for the storms to strengthen. Typically, the surface temperature of the ocean water must be at least 79 degrees Fahrenheit to be favorable for the growth of a hurricane. Hurricanes rapidly weaken once they have moved onto land. The rain bands, which are the outermost part of

the storm, may reach for hundreds of miles and contain thunderstorms and the possibility of tornadoes.

Which statement best summarizes the passage?

A. Ocean waters must generally be at least 79 degrees Fahrenheit in order for a hurricane to form.
B. Hurricanes form and strengthen over warm ocean waters, but quickly weaken once they are over land.
C. Rain bands are the outermost part of a hurricane, and the thunderstorms can reach hundreds of miles.
D. There are five classifications of hurricanes, with Category 1 being the weakest and Category 5 being the strongest.
E. Hurricanes are strong, swirling storms that form over warm ocean waters and have winds of at least 74 miles per hour.

To select the best summary, first identify the main idea:

Hurricanes are large storms that swirl around a center, or eye.

Then, identify the most important supporting details:
- Hurricanes have winds of at least 74 miles per hour.
- Hurricanes form over the warm waters of the ocean.

Now, determine which of the answer choices includes this information. Answer choices A and C are supporting details from the passage. Answer choice B summarizes the second paragraph; however, the correct answer for this question must summarize the entire passage. Likewise, answer choice D summarizes the first paragraph. If you selected answer choice E, you found the best summary of the passage!

Be sure you read each question carefully. You may be asked to choose a statement that summarizes the entire passage, or you

may be asked to choose a statement that summarizes a specific paragraph. Make sure you know exactly what you are being asked to find.

Synthesis

Sometimes, in order to fully comprehend a passage, you will need to consider information given in different parts of the text. **Synthesis** means combining information from various parts of a passage, or from different passages. Let's consider the passage about hurricanes. Suppose you wanted to find the answer to the following question:

> What wind speeds would you expect to find in a Category 1 hurricane that has already reached land?
>
> **A.** 73 mph
> **B.** 95 mph
> **C.** 101 mph
> **D.** 150 mph
> **E.** 164 mph

Take a look at information found in different parts of the passage. The following sentences are from the first paragraph:

> These storms have maximum sustained winds of at least 74 miles per hour. At this speed, a hurricane is classified as a Category 1, the slowest, weakest type of hurricane.

This statement is from the second paragraph:

> Hurricanes rapidly weaken once they have moved onto land.

By synthesizing this information, we know that a Category 1 hurricane has winds of at least 74 mph, but that the storm would weaken quickly once it moves onto land. That tells us that the winds would be slower after the storm has reached land, and that answer choice A would be the most likely wind speed.

As you read, be sure to consider each piece of information as it relates to the rest of the passage. Often, the facts and bits of information will work together to help create a full, complete understanding of the text.

Organizational Structure

When an author writes a passage, he or she arranges the ideas in a particular order, depending on the type of information being included. Not only does this help the information to make sense, but it also helps readers to follow the thoughts and best understand the text.

Organizational structure refers to the way ideas are arranged in a passage. Types of organizational structures that authors often use are listed below.

- Sequence
- Order of importance
- Cause and effect
- Compare and contrast
- Problem and solution
- Description

Being able to recognize organizational structure can help you to make better sense of the material you read. You will be able to mentally organize information as you read it, and anticipate what type of information might be presented in the text. For example, if you are reading a cause and effect passage, you can expect to find events that happen as a result of other occurrences. If you are

reading a passage that uses a problem and solution structure, you will know to look for possible ways to solve a given problem.

Recognizing the organizational structure can also help you to determine the types of questions you might be asked about the passage. If you read a passage that compares and contrasts two countries, for example, you can assume that you will be asked to identify similarities and differences between the places you read about. Knowing this, you can look for the answers to the anticipated questions as you are reading.

Let's review the types of organizational structures you might find on the SHSAT.

Sequence

Authors may choose to use sequence to organize the information in a passage. In this case, ideas and events are arranged in numerical or chronological order. Sequence may also be used to indicate a particular order of steps in completing a process, such as when giving directions or explaining how to do something. Some signal words that indicate a sequential structure are listed below.

after	meanwhile
before	next
during	preceding
finally	second
first	soon
following	then
immediately	today
initially	until
later	when

Dates may also be used to indicate a sequential text structure. Take a look at the sequential paragraph below. Notice the signal words that help readers to recognize the chronological order of the events.

Initially, the United States began with 13 colonies. *During* the early years of our country, the number of states grew quickly. *By 1900,* a total of 45 states were a part of the U.S. *Within the first few years of the twentieth century,* three more states were added: Oklahoma *in 1907* and New Mexico and Arizona *in 1912.* Approximately 47 years *later,* Alaska became the forty-ninth state in *1959,* with Hawaii *following* only a few months *later* to bring the total to the 50 states that are part of our country today.

Order of Importance

Authors may also present ideas in the order of importance. The author may explain the most important idea first, followed by the second most important, and so on, ending with the least significant of the set.

Veterans Day is a day on which we remember all of those who have served our country and honor them for their patriotism and sacrifice. This holiday is celebrated in the United States each year on November 11, the anniversary of the day that World War I fighting ceased. For several years, observation of the holiday was moved to a Monday, like Memorial Day and Presidents' Day; however, the November 11 date was reestablished as Veterans Day in 1978.

In this paragraph, the explanation of Veterans Day contains the most important facts, and it is stated at the beginning. This strategy introduces the strongest facts first and captures the attention of the readers. The ideas that follow are of lesser importance.

On the other hand, the author may begin by explaining the least important idea and build to the idea that he or she feels is most important. This strategy leaves readers with the strongest idea in their minds when they finish reading.

For several years, Veterans Day was observed on a Monday, like Memorial Day and Presidents' Day; however, November 11

was reestablished as Veterans Day in 1978. The holiday is celebrated in the United States on this day each year because it is the anniversary of the day that World War I fighting ceased. Veterans Day is a day on which we remember all of those who have served our country and honor them for their patriotism and sacrifice.

Notice that this paragraph is arranged with the least important fact first, and the most important fact making a final impact on readers.

Cause and Effect

As you know, a *cause* is what makes something happen, or the reason another event takes place. This resulting event is an *effect*.

If you study and answer the practice questions for the SHSAT, you'll be better prepared for the test.

In this example, studying and answering the practice questions are the causes; being prepared for the test is the effect.

When an author uses a cause and effect organizational structure, the causes of an event and the resulting effects are presented in the passage. The text explains how facts, ideas, or events happen or exist because of something else. Words that often signal cause and effect relationships include those in the following list.

accordingly	if . . . then
as a result	in order to
because	leads to
consequently	so that
due to	therefore
for this reason	thus
hence	

A single cause may have more than one effect. Likewise, a single effect may have more than one cause.

Keep in mind that these signal words may not be included to alert you to every cause and effect relationship in a passage. You may have to determine how events are connected based on the facts and the context of the reading.

When you recognize a cause and effect structure in a passage, be on the lookout for ways that one event impacts others, as well as for reasons that events take place. Being able to recognize how events are related will help you anticipate what information will be important, and help you to fully understand the reading. Look at the example below, and determine the cause and effect.

> Abby's current school only includes grades six through eight, so she will be attending a new school next year for ninth grade.

In this example, the word *so* signals a cause and effect relationship. The fact that her current school only goes through eighth grade is the cause, or the reason, she will attend a new school for ninth grade. Attending the new school is the effect. Now, read the following example.

> Abby will be attending a new school next year for ninth grade because her current school only includes grades six through eight.

This sentence explains the same cause and effect relationship; however, the effect is stated first, and the cause is stated afterward. As you can see, either the cause or the effect can come first. Look for signal words and clues about how events are related in order to determine which is the cause and which is the effect.

Compare and Contrast

When an author compares and contrasts, he or she explains similarities and differences between events, concepts, objects, places, or people.

> John F. Kennedy, like Ronald Reagan, was a twentieth-century U.S. president. Unlike Reagan, however, Kennedy was a member of the Democratic Party, whereas Reagan was a Republican.

This example compares ways in which the two men were alike: both were presidents. It also contrasts differences between them: Kennedy was a Democrat, and Reagan was a Republican. When you recognize a compare and contrast organizational structure, look for similarities and differences that can help you fully understand the topic. Some signal words that may be used to indicate this structure are shown below.

although	in contrast
as opposed to	instead of
as well as	nevertheless
both	on the other hand
but	similar to
different from	similarly
however	unlike
in common	yet

There are different ways that authors may arrange texts using a compare and contrast format. Authors may completely discuss one of the subjects, then completely discuss the other. For example, an author might present all of the information about John F. Kennedy, then present all of the information about Ronald Reagan.

On the other hand, the author may compare and contrast the two subjects one characteristic at a time. He or she might tell which state Kennedy was from, then tell which state Reagan was from. The author might then discuss the political affiliation of each of these presidents. Then, he or she might present the most significant achievements of each man.

Now, read the paragraph below. Look for similarities and differences between the topics, and find any signal words that indicate ways in which the two are alike or different.

> Pointillism is an artistic style in which artists, such as Seurat, create paintings using only dots. When seen from a distance, the dots blend together to form the intended colors and images. Surrealism is also an artistic style used by painters, including Dali. However, this style was not intended to provide viewers with a realistic image, but rather a dreamlike creation that does not make logical sense.

You probably recognized that pointillism and surrealism are both artistic styles. The word *also* is used to indicate the similarity between the subjects. You probably also recognized a few differences, such as the fact that Seurat used pointillism, and that the works are created using dots that form the realistic images. Surrealism was used by Dali, and the images are dreamlike and do not make logical sense. Signal words such as *however* and *but* indicate that differences between the styles are being presented.

Problem and Solution

When a problem and solution organizational structure is used, a problem is stated, then one or more solutions to problem are presented. As you read passages with this structure, you can anticipate that possible solutions will be included and can be on the lookout for them.

> Each year, approximately 1,500 people are stung by stingrays in the United States. While most of these cases are not serious, it is possible for the sting to be fatal, and even minor stings can be extremely painful, nonetheless. Since stingrays are often found buried in the sand, beachgoers should shuffle their feet as they enter the water in order to scare the animals away and avoid stepping on them. Divers should avoid hovering above stingrays, which not only can cause the marine creatures to feel trapped, but also places the diver in the ray's strike zone. Also, anyone spending time in the water should be aware of the location of stingray habitats and exercise caution in these areas.

What problem and solution(s) did you notice in this paragraph? You probably recognized that the problem is people being stung by stingrays. After this issue was introduced, several possible solutions were presented: beachgoers should shuffle their feet, divers should not hover above the animals, and anyone near the water should be careful near stingray habitats.

Description

As you might expect, a description describes a topic. The author introduces the topic, then describes it by listing characteristics, attributes, features, or examples. Readers who recognize this organizational structure know to look for details that will help to explain or describe the topic. Signal words to look for in this type of structure are listed below.

above	between	near
along	for example	onto
appears to be	for instance	over
as in	including	such as
beside	looks like	under

Built in 1672, the Castillo de San Marcos is the oldest masonry fort in the United States. Located in St. Augustine, Florida, near the Atlantic Ocean, this fort was built using coquina, a type of stone that was formed from tiny coquina clam shells, and a mortar made from oyster shells, sand, and water. The walls of the fort are 12 feet thick in most places, and 19 feet thick on the sides facing the harbor. The fort, once surrounded by a moat, has two drawbridges, rooms with vaulted ceilings, and three fireplaces, originally used for heat, light, and cooking. On each corner of the fort is a four-sided structure in a shape that looks like a diamond. These allowed soldiers to fire in any direction, if needed. A courtyard in the middle of the fort was once used to protect the townspeople during attacks.

Notice that the first sentence in this paragraph introduces the topic, the Castillo de San Marcos. This is then followed by several features and attributes that explain the fort to readers.

You Try It!

In this chapter you've reviewed a number of reading comprehension skills. Now it's time to put these skills into practice. Use the skills you have learned to select the correct answers to the questions that follow the passage below.

> Arizona's well-known Grand Canyon was created by the flow of the Colorado River; however, another significant U.S. chasm was also the result of many years of extensive erosion. Waimea Canyon, located on the west side of the Hawaiian island of Kauai, is known as "The Grand Canyon of the Pacific." Although it is smaller than the better-known gorge in Arizona, this geological wonder is 14 miles long, one mile wide, and in excess of 3,600 feet deep, making it the largest canyon in the Pacific Ocean.
>
> The island of Kauai is nearly 6 million years old and was originally a volcano. Thousands of years ago, the flood waters and rivers flowing from Mount Wai'ale'ale carved Waimea Canyon, which today is the home of the Waimea River. Weathering and erosion continue to break down the ancient rocks within the canyon, revealing layers of red lava from early volcanic eruptions, and carry these bits of rock into the nearby ocean. This process causes the river water to appear red, giving the name *Waimea* to the river and the canyon: *Wai* means "fresh water," and *mea* means red.
>
> In 1792, an English sea captain, George Vancouver, introduced goats to Kauai. Since that time, the animals have helped to accelerate the erosion of the walls of Waimea Canyon. Today, goats are wild in the canyon. Between wind, water, and

these four-legged creatures, layers upon layers of ancient rock have been revealed along the sides of the canyon. The multicolored cliffs not only include red hues from the lava, but also green and gold from the plant life growing along the sides of the canyon. Visitors to Waimea Canyon can observe this magnificent gorge from three scenic lookout areas. The most striking views occur during the early sunrise hours.

Access to Waimea Canyon allows visitors to get an up-close view of the natural result of erosion, from top to bottom. The road that leads into the canyon reaches nearly 4,000 feet above sea level. Waimea Canyon sits inside of Koke'e State Park, which includes 45 miles of hiking and horse trails, some of which lead to the floor of the valley. One trail in particular, Kukui Trail, offers hikers and horseback riders a two-mile, steep path from an elevation of 2,700 feet down to the bottom of the canyon. From the valley, these visitors can follow the Waimea River out to the Pacific Ocean to see firsthand the path taken by the red waters that carry rock away from the lava-layered walls of the canyon.

1. Which of the following best tells what this passage is about?

A. the history of the Waimea River
B. how erosion created Waimea Canyon
C. hiking and horseback riding in Waimea Canyon
D. the introduction of goats to the Hawaiian Islands
E. the effect of the Colorado River on the Grand Canyon

Remember, the main idea tells what a passage is mostly about. In order to answer question 1, we need to first identify the main idea. In this case, the main idea of the passage is that *Waimea Canyon is a large canyon on Kauai that was created by weathering and erosion of the layers of an ancient volcano*. So the answer choice that best tells what the passage is about will reflect the information in the main idea. You probably recognized that answer choice B best accomplishes this.

So, what makes the other answer choices incorrect? Let's find out. While the Waimea River and the Grand Canyon were mentioned in the passage, neither of these is what the passage is mostly about, so answer choices A and E are incorrect. Answer choice C does tell what the final paragraph is mostly about; however, it does not reflect the main idea of the entire passage. This is a detail that supports the main idea but is not the main idea itself. Similarly, answer choice D is discussed in the third paragraph but is not the topic of the passage as a whole.

Generally, there is not a single correct way to state the main idea. In this case you might have said that the main idea was *Waimea Canyon is a large canyon on Kauai that was created by weathering and erosion of the layers of an ancient volcano.* On the other hand, you might have said the main idea was *Waimea Canyon is the "Grand Canyon of the Pacific" and is the result of many years of wind and water washing away the layers of lava and rock*. Either one of these is correct. Both explain the same basic idea, just in different words.

---

2. Which of the following best summarizes the second paragraph?

F. Waimea Canyon contains many layers of lava rock because the island on which it is found was once a volcano.

G. Waimea Canyon is 14 miles long, one mile wide, and in excess of 3,600 feet deep, making it the largest canyon in the Pacific Ocean.

H. Kauai was once a volcano, so water flowing through and eroding the canyon is red, giving Waimea Canyon a name meaning "red waters."

J. Goats, wind, and water work together to erode the walls of Waimea Canyon, revealing layers of red, which are covered by green and gold plants.

K. This process causes the river water to appear red, giving the

name Waimea to the river and the canyon: *Wai* means “fresh water,” and *mea* means “red.”

As you know, a summary restates the main idea and most important details from a passage. Answer choice F is the main idea of the paragraph and is not a summary of the text. The correct answer will include information from this answer choice, though. Answer choice G contains information found in the first rather than the second paragraph. Also, the majority of this sentence was copied directly from the passage. A summary should restate, or use different words, to give the information. Answer choice J contains the main idea and important details from the third paragraph, and it summarizes that section of the passage rather than the second paragraph. Answer choice K is a fact from the second paragraph but does not summarize all of the information. Also, it directly copies a sentence from the passage rather than restating the ideas. Answer choice H includes the main idea and most important details from the second paragraph. This information is restated, rather than copied, making this the correct answer choice.

3. In what ways are the Grand Canyon and Waimea Canyon similar?

A. Both are the same size.
B. Both are the result of water flow.
C. Both contain plant and animal life.
D. Both offer hiking and horseback trails.
E. Both were created from layers of lava.

In order to answer this question, you had to synthesize facts stated in the first and second paragraphs. By combining information from different locations, you were probably able to determine that both canyons were created by rivers, making answer choice B the correct answer.

The first paragraph explains that Waimea Canyon is much smaller than the Grand Canyon, making answer choice A incorrect. And while plants, animals, and trails may be found in the Grand Canyon, these facts are not mentioned in this particular passage, so answer

choices C and D cannot be the correct choices. Remember, you must base your SHSAT answer choices solely on information given in the passage. Answer choice E must also be incorrect because the passage does not state whether or not the Grand Canyon was created from layers of lava.

4. Which inference can be made based on the passage?
F. Erosion is often accelerated by animal life.
G. Kauai is the oldest of the Hawaiian Islands.
H. Waimea Canyon is the only canyon in the Pacific.
J. The Grand Canyon is more than 3,600 feet deep.
K. More visitors go to the Grand Canyon than Waimea Canyon.

Keep in mind that an inference must be based on information that is stated. Since there is no mention of how often animals affect erosion, answer choice F cannot be correct. The ages of the other islands of Hawaii are not mentioned, nor is there any mention of the number of visitors to either canyon, making answer choices G and K incorrect as well. We do know that Waimea is the largest canyon in the Pacific, but we do not know if it is the only canyon in the Pacific, so answer choice H must be incorrect. What we do know is that Waimea Canyon is 3,600 feet deep, and that it is smaller than the Grand Canyon. Based on this information, we can infer that the Grand Canyon must be more than 3,600 feet deep, making answer choice J the correct option.

5. Which fact best supports the main idea of the passage?

A. The most striking views occur during the early sunrise hours.
B. The Grand Canyon was created by the flow of the Colorado River.
C. From the valley, visitors can follow the Waimea River out to the Pacific Ocean.
D. Flood waters and rivers flowing from Mount Wai’ale’ale carved Waimea Canyon.
E. The road that leads into the canyon reaches nearly 4,000 feet above sea level.

The question asks you to identify a fact that supports the main idea. Remember, a fact can be verified, or proven. The idea in answer choice A is an opinion rather than a fact. Not everyone would agree with this statement, and it cannot be verified, so this is not the correct answer. Answer choice B is a fact; however, it does not support the main idea since it is gives information about the Grand Canyon rather than Waimea Canyon. Answer choices C and E do contain facts about the topic, Waimea Canyon, but these do not strongly support the main idea, which relates to how the canyon was formed. Answer choice D, on the other hand, is a fact that helps to explain how Waimea Canyon was formed, which strongly supports the main idea. So this is the correct choice.

6. Which of the following conveys the same or nearly the same meaning as the word *gorge* in the first paragraph?

F. The stray puppy appeared to gorge himself after not eating in so long.

G. The hawk hunted for its meal, then stored the food in the gorge in its throat.

H. Soldiers were able to enter the fort through the narrow gorge at the rear of the structure.

J. From the bridge spanning the gorge, we could clearly see the base of the valley far below.

K. He caught the front of his coat on the fence, causing the gorge on the left side of the lapel to come unstitched.

In the first paragraph, *gorge* is a noun meaning “a deep, narrow valley with steep rocky sides.” Only answer choice J uses the same meaning. The meaning of *gorge* in answer choice F is “to greedily overeat.” In this sentence, *gorge* is a verb. As you reviewed earlier in the chapter, the correct answer would need to be the same part of speech as the meaning of the word as it was used in the passage. In answer choice G, a gorge is a pouch in the neck of a hawk where it stores food. As it is used in answer choice H, a gorge is a military entrance found in the rear of a fortification. In answer choice K, *gorge* means “the seam found on the front of a coat that joins the lapel to the collar.”

Let's Review!

In this chapter, you've reviewed a number of strategies that can help you to improve your understanding of text and to answer the reading comprehension questions on the SHSAT. Let's take a final look at what you have learned:

- Every passage has a **main idea**, which is the most important information, main message, or key concept that the author wants to share. The main idea can generally be stated in a complete sentence. If a passage has more than one paragraph, each paragraph will have a main idea.
- The main idea is often introduced in the **topic sentence**, located near the beginning of the passage. This sentence tells what the passage will be about.
- The **topic** of a passage is the subject of the material. It may be a single word or a short phrase that tells what the material is about, such as *killer whales*, *dentistry*, or *the Revolutionary War*.
- **Supporting details** are facts, examples, reasons, definitions, or descriptions that help to explain and give more information about the main idea. These supporting details help the reader to gain a more complete understanding of the main idea.
- **Facts** are statements that are true and can be proven or verified. **Opinions**, on the other hand, are someone's personal views or thoughts and can be disputed.
- Authors often expect readers to make **inferences**, or read between the lines to discover information that is implied or suggested in the writing.
- Readers often have to draw **conclusions** about a passage. A conclusion is a reasonable, educated guess based on given information.
- **Context clues** are hints that help readers determine the meanings of unfamiliar words. Types of context clues include:

- Synonyms
- Antonyms
- Definitions
- Explanations
- Examples

- Many words have more than one meaning. Context clues can be helpful in determining which definition of a multiple-meaning word is being used.
- A **summary** is a restatement of the main idea and most important details in a text. To summarize, readers must relay the information using their own words rather than using the words and phrases given by the author.
- In order to gain a full understanding of the text, readers may have to **synthesize**, or combine information from various parts of a passage.
- The **organizational structure**, or the ways ideas are arranged in a passage, can help readers recognize what information will be important and help them to know what types of information to anticipate in the text. Common organizational structures used by authors include
  - Sequence
  - Order of importance
  - Cause and effect
  - Compare and contrast
  - Problem and solution
  - Description

CHAPTER 7
Arithmetic

Some of the questions on the mathematics section of the SHSAT will assess your knowledge of arithmetic. These questions will be mixed in with the other types of mathematics questions: algebra, probability, statistics, and geometry.

As you read this chapter, pay close attention to not only the strategies we review, but also the vocabulary and formulas. You will need to be familiar with these to do well on the test; however, they will not be provided for you on test day.

Number Sense

Let's start by reviewing the types of numbers. Knowing the correct terms for these will be important when answering arithmetic questions.

- **Natural numbers.** These are the counting numbers you first learned when you were much younger. Notice that these start with the number 1: 1, 2, 3, 4, 5, 6, 7 …
- **Whole numbers.** Whole numbers are the natural numbers, along with zero: 0, 1, 2, 3, 4, 5, 6, 7 …
- **Integers.** This group of numbers includes the whole numbers as well as the negatives of the natural numbers: … –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7 …
- **Rational numbers.** Rational numbers include integers as well as fractions that are formed when one integer is divided by another integer. These numbers can be written as a fraction or a ratio. This group includes not only fractions, but also decimals,

repeating decimals, and terminating decimals. A few examples of rational numbers are $\frac{1}{4}$, 0.67, 0.343434, 4.2, $12\frac{15}{16}$, 178.03.

- **Irrational numbers.** Irrational numbers cannot be written as a fraction. Nonterminating decimals and nonrepeating decimals are a part of this group. Any square root that is not a perfect square root is also irrational. Examples of irrational numbers are shown below: pi, 0.745092335071224 …, $\sqrt{3}$.
- **Real numbers.** These are any numbers that can name a position on a number line. This includes all rational and irrational numbers.

Not all numbers with a radical sign are irrational. For example, $\sqrt{16}$ and $\sqrt{25}$ are rational since $\sqrt{16}=4$ and $\sqrt{25}=5$.

Now that you have reviewed the types of numbers within our number system, let's look at different ways numbers may be written.

Expanded Form

Each digit of a number has a different place value name.

24,987,136

As you know, this number is twenty-four million, nine hundred eighty-seven thousand, one hundred thirty-six. In this example, the:

- 2 is in the ten millions' place
- 4 is in the millions' place
- 9 is in the hundred thousands' place
- 8 is in the ten thousands' place
- 7 is in the thousands' place
- 1 is in the hundreds' place

- 3 is in the tens' place
- 6 is in the ones' place

When a number is written as we usually see it, it is in **standard form**:

24,987,136

Expanded form is seen when a number is written as an addition sentence, with each digit showing its place value:

Standard form: 365
Expanded form: $300 + 60 + 5$

Since place value tells us that 365 has 3 hundreds, 6 tens, and 5 ones, we use expanded form to show each of these values.

3 hundreds = 300
6 tens = 60
5 ones = 5

Now you try it. What is the following number, written in expanded form?

73,902

You probably knew that 73,902 has 7 ten thousands, 3 thousands, 9 hundreds, and 2 ones. Notice that there is a zero in the tens' place. That means the number has zero tens. So, in expanded form, the number is

$70{,}000 + 3{,}000 + 900 + 2$

Notice that we simply omitted the number of tens, since there was a 0 in the tens place.

Scientific Notation

Scientific notation is a way to express large numbers as a multiplication sentence including a small number and a power of 10. Each number in scientific notation will have a **coefficient** that is a number greater than or equal to 1 and less than 10. Each number will also have a **base**, which is the number 10. Following the number 10, there will be an **exponent**, or the power of ten.

Let's take a look at an example. The number below is extremely large: 327 billion, to be exact. It is shown in standard form and in scientific notation.

$$327{,}000{,}000{,}000 = 3.27 \times 10^{11}$$

To write 327 billion in scientific notation, we need to place a decimal after the first digit, then drop the zeros.

3.27~~000000000~~

Now, count the number of places from the decimal to the end of the row of zeros. Since there are two digits after the decimal (the 2 and 7) and nine zeros, there are a total of 11 places between the decimal and the end of the number. That means our exponent will be 11.

So, in this example, our coefficient is 3.27, our base is 10, and our exponent is 11.

Now it's your turn. What is the number below, written in scientific notation?

94,500,000

The first step is to place the decimal and determine the coefficient. In this case, the coefficient will be 9.45. Next, count the places between the decimal and the end of the row of zeros. Since there are seven digits, the exponent will be 7. As always, the base is 10. So, in scientific notation, the number is:

$$94{,}500{,}000 = 9.45 \times 10^{7}$$

We know that scientific notation can be used to write large numbers; however, did you know that it can be used to write extremely small numbers as well? The steps in writing a small number are very similar to those used to write large numbers. The only differences are that we will count the numbers to the *left* of the decimal, rather than those to the right, and the exponent will be negative.

$$0.000\ 000\ 000\ 072\ 5 = 7.25 \times 10^{-11}$$

First, move the decimal to the right to create a number between 1 and 10. Then, drop the zeros between the original placement of the decimal and the new placement of the decimal.

0.~~000 000 000 0~~7.2 5

Count the zeros that were crossed off, as well as the number to the left of the decimal. Ten zeros plus the number 7 is a total of 11, so the exponent will be 11. We moved the decimal to the right rather than the left because this is a very small number, so the exponent will be negative.

Let's try one more. Write the following in scientific notation.

0.000 005 46

In this example, the coefficient will be 5.46. There are a total of six digits between the original placement of the decimal point and the placement of the decimal point to the right in the coefficient, so our exponent will be –6. As always, the base will be 10.

$$5.46 \times 10^{-6}$$

What if you need to convert a number from scientific notation to standard notation? Simple! Move the decimal point in the coefficient. The exponent tells how many places to move it. If the exponent is positive, move the decimal to the right to make the number larger. If

the exponent is negative, move the decimal to the left to make the number smaller.

What is the following number in standard notation?

2.9×10^8

Write the coefficient, then move the decimal eight places to the right.

2.9 _ _ _ _ _ _ _

290,000,000.

Operations

In arithmetic, you can expect to perform four basic types of operations on numbers:

- Addition
- Subtraction
- Multiplication
- Division

These are things that you began learning at an early age, and they are the foundation of most of your math skills. These operations are used in nearly every area of math, in one way or another. On the SHSAT, you will be using these skills to answer computation and word problems, not only in the area of arithmetic, but also in algebra, geometry, probability, and statistics.

Choosing an Operation

When you see a computation problem, it is pretty simple to determine which operation, or operations, you will need to use. For example, the addition sign here indicates that you need to add.

734 + 609 =

Word problems, on the other hand, can require a little more thought. Often you will have to read a problem and figure out if you need to add, subtract, multiply, divide, or use a combination of these skills to correctly solve the problem. The good news is there are often hints within the question that can help you select the correct operation to use.

Some key words that may indicate **addition** include:

add	combined	in all	raise
additional	extra	increase	sum
altogether	gain	plus	total
both			

Signal words that often indicate **subtraction** include:

change	fewer	more than
decrease	have left / were left	nearer
difference	less / less than	reduce
drop	loss	remain
fall	lost	take away
farther	minus	

Key words that often indicate the need for **multiplication** include:

a factor of	increased by	times
as much	of	total
by	product	triple
double	multiplied by	twice

Notice that some of the words may also be used to indicate addition. That is because multiplication is the same as repeated addition. Multiplication may be used as a shortcut to find the total of many. Be sure to consider the situation

before selecting an operation, rather than relying completely on the signal words.

Some words that may let you know that **division** is required include:

average	equal parts	per
break up	every	quotient of
cut into	into	ratio of
divide evenly	out of	split into
each		

When choosing an operation, pay attention to the context, and determine what type of answer would make sense. If a larger number is needed, use addition or multiplication. If a smaller number is needed, use subtraction or division. After you have found the answer, check to make sure that the amount is reasonable for the situation. For example, if a pair of shoes costs $35, and you need to find the cost of 7 pairs, $5 would not be a reasonable answer. The correct answer would need to be greater than, not less than, the cost of a single pair of shoes.

Order of Operations

Some arithmetic questions may require a combination of addition, subtraction, multiplication, and division.

$$10 + 5 \times 3 =$$

If we were to complete these steps in order from left to right, the answer would be 45.

$$10 + 5 \times 3 = 15 \times 3 = 45$$

The only problem is, this is not the correct answer! There is a set of rules, known as the **order of operations**, that states the order in

which the operations in math problems must be solved. According to these rules, math problems must be solved according to the following sequence:

1. Parentheses
2. Exponents
3. Multiplication and division (from left to right)
4. Addition and subtraction (from left to right)

So, according to this set of rules, how would we correctly solve the example?

$$10 + 5 \times 3 =$$

Since there are no parentheses or exponents, we'll start with the multiplication, then complete the addition.

$$10 + \underline{5 \times 3} = 10 + 15 = 25$$

> To help you remember the order of operations, take a look at the first letter in each of the steps: P, E, M, D, A, S. These are also the first letters of the words in the phrase *Please excuse my dear Aunt Sally*. Keep this phrase in mind, and you'll be sure to solve the operations in the correct order!

Let's try another one. Remember to use the order of operations to solve the steps in the proper sequence.

$$39 \div (15 - 2) + 4^2 =$$

First, solve anything in parentheses.

$39 \div \underline{(15 - 2)} + 4^2 = 39 \div \underline{13} + 4^2$

Next, solve the exponent.

$39 \div 13 + \underline{4^2} = 39 \div 13 + \underline{16}$

Now, complete any multiplication or division. In this example, there is only one division step. If there were more than one multiplication or division step, we would solve them from left to right.

$\underline{39 \div 13} + 16 = \underline{3} + 16$

Finally, complete any addition or subtraction. Again, if there were more than one addition or subtraction problem to solve, we would begin at the left of the equation and work toward the right.

$3 + 16 = 19$

It's your turn. Use the order of operations to solve the problem below.

$8 \times 7 - (2 + 4)^2 \div 9 =$

A. 2.2
B. 7.7
C. 24
D. 52
E. 54

Although there are many ways to solve this problem, only one way is correct. First, solve what is in the parentheses.

$8 \times 7 - \underline{(2 + 4)}^2 \div 9 = 8 \times 7 - \underline{6}^2 \div 9$

Next, solve the exponent.

$$8 \times 7 - \underline{6^2} \div 9 = 8 \times 7 - \underline{36} \div 9$$

This leaves multiplication, subtraction, and division. According to the order of operations, we must solve the multiplication and division next, working from left to right.

$$\underline{8 \times 7} - 36 \div 9 = \underline{56} - 36 \div 9$$

$$56 - \underline{36 \div 9} = 56 - \underline{4}$$

Finally, we can solve the subtraction.

$$\underline{56} - \underline{4} = \underline{52}$$

Did you select answer choice D? Awesome!

Estimation

At times, it is not necessary to find an exact answer to a math problem. In these circumstances, we can use **estimation**, which is an educated guess of about what the actual answer is.

> A computer costs $1,470. A business owner wants to purchase 105 of the computers for his employees. About how much will the computers cost?

The words *about how much* tell you that you can estimate the amount rather than try to find the exact cost. Rounding is one strategy you can use to estimate. The price of each computer is about $1,500, and the business owner needs about 100 computers.

$$\$1{,}500 \times 100 = \$150{,}000$$

So the total cost of the computers will be about $150,000.

Estimation is a great way to quickly determine whether your answer to a math problem is reasonable or not. If estimation shows that your answer is not reasonable, you know to go back and look for mistakes.

Multiplication

You probably learned how to multiply when you were in the second or third grade, so by now you're a pro! But it's a good idea to refresh your memory on a few things related to multiplication that you will need to know before taking the SHSAT.

Prime Numbers

As you learned when you were much younger, *factors* are the numbers being multiplied, and the *product* is the answer.

$$5 \times 7 = 35$$

In this example, 5 and 7 are the factors, and 35 is the product.

Prime numbers are those numbers greater than 1 whose only factors are 1 and the number itself. These numbers cannot be evenly divided by any other numbers. For example, what numbers could be multiplied in order to have a product of 3? Only 1 and 3. That means 3 is a prime number since the only factors are 1 and the number itself, 3. That is, 3 is only evenly divisible by 1 and 3.

The prime numbers between 0 and 100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Notice that 2 is the first number in the list. Since 1 is not greater than 1, it is not considered prime. The only factors of 2 are 1 and 2, so 2 is a prime number. Obviously, since 2 is a factor of any even number, all other prime numbers are odd.

Is 133 a prime number? No, because $7 \times 19 = 133$. And $133 \div 19 = 7$.

Is 157 a prime number? Yes, because it can only be evenly divided by 1 and 157.

Composite Numbers

Composite numbers, unlike prime numbers, have more than two factors. For example, 12 is a composite number. Factors of 12 include 1, 2, 3, 4, 6, and 12.

You just learned that 1 is not a prime number. So does that make it a composite number? Well, let's take another look at the definition: *composite numbers have more than two factors*. How many factors does 1 have? One. So 1 is not a composite number or a prime number.

To determine whether a number is prime or composite, you may have to see whether or not it is divisible by another number. But here are a few ways to quickly identify composite numbers.

- All even numbers, with the exception of 2, are composite. Even numbers are all divisible by at least 1, 2, and the number itself.
- All numbers with a 5 in the ones place are composite. Any number that ends with a 5 is a multiple of 5 and is divisible by at least 1, 5, and the number itself. Think about the factors of 25, 175, and 495 as examples.
- All numbers whose digits have a sum that is divisible by 9 are composite. If the sum of the digits is divisible by 9, the number itself is divisible by 9. For example, 243 is divisible by 9 because $2 + 4 + 3 = 9$. And $243 \div 9 = 27$. Also, 685 is divisible by 9 because $5 + 6 + 7 = 18$, which is divisible by 9: $567 \div 9 = 63$. So any of these numbers are at least divisible by 1, 3, 9, and the number itself.
- Likewise, all numbers whose digits have a sum that is divisible by 3 are composite. If the sum of the digits is divisible by 3, the number itself is divisible by 3. For example, 102 is divisible by 3 because $1 + 0 + 2 = 3$. So $102 \div 3 = 34$. Also, 411 is divisible by 3 because $4 + 1 + 1 = 6$, and 6 is divisible by 3. So $411 \div 3 = 137$.

It is true that if the sum of the digits of a number is divisible by 9, the number itself is divisible by 9, and if the sum of the digits of a number is divisible by 3, the number itself is divisible by 3. However, this trick does not work with other numbers. Just because the sum of the digits of a number is divisible by 7, that does not mean that the number is divisible by 7. This only works with multiples of 9 and multiples of 3, but it can come in very handy when determining composite numbers.

You know that even numbers are composite. So, to find out if an odd number is prime or composite:

1. See if it is divisible by 3 by finding the sum of the digits.
2. See if it is divisible by 5 by looking at the number in the ones' place.
3. See if it is divisible by 9 by finding the sum of the digits.
4. If the number is not divisible by 3, 5, or 9, try dividing it by the only remaining odd factor: 7.

If the number is evenly divisible by 7, it is composite. If not, it is a prime number.

Use what you've learned so far to answer the following question.

Which is the greatest odd composite integer between 485 and 499?

A. 498
B. 497
C. 495
D. 493
E. 489

To answer this, we have to check each of the answer choices to determine if it meets the requirements of the question. Answer

choice A, 498, is a composite number; however, it is even. Since the question asks for an odd integer, this is incorrect.

Take a look at answer choice B, 497. This number does not have a 5 or 0 in the ones' place, so it is not divisible by 5. The sum of the digits is not divisible by 3 or 9, so the number itself is not divisible by 3 or 9. To determine if it is composite, divide 497 by 7. Since $497 \div 7 = 71$, this is the greatest odd composite integer between 450 and 499.

Let's check the other answer choices as well. Since answer C, 495, has a 5 in the ones' place, this number is divisible by 5 and is a composite number. However, since 495 is not greater than 497, which is also composite, this is not the correct answer choice.

Answer choice D, 493, is the greatest prime number in the list, but since we are looking for the greatest composite number, this is not the correct option.

The sum of the digits in answer choice E is 21; $4 + 8 + 9 = 21$. Since 21 is divisible by 3, the number itself, 489, is also divisible by 3; $489 \div 3 = 163$. That means 489 is composite. But 489 is the lowest, rather than the greatest composite number in the list, so this answer choice is incorrect.

Greatest Common Factor

The **greatest common factor**, also known as the GCF, is the largest number that is a factor of two or more numbers. To find the GCF of a pair of numbers, list all of the prime factors of each number. Then, multiply the factors that both numbers have in common.

Let's find the GCF of 12 and 18. First, list all of the prime factors of both numbers.

Prime factors of 12: $2 \times 2 \times 3$

Prime factors of 18: $2 \times 3 \times 3$

Next, identify which factors are shared by both numbers.

Prime factors of 12: $\underline{2} \times 2 \times \underline{3}$

Prime factors of 18: $\underline{2} \times \underline{3} \times 3$

These numbers share two factors: 2 and 3. So, we'll multiply 2 and 3 to find the greatest common factor.

$$2 \times 3 = 6$$

So, 6 is the greatest common factor of 12 and 18.

Now you try it. Find the GCF and select the correct answer.

What is the greatest common factor of 72 and 90?

A. 2
B. 3
C. 6
D. 18
E. 36

First, list the prime factors of 72 and 90. Then identify which factors the numbers have in common.

Prime factors of 72: $\underline{2} \times 2 \times 2 \times \underline{3} \times \underline{3}$
Prime factors of 90: $\underline{2} \times \underline{3} \times \underline{3} \times 5$

Both numbers have at least one 2, and two 3s. Notice that the prime factorization of 72 has two 2s, and the prime factorization of 90 only has one 2. We have to find how many factors 72 and 90 have in common, so we only underlined one 2 in each list of factors. However, since both 72 and 90 have two 3s, we were able to underline both 3s in each list.

Now, we can multiply the factors that are shared by both numbers.

$$2 \times 3 \times 3 = 18$$

So, the GCF of 72 and 90 is 18. Did you select answer choice D? Great job!

If two numbers do not share any common factors, the GCF is 1, because 1 is a factor of all numbers.

Least Common Multiple

A **multiple** is a number that can be divided evenly by another number. For example, multiples of 4 include 4, 8, 12, 16, 20, 24, 28, 32 … because each of these numbers can be evenly divided by 4.

A **common multiple** is a number that is evenly divisible by two or more given numbers. For example, 24 is a common multiple of both 3 and 4 since it can be evenly divided by both of these numbers.

The **least common multiple,** or LCM, is the lowest common multiple shared by two or more numbers. We already know that 24 is a common multiple of 3 and 4; however, it is not the least common multiple because there is a smaller number that is a multiple of both.

To find the least common multiple, list the multiples of both numbers. Look for numbers that appear on both lists.

Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27…
Multiples of 4: 4, 8, 12, 16, 20, 24, 28…

Since 12 is the lowest number that is included on both lists, it is the least common multiple of 3 and 4.

Notice that when you list multiples of the two numbers, you only have to write numbers until you find one that is on both lists. There really is no reason to continue listing extra numbers once you find a match.

What if you need to find the LCM of more than two numbers? Follow the same steps, just be sure to include lists of multiples of all of the numbers in question. See how well you can do answering the following question.

What is the lowest common multiple of 8, 12, and 15?

A. 4

B. 24

C. 60

D. 96

E. 120

The first step in solving this is to list the multiples of each number.

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120

Multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120

Multiples of 15: 15, 30, 45, 60, 75, 90, 105, 120

Notice that the smallest number to appear in each of the lists is 120, so E is the correct answer. Answer choice A is the greatest common factor of 8 and 12. Answer choice B is the lowest common multiple of 8 and 12, but it is not a multiple of 15, so this choice is incorrect. Answer choice C is the lowest common multiple of 12 and 15, but it is not a multiple of 8, so it is also incorrect. Answer choice D is a multiple of 8 and 12, but it is not the lowest common multiple, nor is it a multiple of 15.

Let's check out another way to find the least common multiple of 8, 12, and 15. First, list the prime factors of each of the numbers.

Prime factorization of 8: $2 \times 2 \times 2$

Prime factorization of 12: $2 \times 2 \times 3$

Prime factorization of 15: 3×5

Take a look at the prime factors. The least common multiple of 8, 12, and 15 will be the product of these factors, multiplied by the greatest number of times it appears on any list. For example, 2 is a factor of both 8 and 12. It appears three times in the prime factorization of 8 and twice in the prime factorization of 12. Since the most times it appears in a list is three, we will include it three times. Since the most times 3 appears on any list is once, we will include one 3. And,

since the most times 5 appears on any list is once, we will include one 5.

Prime factorization of $8: \underline{2} \times \underline{2} \times \underline{2}$
Prime factorization of $12: 2 \times 2 \times \underline{3}$
Prime factorization of $15: 3 \times \underline{5}$

Now, let's multiply these factors.

$$\underline{2} \times \underline{2} \times \underline{2} \times \underline{3} \times \underline{5} = 120$$

This shows us that the least common multiple of 8, 12, and 15 is 120.

Exponents

An **exponent** indicates repeated multiplication.

$$4^2 = 4 \times 4$$

$$5^3 = 5 \times 5 \times 5$$

As you can see in these examples, when the exponent is 2, we multiply the number by itself 2 times. When the exponent is 3, we multiply the number by itself 3 times.

When the exponent in an expression is 2, we say the number is *raised to the second power*. Another way to say this is to say that the number is *squared*. For example, 4^2 is *four to the second power*, or *four squared*.

When the exponent is 3, we say that the number is *raised to the third power*, or we can say that the number is *cubed*. So, 5^3 is *five to the third power*, or *five cubed*.

So, what is the correct answer to the following example?

$$15^2 = ?$$

To find this answer, multiply 15×15. So, $15^2 = 15 \times 15 = 225$.

What if a term has a zero exponent? Anything to the power of zero equals 1.

$$23^0 = 1$$

Sometimes in math, we need to multiply terms that have exponents.

$$y^4 \times y^6 =$$

In this example, both terms have the same base, *y*. So these can be multiplied. To multiply terms that have the same base, we simply add the exponents.

$$y^4 \times y^6 = y^{4+6}$$
$$y^{4+6} = y^{10}$$

Let's take a closer look at how that works. Remember, exponents stand for repeated multiplication.

$$y^4 = y \times y \times y \times y$$
$$y^6 = y \times y \times y \times y \times y \times y$$

So, when we multiply $y^4 \times y^6$, we are really multiplying 4 *y*'s and 6 *y*'s.

$$y^4 \times y^6 = y \times y \times y \times y \times y \times y \times y \times y \times y \times y$$

That shows *y* multiplied by itself 10 times. This is equal to y^{10}, which is the same result we found by adding the exponents.

So, how do we divide terms with exponents? Well, we already know that division is the opposite of multiplication. Since we multiply

terms with exponents by adding, we divide terms with exponents by subtracting.

$$m^9 \div m^2 = m^{9-2}$$

$$m^9 \div m^2 = m^7$$

Remember, to multiply or divide terms with exponents, the terms must have the same base.

$$a^3 \times b^5 = a^3 \times b^5$$

Since a and b are not the same base, we cannot add these exponents to multiply.

Roots

Roots, also known as **radicals**, are the opposite of exponents. For example, $5^2 = 25$, or *five squared equals 25*. Since 5 times itself equals 25, 5 is the **square root** of 25.

$$\sqrt{25} = 5$$

The symbol used to indicate square root is known as a *radical symbol*. The term $\sqrt{25}$ is read *the square root of 25*.

What is $\sqrt{81}$?

We know that $9 \times 9 = 81$. That means that 9 is the square root of 81.

$$9^2 = 81$$

$$\sqrt{81} = 9$$

Simplifying Radicals

To simplify a radical, or find the square root, we take any perfect squares out from under the radical sign.

$$\sqrt{144} = \sqrt{12^2} = 12$$

Not all numbers under a radical sign will be perfect squares. That means, they are not the result of multiplying a whole number by itself. For example, $\sqrt{48}$ is not a whole number. We know that $6 \times 6 = 36$ and $7 \times 7 = 49$, but no number times itself equals 48. So how do we solve $\sqrt{48}$? We need to find a factor of 48 that is a perfect square. Since 16 is a perfect square $(4^2 = 16)$ and is a factor of 48, we can use it to help us find the square root of 48.

$$\sqrt{48} = \sqrt{16 \times 3}$$

Next, we find the square root of the factor that is a perfect square, 16. We will move the square root of 16 out from under the radical sign.

$$\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$$

Since 3 is not a perfect square, and no factor of 3 is a perfect square, 3 will stay under the radical sign. The answer, $4\sqrt{3}$, is read *four times the square root of three*.

If you haven't already done so, it would be a great idea to memorize all of the perfect squares from $2^2 = 4$ through $15^2 = 225$. Knowing these will help you find square roots in the same way that memorizing multiplication facts helps you solve division problems.

Now you try it!

Simplify the expression $\sqrt{150}$.

First, determine whether the number under the radical sign is a perfect square. Since no number times itself equals 150, determine whether any factor of 150 is a perfect square. Think of the perfect square numbers, and figure out if they are factors of 150: 4, 9, and 16 are not; however, 25 is a factor of 150, so we can use that.

$$\sqrt{150} = \sqrt{25 \times 6}$$

Now move the root of the perfect square out from under the radical sign.

$$\sqrt{150} = \sqrt{25 \times 6} = 5\sqrt{6}$$

Since 6 does not have a factor that is a perfect square, we have simplified the expression.

Multiplying Square Roots

Sometimes, to simplify an expression, we will need to multiply radicals.

$$(\sqrt{20})(\sqrt{5})$$

Neither of these factors is a perfect square, so let's multiply them together first, then find the square root of the product.

$$(\sqrt{20})(\sqrt{5}) = \sqrt{100}$$

$$\sqrt{100} = \sqrt{10^2} = 10$$

Let's try another one. Use multiplication to simplify the following expression.

$$(\sqrt{24})(\sqrt{10})(\sqrt{3})$$

Start by multiplying the numbers and writing the expression using a single radical symbol.

$$(\sqrt{24})(\sqrt{10})(\sqrt{3}) = \sqrt{720}$$

Next, find factors that are perfect squares. You probably recognized that since 72 is divisible by 9, then 720 is as well. And since 9 is a perfect square, we can start simplifying by using that number.

$$\sqrt{720} = \sqrt{9 \times 80}$$

Now, we can move the square root of 9 out from under the radical symbol.

$$\sqrt{9 \times 80} = 3\sqrt{80}$$

Next, determine whether any factors of 80 are a perfect square. Since 16 is both a factor and a perfect square, we can use that to further simplify the expression.

$$3\sqrt{80} = 3\sqrt{16 \times 5}$$

When we move the square root of 16 out from under the radical sign, we will multiply it by the number that is already outside of the radical, 3.

$$3\sqrt{16 \times 5} = 4 \times 3\sqrt{5} = 12\sqrt{5}$$

Since 5 does not have any perfect square factors, the expression has been completely simplified.

Adding and Subtracting Radicals

As we just reviewed, square roots can be multiplied. Now let's review how to add and subtract these numbers. In order to add and subtract square roots, the radicals must be the same.

$$3\sqrt{7}+5\sqrt{7}$$

Since both terms include the same number under the radical sign, we can simply add the numbers by which square roots are being multiplied.

$$\underline{3}\sqrt{7}+\underline{5}\sqrt{7}=\underline{8}\sqrt{7}$$

Square roots can be subtracted in the same way.

$$12\sqrt{15}-2\sqrt{15}=10\sqrt{15}$$

Take a look at the equation below. Notice that the middle term does not have a number outside of the radical sign. That means that there is *one* of that radical. In other words, the term is being multiplied by 1. When this happens, we will add or subtract 1 as needed.

$$20\sqrt{2}+\sqrt{2}-14\sqrt{2}=21\sqrt{2}-14\sqrt{2}=7\sqrt{2}$$

Often before being able to add or subtract square roots, the expressions must first be simplified. After that, if the radicals are the same, the terms can be added or subtracted.

$$\sqrt{27}+\sqrt{75}=$$

First simplify the radicals.

$$\sqrt{27}+\sqrt{75}=\sqrt{9\times3}+\sqrt{25\times3}=3\sqrt{3}+5\sqrt{3}$$

The terms have the same square root, so now we can add.

$$3\sqrt{3}+5\sqrt{3}=8\sqrt{3}$$

Now it's your turn. Simplify the expression.

$$4\sqrt{54}+\sqrt{24}+\sqrt{98}$$

Remember, the first step is to simplify each of the terms. If there is already a term outside of the radical sign, as seen in the first term in this example, be sure to multiply this number by the square root that you take out from under the radical sign.

$$4\sqrt{54}+\sqrt{24}+\sqrt{98}=$$
$$4\sqrt{9\times6}+\sqrt{4\times6}+\sqrt{49\times2}$$

$$(3\times4)\sqrt{6}+2\sqrt{6}+7\sqrt{2}=$$
$$12\sqrt{6}+2\sqrt{6}+7\sqrt{2}$$

Now, add to combine like radicals. Notice that two of the terms have the same number under the radical sign. These can be combined. However, the unlike term cannot be combined with the others.

$$\underline{12\sqrt{6}}+\underline{2\sqrt{6}}+7\sqrt{2}=14\sqrt{6}+7\sqrt{2}$$

Fractions, Mixed Numbers, and Decimals

Let's review a few things you probably learned about fractions during elementary school. As you know, a **fraction** names parts of a whole. The number on the top of the fraction is the **numerator**, and the number on the bottom is the **denominator**. For example, if you ate

$\frac{3}{8}$ of a pizza, the denominator tells that the pizza was divided into 8 equal parts, and the numerator tells that you ate 3 of those equal parts.

Equivalent fractions are fractions that name equal amounts, such as $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{8}{16}$.

A **mixed number** is a whole number and a fraction combined. For example, if you studied at the library for $2\frac{1}{3}$ hours, you were there for 2 whole hours plus $\frac{1}{3}$ of another hour.

On the SHSAT, you may need to add, subtract, multiply, or divide fractions. We'll review how to perform each of these operations.

Adding and Subtracting Fractions

To add or subtract fractions with like denominators, we simply add or subtract the numerators. The denominator stays the same.

$$\frac{5}{12}+\frac{6}{12}=\frac{11}{12}$$

If the fractions have unlike denominators, we must first find a common denominator.

$$\frac{1}{3}+\frac{5}{12}=$$

In this case, we can multiply the first fraction by $\frac{4}{4}$ to make the denominator the same as the second fraction. Since $\frac{4}{4}$ is equal to 1, the value of the fraction does not change; the new fraction will be equivalent to the original.

$$\frac{1}{3}\left(\frac{4}{4}\right)=\frac{4}{12}$$

Now we can add.

$$\frac{4}{12}+\frac{5}{12}=\frac{9}{12}$$

The numerator and denominator are both multiples of 3, so we can simplify the fraction by dividing the numerator and denominator both by 3.

$$\frac{9}{12}=\frac{3}{4}$$

Let's subtract fractions.

$$\frac{5}{7}-\frac{1}{6}=$$

Before we can subtract, we must first find a common denominator. Here is where finding the lowest common multiple comes in handy. Both denominators can evenly divide 42, so we will multiply the first fraction by $\frac{6}{6}$ and the second by $\frac{7}{7}$ to give them the same denominator. Then we can subtract.

$$\frac{5}{7}\left(\frac{6}{6}\right)-\frac{1}{6}\left(\frac{7}{7}\right)=$$

$$\frac{30}{42}-\frac{7}{42}=\frac{23}{42}$$

When performing operations involving a fraction and a whole number, be sure to write the whole number as a fraction with

a denominator of 1. For example, 12 is the same as $\frac{12}{1}$

Multiplying and Dividing Fractions

To multiply fractions, the denominators do not have to be the same. Simply multiply the numerators, multiply the denominators, and simplify the product.

$$\frac{3}{16}\times\frac{2}{3}=\frac{6}{48}$$

$$\frac{6}{48}=\frac{1}{8}$$

To divide fractions, invert the second fraction and multiply.

$$\frac{5}{8}\div\frac{15}{16}=\frac{5}{8}\times\frac{16}{15}=\frac{80}{120}=\frac{2}{3}$$

Let's try multiplying a fraction and a mixed number.

$$\frac{7}{9}\times 3\frac{1}{12}=$$

First, convert the mixed number into an **improper fraction**. An improper fraction is simply a fraction that has a numerator that is greater than the denominator. To change a mixed number to an improper fraction, multiply the denominator by the whole number, then add the numerator. The result is the new numerator. The denominator stays the same. So, to change $3\frac{1}{2}$ to an improper fraction, multiply 12×3, then add the numerator, 1. The result is 37, which is the numerator of the improper fraction. The denominator is still 12.

Now multiply.

$$\frac{7}{9} \times \frac{37}{12} = \frac{259}{108}$$

The result is an improper fraction. To simplify this, convert it back to a mixed number. First, divide the numerator by the denominator to find the whole number. Then, write the remainder as the numerator of the fraction.

$$\frac{259}{108} = 2\frac{43}{108}$$

> Remember to do all of your calculations on scrap paper or in your SHSAT test book. Do not work any of the problems on the answer sheet. This sheet should only be used to mark your final answer.

Now you try it. Use what you know about fractions to answer the question below.

For her science fair project, Nicolette is testing the effect of adding plant food to potted vegetables. She has $3\frac{7}{8}$ pounds of plant food and will dissolve $\frac{1}{2}$ ounce of the food in one cup of water daily for each plant. How many servings of plant food will she be able to make?

A. $1\frac{15}{16}$

B. $7\frac{3}{4}$

C. 31

D. 96

E. 124

Let's see how well you did! First, we have to find how many ounces of plant food she has. Notice that the item states that she has $3\frac{7}{8}$ *pounds* of the food. We need to find how many *ounces*. To do this, we need to multiply $3\frac{7}{8}$ by 16 since there are 16 ounces in a pound. So, convert $3\frac{7}{8}$ to an improper fraction, then multiply it by 16.

$$3\frac{7}{8}\times 16=\frac{31}{8}\times\frac{16}{1}=\frac{496}{8}=62$$

So, Nicolette has 62 ounces of plant food. Next, divide this amount into $\frac{1}{2}$ ounce servings.

$$\frac{62}{1}\div\frac{1}{2}=\frac{62}{1}\times\frac{2}{1}=124$$

She can make 124 servings of plant food for her project.

Converting Fractions and Decimals

Decimals are another way to write amounts that are part of a whole.

$$\frac{1}{4} = 0.25$$

$$\frac{1}{2} = 0.5$$

$$\frac{60}{10} = 0.6$$

$$\frac{3}{4} = 0.75\ ¾$$

$$\frac{95}{100} = 0.95$$

$$1\frac{1}{2} = 1.5$$

To convert a fraction to a decimal, divide the numerator by the denominator.

Convert $\frac{3}{4}$ to a decimal.

$$3 \div 4 = 0.75$$

$$\frac{3}{4} = 0.75$$

To convert a decimal to a fraction, place the decimal as the numerator over a denominator of 100. Then, simplify the fraction.

Convert 0.35 to a fraction.

$$0.35 = \frac{35}{100} = \frac{7}{20}$$

Percentages

Like fractions and decimals, percentages also refer to part of a whole. However, **percent** means “out of 100” or “per 100.” For example, 85 percent, or 85%, means 85 out of 100.

$$85\% = \frac{85}{100} = 0.85$$

Changing a fraction to a percent is the same as changing a fraction to a decimal, only you will use a percent symbol (%) rather than a decimal point.

Mei Ling scored $\frac{32}{40}$ on her math quiz. What percent of the questions did she answer correctly?

To solve this, divide the numerator by the denominator.

$$\frac{32}{40} = 32 \div 40 = 0.8 = 80\%$$

Be careful not to confuse decimals that only have one digit to the right of the decimal point. Remember that the first place value is tenths, then hundredths. That means $0.8 = 80\%$ and $0.08 = 8\%$.

To convert a percent to a decimal, move the decimal two places to the left.

$43\% = 0.43$

$7\% = 0.07$

$135\% = 1.35$

Notice that percents can be greater than 100%. Take a look at the following example.

The number of students attending Lincoln Junior High School has increased by 178% over the past 10 years. If there were 650 students originally, how many students currently attend the school?

To solve this, we need to first change the percent to a decimal.

$$178\% = 1.78$$

Next, we can multiply the original number by the decimal number.

$$650 \times 1.78 = 1{,}157$$

Over the past 10 years, the number of students has increased from 650 to 1,157.

Very small amounts may be described as percents that are less than 1%.

A survey of 800 students showed that only 0.5% of students would like to have more homework. How many students surveyed responded that they would like more homework?

To solve this, first change the percent to a decimal by moving the decimal point two places to the left.

$$0.5\% = 0.005$$

Now, multiply the number of students surveyed by the decimal number.

$$800 \times 0.005 = 4$$

So only 4 students out of 800 surveyed would like more homework.

Let's use what we've reviewed about percents to solve the following example.

A pair of jeans is priced $85. They are on sale for 30% off. What is the final price of the jeans, including 6% sales tax?

First, we need to find the discounted price. To do this, we need to find 30% of $85. Convert 30% to a decimal and multiply by the price of the jeans.

$$30\% = 0.30$$

$$0.30 \times 85 = 25.5$$

So, the discount is $25.50. Next, we need to subtract the discount from the original price in order to find the sale price.

$$85 - 25.5 = 59.50$$

Next, to find the sales tax, we need to determine 6% of 59.50. Again, convert the percent to a decimal, then multiply.

$$6\% = 0.06$$

$$59.50 \times 0.06 = 3.57$$

Finally, add the sales tax to the discounted price to find the total cost.

$$59.50 + 3.57 = 63.07$$

So the final price, including the discount and sales tax, is $63.07.

Ratios and Proportions

A **ratio** is a way to compare two different numbers. For example, suppose there are 25 students in your math class. Of those, 15 are boys and 10 are girls. The ratio of boys to girls is 15:10. In the statement *ratio of boys to girls*, the word *boys* was listed first,

therefore, the number of boys must be listed first in the ratio. Ratios can be stated in words, numbers, or as a fraction.

There were 15 boys and 10 girls.
The ratio of boys to girls is 15 to 10.

15:10

$$\frac{15}{10}$$

Notice that the numbers included in the ratio are the number of boys and the number of girls. The total number of students is not included.

Just like fractions, ratios can also be reduced, or simplified.

The ratio of boys to girls is $\frac{15}{10}$, or 15:10

The ratio of boys to girls is $\frac{3}{2}$, or 3:2.

This shows that for every three boys there are two girls.

Let's use what we've discussed about ratios to solve a problem.

In Mr. O'Rourke's third period class, the ratio of freshmen to sophomores is 7:5. How many of the 24 students are freshmen?

The ratio 7:5 indicates that for every 7 freshmen, there are 5 sophomores. That means that in every 12 students, 7 are freshmen and 5 are sophomores. So, the fraction $\frac{7}{12}$ shows that 7 out of 12 students are freshmen. We can multiply this fraction by the total number of students to find how many freshmen are in the class.

$$\frac{7}{12} \times 24 = 14$$

So, there are 14 freshmen. That also means that since there are a total of 24 students, the remaining 10 are sophomores.

Ratios are important to properly setting up and solving proportions. A **proportion** is an equation that has equivalent ratios on each side.

$$\frac{2}{3}=\frac{4}{6}$$

Since the two ratios are equal, this is a proportion. When one value in a ratio is missing, solving a proportion is a way to determine the unknown number.

$$\frac{x}{12}=\frac{1}{2}$$

Since you know that $\frac{6}{12}$ is equivalent to $\frac{1}{2}$, you probably already figured out that the missing number in this proportion is 6. One way to solve a proportion is to cross multiply. This means, multiply the denominator on the left by the numerator on the right. Then, multiply the numerator on the left by the denominator on the right. Basically, you multiply across the equals sign.

$$\frac{x}{12}=\frac{1}{2}$$
$$x(2)=12(1)$$
$$2x=12$$
$$x=6$$

Let's try another example.

The ratio of teachers to students at West Lake Junior High School is 1:14. How many teachers are there if the school has a total of 882 students?

First, write the ratio of teachers to students as a fraction. Remember, since teachers are listed first, the number of teachers comes first.

$$1:14 = \frac{1}{14}$$

Now write the ratio of the total number of teachers to the total number of students. Since the actual number of teachers is unknown, use a variable in the ratio.

$$\frac{t}{882}$$

Next, set up the ratios as a proportion. Remember, the ratios are equivalent.

$$\frac{t}{882} = \frac{1}{14}$$

Cross multiply to solve for *t* and find the total number of teachers.

$$\begin{aligned} \frac{t}{882} &= \frac{1}{14} \\ 14(t) &= (1)882 \\ 14t &= 882 \\ 14t \div 14 &= 882 \div 14 \\ t &= 63 \end{aligned}$$

So, the total number of teachers at the school is 63.

You Try It!

In this chapter, you've reviewed a number of skills that will help you answer the arithmetic questions on the SHSAT. Use what you have learned to answer the questions below.

1. What is the greatest composite odd integer less than 50 that is not a perfect square?

A. 49
B. 47
C. 45
D. 39
E. 37

To find the correct answer, let's start with the greatest odd integer less than 50, which is 49. Since 49 is a perfect square ($7^2 = 49$), this is not the correct answer. Since 47 is a prime number, it is not correct either. Answer choice C, 45, is correct because it is composite (factors of 45 include 1, 3, 5, 9, 15, and 45), odd, and not a perfect square. Answer choice D, 39, is also composite; however, it is not the greatest composite odd integer. And since choice E, 37, is prime, it is also incorrect.

2. Elliot began a new job working in his aunt's day care center. He will work $12\frac{1}{2}$ hours each week and will earn $8.60 per hour for the first three weeks. He will earn a 20% raise after that. How much will Elliot's paycheck be for his work the fourth week?

A. $21.50
B. $86.00
C. $107.50
D. $129.00
E. $161.25

First, convert $12\frac{1}{2}$ to a decimal, 12.5. Next, multiply the hours Elliot will work by his rate per hour, $8.60.

$$12.5 \times 8.6 = 107.5$$

So, we know he will make $107.50 per week for the first three weeks. Next, find 20% of this amount to determine the amount of his raise. Convert 20% to a decimal, 0.2, then multiply by the amount he earned prior to the raise.

$$107.5 \times 0.2 = 21.5$$

He will earn an additional $21.50 per week. Add this to his previous wages to find the amount of his paycheck for the fourth week.

$$\$107.50 + \$21.50 = \$129.00$$

3. $(\sqrt{45})(\sqrt{5})(\sqrt{32}) =$

A. $12\sqrt{10}$

B. $60\sqrt{2}$

C. $72\sqrt{10}$

D. $100\sqrt{72}$

E. $120\sqrt{2}$

To simplify square roots, multiply the numbers underneath the radical sign, then look for factors that are perfect squares.

$$(\sqrt{45})(\sqrt{5})(\sqrt{32}) = \sqrt{7{,}200} = \sqrt{72 \times 100}$$

Pull the square root of the perfect square to the outside of the radical sign, and look for perfect square factors of the number remaining under the radical sign.

$$10\sqrt{72} = 10\sqrt{36 \times 2}$$

Move the square root of the perfect square to the outside of the radical sign, and multiply it by the factor that is already there.

$$10\sqrt{36 \times 2} = 6 \times 10\sqrt{2} = 60\sqrt{2}$$

4. Marquez is making a scale model of his school for a project. According to his scale, 2 cm is equal to 3 meters. If the length

of the building is 120 meters, what will be the length of the model, in cm?

A. 40

B. 60

C. 80

D. 120

E. 240

To find the length of the model, we will need to set up a proportion. First, find the ratio of cm to meters Marquez is using for his scale. According to the scale, the ratio of cm to meters is 2:3.

$$\frac{2}{3}$$

We do not know how many cm the model will be in length, so let's use the variable *s* to represent the unknown value. We do know that the building is 120 meters long, so this will be the bottom number of the ratio.

$$\frac{s}{120} = \frac{2}{3}$$

Cross multiply to find the value of the variable.

$$3(s) = 2\,(120)$$

$$3s = 240$$

$$3s \div 3 = 240 \div 3$$

$$s = 80$$

So Marquez's model will have a length of 80 cm.

Let's Review!

In this chapter, you've reviewed arithmetic vocabulary and skills that will be helpful for correctly answering questions on the SHSAT Math test. Let's take another quick look at some of the information that will help you do your best on this section of the test.

- **Expanded form** uses the place values of the digits to write a number as an addition sentence: $4{,}376 = 4{,}000 + 300 + 70 + 6$.
- **Scientific notation** is a way to write very large or very small numbers as a multiplication sentence including a number between 1 and 10 and a power of 10: $920{,}000{,}000{,}000 = 9.2 \times 10^{11}$. In this example, 9.2 is the **coefficient**, 10 is the **base**, and 11 is the **exponent**. Remember to move the decimal point to the left when converting large numbers to scientific notation, and to the right when converting small numbers.
- Look for key words when selecting which operation is required to solve word problems. Clues such as *both*, *combined*, and *total* often indicate that you need to add. *Decrease*, *difference*, and *fewer* often suggest subtraction. *Times*, *product*, and *triple* may tell you to multiply. *Average*, *quotient of*, and *per* sometimes indicate division.
- The **order of operations** is a set of rules that tell you the sequence in which operations must be completed. Always solve math problems according to the following order: parentheses, exponents, multiplication and division, addition and subtraction.
- **Prime numbers** are those that have exactly two factors: 1 and the number itself. Examples of prime numbers include 2, 3, 5, and 7. On the other hand, composite numbers have three or more factors. Examples of **composite numbers** include 6, 10, and 15. All even numbers, with the exception of 2, are composite.
- The **greatest common factor** (GCF) is the largest number that is a factor of two or more numbers. The **least common multiple** (LCM) is the lowest multiple shared by two or more numbers.
- An **exponent** indicates repeated multiplication: $7^4 = 7 \times 7 \times 7 \times 7$. In this example, the exponent, 4, tells that 7 is multiplied by itself 4 times. This is read *seven to the fourth power*. When a number has

an exponent of 2, we say the number is *squared*. When the exponent is 3, we say the number is *cubed*.

- To multiply terms containing exponents, first make sure they have the same base, then add the exponents. To divide terms with the same base, subtract the exponents.

$$q^4 \times q^8 = q^{12}$$
$$w^9 \div w^7 = w^2$$

- **Roots**, or **radicals**, are the opposite of exponents. To find the square root of a number, determine what number when multiplied by itself equals the number under the radical sign.

$$\sqrt{144} = 12$$

- To simplify radicals, find any factors of the number that are perfect squares, and move the square root of the factor to the outside of the radical sign.

$$\sqrt{300} = \sqrt{100 \times 3} = 10\sqrt{3}$$

To multiply radicals, multiply the numbers under the radical signs first, then find the square root of the product.

- Terms that have the same number under the radical sign can be added or subtracted. We cannot add or subtract terms with different numbers under the radical sign.
- **Fractions** name parts of a whole. To add or subtract fractions, first find a common denominator. To multiply fractions, multiply the numerators, then multiply the denominators. To divide fractions, invert one of the fractions, then multiply.
- **Decimals** also name part of a whole. To convert fractions to decimals, divide the numerator by the denominator.
- Percents refer to part of 100. For example, 76% means 76 out of 100, or 76 per 100. Percents can be written as a decimal:

$$\frac{76}{100} = 76\% - 0.76.$$

- **Ratios** compare two different numbers. A **proportion** is an equation that sets two ratios equal to one another. Cross multiplication is one way to solve a proportion containing a ratio with an unknown value.

CHAPTER 8
Algebra

The Math section of the SHSAT will include questions to assess your knowledge of arithmetic, geometry, probability and statistics, and algebra. In this chapter, we will review some of the skills that will help you answer the algebra questions. Keep in mind that as you work through algebra questions, you may need to use skills reviewed in other chapters. For example, we will be using exponents to solve some of the algebra questions. Exponents are reviewed in the arithmetic chapter. Be sure to refer to other sections of the book as needed.

As you work through the chapter, also remember that any vocabulary or formulas you need to solve the problems must be memorized. You will be expected to use these to answer the questions; however, they will not be provided for you on the test.

Variables

When you think of algebra, the first thing that probably comes to mind is letters. **Variables** are letters or symbols that take the place of an unknown value in math.

$$7x + 3y = 15$$

In this equation, the variables *x* and *y* represent unknown values. In this chapter, we will review strategies for solving equations, expressions, and inequalities that include variables. In other words, we'll review ways to figure out what values are represented by these letters or determine the values of the expressions in which the variables are included.

Sometimes the values of the variables are given, and we can use substitution to answer the question.

What is the value of $4x - 2y$ if $x = 7$ and $y = 3$?

In this question, the values of the variables are given. All we have to do is substitute these values into the expression.

$x = 7$ and $y = 3$

$$4x - 2y = 4(7) - 2(3) = 28 - 6 = 22$$

Let's try another one.

What is the value of $5\,|w| \div 10\,|q|$ if $w = 8$ and $q = -1$?

First, take a look at the terms $|w|$ and $|q|$. The vertical lines on either side of the variables indicate absolute value. The **absolute value** of a number is its distance from zero on a number line. For example, –3 is three units from zero, so the absolute value of –3 is 3. Absolute value is always positive.

So, to solve the example, the absolute value of the variables will be positive numbers. Let's start by substituting the values into the expression, then solve the expression.

$w = 8$ and $q = -1$

$$5\,|w| \div 10\,|q| = 5\,|8| \div 10\,|-1| = 5(8) \div 10(1) = 40 \div 10 = 4$$

Now you try one.

If $a = 3$ and $b = 6$, what is the value of $\frac{a^b}{b^a}$?

A. $\frac{1}{12}$

B. $\frac{4}{7}$

C. $\frac{8}{27}$

D. 1

E. $3\frac{3}{8}$

First, substitute the given values for each variable.

$$\frac{a^b}{b^a}=\frac{3^6}{6^3}$$

Then solve the expression.

$$\frac{3^6}{6^3}=\frac{729}{216}$$

Both the numerator and the denominator are divisible by 9, so simplify the fraction.

$$\frac{729}{216}=\frac{81}{24}$$

Since both 81 and 24 are divisible by 3, reduce the fraction further. Then, convert the improper fraction to a mixed number.

$$\frac{81}{24}=\frac{27}{8}=3\frac{3}{8}$$

Solving Algebraic Equations

When we use substitution, the values of the variables are given. Many times, though, this is not the case. Take a look at the equation below.

$$7 + x = 15$$

Here, rather than being told the value of the variable, we have to figure that out. At a quick glance, you can probably tell that $x = 8$ because you already know that $7 + 8 = 15$. But many equations are not solved that easily.

In order to solve equations, we must isolate the variable. That means we have to get the variable alone on one side of the equation.

As you know, the values on either side of the equals sign are the same.

$$7 + x = 15$$

$$7 + 8 = 15$$

As we work to isolate the variable, the values on both sides of the equation must remain equal. That means the same operations must be performed to both sides. So how do we know what operations to perform? In order to solve equations, we will use the operations that are opposite of those that are included in the equation. For example, addition and subtraction are opposite operations because we can use addition to undo subtraction.

$$7 + 8 = 15$$

$$15 - 8 = 7$$

Multiplication and division are also opposite operations.

$$3 \times 4 = 12$$

$$12 \div 4 = 3$$

Let's go back to our original example.

$$7 + x = 15$$

This equation uses addition. In order to isolate the variable, we will use the opposite operation, subtraction. And we will perform the same subtraction to both sides of the equation so the two sides remain equal. In order to get the *x* by itself on one side of the equation, we need to subtract 7. So we will subtract 7 from both sides in order to maintain equality.

$$7 + x - 7 = 15 - 7$$

$$x = 8$$

By subtracting 7 from both sides, not only do we isolate the variable, we also discover the value of *x*.

Let's try another one.

$$9y = 108$$

This equation involves multiplication because 9*y* means 9 × *y*. Since division is the opposite of multiplication, we will divide both sides of the equation by 9 to isolate the variable.

$$\frac{9y}{9} = \frac{108}{9}$$

$$y = 12$$

To check your work, substitute the value of the variable back into the original equation.

$$9y = 108$$
$$9(12) = 108$$
$$108 = 108$$

Since the two sides of the equation are equal, we know we solved the equation correctly and found the value of the variable.

Let's try an equation that is a little more complex.

What is the positive value of the variable in the equation $\frac{k^2}{40} = \frac{9}{10}$?

Since the variable is part of a division expression, we must use multiplication to isolate it. If we multiply both sides of the equation by 40, we can eliminate the division on the right side of the equals sign.

$$\frac{k^2}{40} = \frac{9}{10}$$
$$40 \times \frac{k^2}{40} = 40 \times \frac{9}{10}$$
$$k^2 = \frac{360}{10}$$
$$k^2 = 36$$

Since the variable is squared, the next step is to eliminate the exponent. Finding the square root will isolate the variable. Be sure to find the square root of the values on both sides of the equation in order to maintain equality.

$$\sqrt{k} = \sqrt{36}$$
$$k = 6$$

In order to make sure we solved the equation correctly, let's plug the value of the variable back into the original equation.

$$\frac{k^2}{40} = \frac{9}{10}$$

Substituting 6 for *k*:

$$\frac{6^2}{40} = \frac{9}{10}$$
$$\frac{36}{40} = \frac{9}{10}$$
$$\frac{9}{10} = \frac{9}{10}$$

Perfect! The value of *k* is 6!

Let's use what you've reviewed about algebraic expressions to solve an equation that involves substitution as well as finding the unknown value of a variable.

If $s = 5$ and $2s(8t - 4s) = 40$, what is the value of *t*?

In this case, the value of *s* is given, so begin by substituting this value into the equation.

$$2s(8t - 4s) = 40$$
$$2 \times 5(8t - 4 \times 5) = 40$$
$$10(8t - 20) = 40$$

Now, we can work on solving for *t*. Since the variable is part of a multiplication expression, use division as the first step in getting the variable alone on the left side of the equation. Be sure to divide both sides by 10 to keep them equal.

$$10(8t-20)=40$$
$$\frac{10(8t-20)}{10}=\frac{40}{10}$$
$$8t-20=4$$

Look at the left side of the equation. It involves subtracting 20. We can add 20 to both sides to isolate the term 8*t*.

$$8t-20=4$$
$$8t-20+20=4+20$$
$$8t=24$$

We know that 8*t* means *8 multiplied by t*, so we will use division to undo the multiplication. So the next step is to divide both sides of the equation by 8.

$$\frac{8t}{8}=\frac{24}{8}$$
$$t=3$$

Let's check our work. Substitute the value of *t* back into the original equation. We know that $s=5$ and $t=3$.

$$2s(8t-4s)=40$$
$$2\times5(8\times3-4\times5)=40$$
$$10(24-20)=40$$
$$10(4)=40$$
$$40=40$$

Since $40=40$ is true, we know that the equation was solved correctly!

Now you try it! Use what you have learned about solving equations to answer the question below.

For what value of *m* is $4(m+6)=9(m-4)$?

A. 2
B. 5
C. 10
D. 12
E. 15

Let's solve the equation to check your work. The 4 outside of the parentheses on the left side of the equation means we have to multiply 4 by each of the terms inside those parentheses. So, we will multiply $4\times m$, then multiply 4×6. The same is true of the 9 on the right side of the equation. The 9 must be multiplied by *m*, then by 4.

$$4(m+6)=9(m-4)$$

$$4m+24=9m-36$$

Next, we need to isolate the terms with variables on the same side of the equation. Let's subtract 4*m* from both sides of the equation. That will eliminate the variable from the left side, and place the only terms with a variable on the right.

$$4m+24\underline{-4m}=9m-36\underline{-4m}$$

$$24=5m-36$$

When isolating a variable, it does not matter which side of the equation contains the variable. However, it is usually easier to work with positive variables than negative. When deciding where to isolate the variable, take a look at which side of the equation contains the greatest coefficient. By subtracting the lesser coefficient from both sides, the variable ends up on the side with the greatest coefficient, and the term is positive.

Now, the only term with a variable is on the right side of the equation. To isolate the variable, we need to move the constant, 36, to the left side of the equation. Since 36 is negative, we need to add 36 to both sides of the equation, since $-36 + 36 = 0$. That way, the only constants are on the left of the equation.

$$24 \underline{+ 36} = 5m - 36 \underline{+ 36}$$

$$60 = 5m$$

We're almost there! The term *5m* is the same as *5 multiplied by m*. That means, we need to use division to eliminate the coefficient and get the variable alone. So, we will divide both sides of the equation by 5.

$$\frac{6}{5} = \frac{5m}{5}$$

$$12 = m$$

So the value of *m* is 12. Did you select answer choice D? Great job! Remember to substitute the value of the variable into the original equation to check your work.

$$4(m + 6) = 9(m - 4)$$

$$4(12 + 6) = 9(12 - 4)$$

$$4(18) = 9(8)$$

$$72 = 72$$

The equation is true, so we know the answer is correct!

If you are completely stuck on an SHSAT question asking you to find the value of a variable, try substituting each of the answer choices into the equation to see which makes the equation true. This will probably take more time than solving

the equation as you've learned here; however, the strategy can be useful in finding the correct answer if you are confused and worried about getting the question wrong.

Monomials and Polynomials

In the previous section, we worked with variables and exponents. Now let's review some definitions that will come in handy on the SHSAT. In algebra, you will work with monomials and polynomials. A **monomial** is one term and may contain a constant, a variable, an exponent, or the product of a constant and one or more variables:

$3x^2$

A **constant** is a single number. In this example, the constant is 3. As you know, the variable is a letter or symbol, which in this example is *x*. A monomial can contain more than one variable as long as they are being multiplied. Here are some examples of monomials:

$4mn$

xy^3z

Notice that monomials do not contain addition or subtraction signs. A **polynomial**, on the other hand, is the sum or difference of more than one term.

$6y^2 + 3$

$7qrs^3 - 2s + 8$

$9a^2 + 4b + e$

Notice that each of these polynomials contains plus and/or minus signs. Polynomials can also contain positive exponents, but they do not contain negative exponents or square roots. The typical format of

a polynomial lists terms with the greatest exponents first, followed by other exponents in decreasing order.

$$12x^3 + 2x^2 - 4x + 5$$

Each part of the polynomial is called a **term**. The example above has four terms, each separated by a plus or minus sign.

Monomials

A monomial can contain multiplication. For example, $10b^2$ means $10 \times b^2$. Also, $6st$ means $6 \times s \times t$. Suppose we need to multiply two monomials.

$$4c^3 \times 5d^2$$

What this really means is $4 \times c^3 \times 5 \times d^2$.

To solve this, multiply the coefficients. Since we can't combine unlike terms, we will list the variables beside each other, which indicates multiplication.

$$4c^3 \times 5d^2 = 20c^3d^2$$

Let's try another example. Simplify:

$$3(a^4)(-6a^2)$$

As you know, terms beside each other in parentheses indicate multiplication. First, multiply the coefficients, keeping in mind that multiplying a positive by a negative results in a negative product. Also, remember that to multiply like terms containing exponents, add the exponents.

$$3(a^4)(-6a^2) = -18a^{4+2} = -18a^6$$

Polynomials

In order to evaluate polynomials, we will combine like terms. For terms to be alike, the variable part must be exactly the same.

$$5x^2 + 7x^2 =$$

Since the variable part of the first two terms is the same, x^2, these terms can be combined by adding the coefficients.

$$5x^2 + 7x^2 = 12x^2$$

At times, some terms in a polynomial will be alike and can be combined, while others can't.

$$8y^3 + 2y^3 + 4xy =$$

In this example, the first two terms can be combined because the variable part is exactly the same, y^3. However, the third term has a different variable part, so it cannot be combined.

$$8y^3 + 2y^3 + 4xy = 11y^3 + 4xy$$

Let's simplify the following by combining like terms.

$$10a^2 + 3a^2 - a^2$$

Since all of the variable parts of these terms are the same, a^2, we can add and subtract the coefficients to combine the like terms. The third term does not include a coefficient. In this case, it is understood that the coefficient is 1 because $1 \times a^2 = a^2$.

$$10a^2 + 3a^2 - a^2 = 12a^2$$

Let's try another one. Remember, we can only combine terms that are exactly alike to simplify the polynomial.

$$5k^2 + 7k - 3 - 2k + 4k^2 + 6$$

First, group the like terms together. Keep the plus and minus signs with the term that follows them. For example, since the polynomial above indicates that we should subtract $2k$, we will move the minus sign with the $2k$ when we group the like terms.

$$(5k^2 + 4k^2) + (7k - 2k) + (-3 + 6)$$

$$9k^2 + 5k + 3$$

Remember to arrange the terms in order of descending exponents.

It's your turn. Combine like terms in the equation below.

$$(15 + 5z) - (7 - 2z) =$$

A. $8 + 7z$
B. $8 + 3z$
C. $8 - 3z$
D. $22 - 7z$
E. $22 + 3z$

Did you select answer choice A? Great job! Let's review why this is the correct answer choice. To combine the like terms, first remove the parentheses. Keep in mind that the negative sign between the sets of parentheses must be applied to both terms within the second set. So $7 - 2z$ becomes $-7 + 2z$.

$$(15 + 5z) - (7 - 2z) = 15 + 5z - 7 + 2z$$

Next, combine the like terms.

$$15 + 5z - 7 + 2z = 5z + 2z + 15 - 7 = 7z + 8$$

Now that we've reviewed adding and subtracting polynomials, let's try multiplying a monomial and a polynomial. In this case, there are

two factors: one outside the parentheses and one inside.

$$4w(5w^2 + 2w - 3)$$

This multiplication sentence has two factors. The first factor is $4w$. The second factor is $(5w^2 + 2w - 3)$. Notice that the term $4w$ is outside of the parentheses, and the other terms are inside. This means that $4w$ must be multiplied by each of the terms within the parentheses. Start by multiplying $4w$ by the first term, $5w^2$. Then multiply $4w$ by each of the other terms.

$$4w(5w^2 + 2w - 3) = (4w \times 5w^2) + (4w \times 2w) + (4w \times -3)$$

$$20w^3 + 8w^2 - 12w$$

Now you try it. Remember to multiply the monomial by each of the terms of the polynomial.

Simplify $-7q^2(4q^3 - 9q^2 + q - 3)$.

A. $-28q^6 + 63q^4 - 7q^2 + 21q^2$

B. $-28q^5 + 63q^4 - 7q^3 + 21q^2$

C. $-28q^5 - 63q^4 + 7q^3 - 21q^2$

D. $-28q^4 + 63q^3 - 7q^2 + 21q$

E. $-28q^3 + 63q^2 - 7q + 21$

Did you choose answer choice B? Awesome! Let's check to see why this was the correct answer. Start by multiplying $-7q^2$ by each of the

terms of the other factor.

$$-7q^2(4q^3 - 9q^2 + q - 3) = (-7q^2 \times 4q^3) + (-7q^2 \times -9q^2) + (-7q^2 \times q) + (-7q^2 \times -3)$$

Remember that to multiply terms with exponents, the exponents are added.

$$(-7q^2 \times 4q^3) + (-7q^2 \times -9q^2) + (-7q^2 \times q) + (-7q^2 \times -3) = -28q^5 + 63q^4 - 7q^3 + 21q^2$$

Solving Inequalities

The symbols >, <, ≥, and ≤ are often used to compare values:

	MEANING
>	is greater than
<	is less than
≥	is greater than or equal to
≤	is less than or equal to

Inequalities use greater than and less than symbols to compare values that are not equal.

$6 > 3$

$-4 < 2 < 7$

$x > 9$

$y \leq 14$

Each of these inequalities compares values that are not the same. For example, the first inequality tells that *6 is greater than 3*. The next tells *that –4 is less than 2, and 2 is less than 7*. The third tells

that *the value of* × *is greater than 9*. The last example tells that *the value of* y *is less than or equal to 14*.

Inequalities are solved in much the same way as equations. We are trying to find the value of the variable. The only difference is that the result will not tell the exact value of the variable, rather it will tell that the value is greater than or less than the solution. Take a look at the following inequality.

$$n + 7 > 10$$

To solve the inequality, isolate the variable. Since 7 is added to *n*, we will subtract 7 from both sides of the inequality.

$$\begin{aligned} n + 7 &> 10 \\ n + 7 \underline{- 7} &> 10 \underline{- 7} \\ n &> 3 \end{aligned}$$

While the solution to this inequality does not give us an exact value for *n*, we do know that the value of *n* is greater than 3.

Let's try another example.

Which is equivalent to the following inequality?

$$15 \leq d - 9$$

A. $6 \leq d$

B. $6 \geq d$

C. $24 \geq d$

D. $d \geq 24$

E. $d \leq -24$

Since 9 is subtracted from the value on the right side of the inequality, we will add 9 to both sides to isolate the variable.

$$15 \leq d - 9$$
$$15 + 9 \leq d - 9 + 9$$
$$24 \leq d$$

Notice that $24 \leq \mathrm{d}$ is not one of the answer choices; however, $\mathrm{d} \geq 24$ is. Answer choice D means d *is greater than or equal to 24*. The solution we found when solving the inequality was *24 is less than or equal to* d. Both of these have the same meaning, since the terms are reversed as well as the inequality symbol. Watch carefully! Make sure your answer has the same meaning as the answer choice you select.

As you can see, solving inequalities is very much like solving equations. The only difference occurs when the inequality involves multiplying or dividing negative numbers. Any time we have to multiply or divide the inequality by a negative number, the inequality sign reverses. Check out the example below.

$$6x > 18$$

To solve this inequality, we will divide both sides by 6.

$$6x > 18$$
$$\frac{6x}{6} > \frac{18}{6}$$
$$x > 3$$

Now let's solve a similar inequality that includes *negative* 6*x* rather than positive 6*x*.

$$-6x > 18$$

In order isolate the variable and solve this inequality, we need to divide both sides by –6. When dividing or multiplying an inequality by a negative number, the inequality symbol reverses. Take a look.

$$-6x > 18$$

$$\frac{-6x}{-6} < \frac{18}{-6}$$

$$x < -3$$

We solved the inequality in the same way, with the exception of reversing the symbol. Let's try another example.

$$\frac{5y}{-3} + 4 < 19$$

First, subtract 4 from both sides so the term with the variable is alone on the left side of the inequality.

$$\frac{5y}{-3} + 4 < 19$$

$$\frac{5y}{-3} + 4 - 4 < 19 - 4$$

$$\frac{5y}{-3} < 15$$

The term $\frac{5y}{-3}$ involves division. This term means $5y \div -3$. In order to undo the division, we need to multiply both sides of the inequality by –3, which reverses the inequality sign.

$$\frac{5y}{-3} < 15$$
$$\frac{(-3)5y}{-3} > (-3)15$$
$$5y > -45$$

Notice that the inequality symbol reversed when we multiplied by a negative number. Now we can finish solving the inequality by dividing both sides by 5. Since we are now dividing by a positive number, the inequality symbol does not change again.

$$\frac{5y}{5} > \frac{-45}{5}$$
$$y > 9$$

In order to isolate the variable in $\frac{5y}{-3} < 15$, we could multiply both sides by the fraction $\frac{-3}{5}$, since this is the inverse of the fraction in the inequality. Multiplying by the inverse results in a product of 1. Since we would have been multiplying by a negative number, the inequality sign would still have been reversed following this step.

Writing Algebraic Equations

At times, in order to solve word problems, the problems need to be written as algebraic expressions. The key words that suggest which operation to use in solving the problem can be helpful in correctly writing the expression. Several examples of how to translate words into expressions are shown below.

Addition	the sum of 9 and x 10 more than s	$9 + x$ $s + 10$
Subtraction	7 less than q the difference of 8 and d	$q - 7$ $8 - d$
Multiplication	y multiplied by 12 the product of 6 and h	$12y$ $6h$
Division	the quotient of t and 5 k divided by 3	$\frac{t}{5}$ $\frac{k}{3}$

In a word problem, the word *is* often means *equals*. The word *of* often indicates multiplication. For example, *half of twelve is six* translates to $\frac{1}{2} \times 12 = 6$.

In algebra, it is common for word problems to require that they be written as algebraic expressions in order to be solved. Don't let this make you nervous. Just think about what the question is asking, and pay close attention to clues given in the words.

Write an expression to show that q is equal to three times the value of r.

The phrase *three times the value of* r tells that we need to multiply by 3 and r. So we know that q equals 3 multiplied by r.

$$q = 3r$$

Let's try another example.

The quotient of a and 9 is less than the sum of 8 and b.

The word *quotient* indicates division, and the word *sum* indicates addition. Notice that we are told that one of the values *is less than*

the other, rather than the values being equal. That means we will be using an inequality symbol, not an equals sign.

Let's look at each part of the statement separately.

> *The quotient of* a *and 9* translates to $\frac{a}{9}$. … *is less than* translates to the symbol <. … *the sum of 8 and* b translates to $8 + b$.

So *the quotient of* a *and 9 is less than the sum of 8 and* b translates to the inequality below.

$$\frac{a}{9} < 8 + b$$

Now you try it. Look for key words that indicate which operations and symbols to include.

> Which shows that one-third the value of *h* is less than the value of *j*, and *j* is less than or equal to 7?

A. $\frac{h}{3} < j \leq 7$

B. $\frac{h}{3} > j \geq 7$

C. $\frac{h}{3} \leq j < 7$

D. $3h < j \leq 7$

E. $3h > j \geq 7$

This problem involves two inequalities, so let's take a look at them one at a time. The first part of the problem states that *one-third the value of* h *is less than the value of j.* One-third the value of *h* is $\frac{1}{3}$ of *h*. This can be expressed as $\left(\frac{1}{3}\right)h$ or $\frac{h}{3}$. So *one-third the value of* h *is less than the value of* j is expressed as $\frac{h}{3}<j$. The first half of the correct inequality is shown below.

$$\frac{h}{3}<j$$

The second part of the problem includes other inequality. It tells that j *is less than or equal to 7*. This is expressed as $j \leq 7$. The second half of the inequality is shown below.

$$j \leq 7$$

So *one-third the value of* h *is less than the value of* j, *and* j *is less than or equal to 7* is expressed as the inequality below, which is answer choice A.

$$\frac{h}{3}<j \leq 7$$

In the examples we just reviewed, you had to translate a word problem into an algebraic expression. Sometimes, you will need to not only write the expression, but also find the value of the variable in order to answer a question. Take a look at the following example.

> Mackenzie is twice as old as Luke. Luke is 8 years old. How old is Mackenzie?

To solve this problem, you will write an algebraic expression to represent the situation, then solve the expression to answer the

question:

- First, think about what we know: Luke is 8 years old. Mackenzie is twice as old as Luke.
- Next, think about what we need to find out: How old is Mackenzie?
- Now let's use the information to write an expression. We'll use the variable *M* to represent Mackenzie's age. The words *twice as old* tell us that we will need to multiply Luke's age by 2 to find Mackenzie's age. Let's use the variable *L* to stand for Luke's age.

$$M = 2L$$

This equation tells us that Mackenzie's age is twice Luke's age. Since we know his age already, we can substitute this value for *L*.

$$M = 2L$$

$$M = 2(8)$$

Now, we can solve to find the value of *M*.

$$M = 2(8)$$

$$M = 16$$

So Mackenzie is 16 years old.

Let's try another one.

Reggie spent $\frac{5}{6}$ of an hour working on his math homework. Suri spent $2\frac{1}{2}$ times as many hours completing her homework. How many hours did Suri spend doing homework?

What we know is that:

- Reggie spent $\frac{5}{6}$ of an hour doing homework.
- Suri spent $2\frac{1}{2}$ times as long doing her homework.

What you need to find is how many hours Suri spent on homework.

The key words $2\frac{1}{2}$ *times* indicate that you will need to multiply. Let *R* stand for the time Reggie spent, and *S* represent the time Suri spent.

Suri's time is equal to $2\frac{1}{2}$ times as long as Reggie spent.

$$S = \left(2\frac{1}{2}\right)R$$

Since we know how long Reggie spent, let's substitute his time for the variable *R*.

$$S = \left(2\frac{1}{2}\right)R$$

$$S = \left(2\frac{1}{2}\right)\left(\frac{5}{6}\right)$$

Now we can multiply the fractions to solve for *S*.

$$S = \left(2\frac{1}{2}\right)\left(\frac{5}{6}\right)$$

$$S = \left(\frac{5}{2}\right)\left(\frac{5}{6}\right)$$

$$S = \frac{25}{12}$$

$$S = 2\frac{1}{12}$$

So Suri spent $2\frac{1}{12}$ hours doing her homework.

> Remember that when multiplying mixed numbers, such as $2\frac{1}{12}$, the mixed number needs to first be converted to an improper fraction.

Each of the previous two examples involved only multiplication. Some word problems will require algebraic expressions involving more than one step. Read the problem below, and determine how to write the information as an algebraic expression in order to answer the question.

> Marco and Adam play on the same basketball team. The number of points Marco scored in the district tournament was 6 less than 4 times the number of points Adam scored. If Adam scored 9 points, how many points did Marco score?

We know that:

- Marco scored 6 less than 4 times the number of points Adam scored.
- Adam scored 9 points.

We need to find how many points Marco scored.

> It can be helpful to underline key information in a math problem. On the SHSAT, feel free to underline, circle, or put stars by important facts and numbers in the test booklet. Just be careful not to make any extra marks on your answer sheet.

Let's use the information to write an algebraic expression. We can use the variable *M* for Marco's points and *A* for Adam's points. The phrase *6 less than* tells that we will need to subtract 6. The phrase *4 times* indicates multiplying by 4. Take another look at this sentence from the problem:

> The number of points Marco scored in the district tournament was 6 less than 4 times the number of points Adam scored.

So Marco's points are equal to 6 less than 4 times Adam's points.

$$M = 4A - 6$$

Since we know that Adam scored 9 points, we can substitute this value for *A*.

$$M = 4(9) - 6$$

Now we can solve the equation to find the value of *M*.

$$M = 4(9) - 6$$
$$M = 36 - 6$$
$$M = 30$$

Marco scored 30 points in the tournament. Substitute this value back into the equation to make sure the answer is correct.

$$M = 4A - 6$$
$$30 = 4(9) - 6$$
$$30 = 36 - 6$$
$$30 = 30$$

The equation is true, so we solved it correctly!

Not all word problems that require writing an algebraic expression will involve a real-life scenario. Sometimes the problems will simply be mathematical situations. Take a look at the following example.

> Three consecutive multiples of 5 have a sum of 240. What is the greatest of these numbers?

To write an expression that will help us solve the problem, let's start by figuring out what we know. *Three consecutive multiples of 5* tells us that each of the numbers is divisible by five and is five more than the previous number. For example, the numbers could be 5, 10, and 15 or possibly 25, 30, and 35. Let's call the first of the consecutive numbers *x*. The second of the consecutive numbers is five more than the first, so its value is $x + 5$. The third of the consecutive numbers is five more than that, or $x + 5 + 5$, which is the same as $x + 10$. So our three numbers are *x*, $x + 5$, and $x + 10$.

The problem states that these three consecutive multiples of five, which we are calling *x*, $x + 5$, and $x + 10$, have a sum of 240. The word *sum* indicates that the numbers are added together. So we can write the equation shown here.

$$x + (x + 5) + (x + 10) = 240$$

The question asks what the greatest of the numbers is. We have named the greatest number $x + 10$. To find its value, we need to first solve for *x*, then add 10.

To solve for x, start by combining like terms.

$$\begin{aligned} x + (x + 5) + (x + 10) &= 240 \\ x + x + x + 5 + 10 &= 240 \\ 3x + 15 &= 240 \end{aligned}$$

Next, subtract 15 from both sides to leave the term containing the variable alone on the left side of the equation.

$$3x + 15 - 15 = 240 - 15$$

$$3x = 225$$

Now we can divide both sides by 3 to isolate the variable.

$$\frac{3x}{3} = \frac{225}{3}$$

$$x = 75$$

Now we know that the value of *x* is 75. So the least of the consecutive multiples of 5 is 75. The question asked for the greatest of these numbers, which is $x = 10$.

$$x + 10 = 75 + 10 = 85$$

So the correct answer is 85.

You Try It!

In this chapter, you've reviewed a number of skills that will be helpful in answering the algebra questions on the SHSAT. Now, use what you have learned to select the best answer choice to each of the questions below.

1. What is the value of $f + g(g + 2f)f$ when $f = 3$ and $g = 5$?

A. 34
B. 44
C. 53
D. 58
E. 88

Did you choose D? Excellent! To find the value of the expression, substitute the given values of the variables.

$$f + g(g + 2f) = 3 + 5(5 + 2 \times 3)$$

Remember that the order of operations states that operations within parentheses must be completed first, and that multiplication and division are applied before addition and subtraction. That means the first step is to multiply 2×3 in the parentheses, then add the 5.

$$3 + 5(5 + 2 \times 3) = 3 + 5(5 + 6) = 3 + 5(11)$$

Now multiply 5×11, then add 3 to the product.

$$3 + 5(11) = 3 + 55 = 58$$

2. What is the value of $15(q - 7) = 4(q + 4)$?

A. 0
B. 1
C. 8
D. 11
E. 16

Let's solve the equation to find the value of the variable. First, multiply the constant outside of each set of parentheses by each term inside the parentheses.

$$15(q - 7) = 4(q + 4) = 15q - 105 = 4q + 16$$

Since the largest term containing the variable is on the left of the equals sign, let's move all of the variable terms to that side. To do so, subtract $4q$ from both sides of the equation. Then combine like terms.

$$\begin{aligned} 15q - 105 &= 4q + 16 \\ 15q - 105 - 4q &= 4q + 16 - 4q \\ 11q - 105 &= 16 \end{aligned}$$

Next, add 105 to both sides of the equation to isolate $11q$ on the left.

$$\begin{aligned} 11q - 105 &= 16 \\ 11q - 105 + 105 &= 16 + 105 \\ 11q &= 121 \end{aligned}$$

Now divide both sides of the equation by 11 to find the value of q.

$$\begin{aligned} 11q &= 121 \\ \frac{11q}{11} &= \frac{121}{11} \\ q &= 11 \end{aligned}$$

So the value of q is 11. Substitute this back into the original equation to verify the answer.

$$\begin{aligned} 15(q - 7) &= 4(q + 4) \\ 15(11 - 7) &= 4(11 + 4) \\ 15(4) &= 4\ (15) \\ 60 &= 60 \end{aligned}$$

The equation is true, so the answer was correct! The value of the variable is 11.

3. Simplify $(24c^2 + 9) - (-6 + 3c^2) + 2c$.

A. $24c^2 + 2c + 17$

B. $21c^2 + 2c + 17$

C. $21c^2 + 2c + 3$

D. $29c^2 + 3$

E. $23c^2 + 17$

Start by removing the parentheses. Be sure to apply the negative sign to all terms in the second set of parentheses.

$$(24c^2 + 9) - (-6 + 3c^2) + 2c =$$

$$24c^2 + 9 + 6 - 3c^2 + 2c$$

To simplify the expression, combine like terms. Remember, for terms to be alike, they must have the exact same variable and exponent. Remember to arrange the terms in order of descending exponents.

$$24c^2 + 9 + 6 - 3c^2 + 2c$$

$$24c^2 - 3c^2 + 2c + 9 + 6$$

$$21c^2 + 2c + 17$$

So answer choice B is correct.

4. Simplify $-12x + 4 < -9x + 25$.

A. $x > -7$

B. $x < -7$

C. $x < 7$

D. $x > 7$

E. $x = 7$

The symbol for *is less than* (<) tells that this is an inequality. As you know, inequalities are solved in much the same way as equations. First, use opposite operations to collect like terms on either side of the inequality. Let's move the terms with variables to the left, and the terms without variables to the right.

$$-12x + 4 < -9x + 25$$
$$-12x + 4 + 9x < -9x + 25 + 9x$$
$$-3x + 4 < 25$$
$$-3x + 4 - 4 < 25 - 4$$
$$-3x < 21$$

Next, we have to divide both sides of the inequality by –3 in order to isolate the variable. This is the step where things are slightly different from solving an equation. Since we will be dividing by a negative number, the inequality sign will reverse. So, instead of being the symbol for *is less than*, it will become the symbol for *is greater than* (>).

$$-3x < 21$$
$$\frac{-3x}{-3} > \frac{21}{-3}$$
$$x > -7$$

5. Three friends each earned babysitting money over the weekend. Keisha earned $2.50 more than Chloe. Chloe earned three times as much as Sydney. If the total the friends earned was $46.25, how much did Keisha earn?

A. $6.25
B. $8.75
C. $18.75
D. $21.25

E. $26.25

In order to solve this word problem, we need to write it as an algebraic expression. First, think about what we know:

- Keisha earned $2.50 more than Chloe.
- Chloe earned 3 times more than Sydney.
- The total earned was $46.25.

We don't know how much Sydney earned, so we'll use the variable *s* to represent the amount of money she made. We do know that Chloe earned 3 times as much as Sydney. The words *3 times as much* indicate that we will multiply Sydney's earnings by 3 to find Chloe's earnings. So, Chloe's money is represented by $3 \times s$, or $3s$. Keisha earned $2.50 more than Chloe. The phrase *$2.50 more than* tells that we will add this amount to Chloe's total. So, Keisha made $3s + 2.50$.

The total the friends earned was $46.25. That means Sydney's money + Chloe's money + Keisha's money = $46.25.

$$s + 3s + (3s + 2.50) = 46.25$$

Now we can combine like terms and solve for the variable. First, combine all terms containing the variable.

$$\begin{aligned} s + 3s + (3s + 2.50) &= 46.25 \\ s + 3s + 3s + 2.50 &= 46.25 \\ 7s + 2.50 &= 46.25 \end{aligned}$$

Next, subtract 2.50 from both sides to get the variable term alone on the left side of the equation.

$$7s + 2.50 = 46.25$$

$$7s + 2.50 - 2.50 = 46.25 - 2.50$$

$$7s = 43.75$$

Finally, divide both sides of the equation by 7.

$$7s = 43.75$$

$$\frac{7s}{7} = \frac{43.75}{7}$$

$$s = 6.25$$

We have found the value of the variable!

However, we're not quite finished. The question asked how much Keisha earned. The variable represents the amount Sydney earned. Take another look at the information given in the problem.

> Keisha earned $2.50 more than Chloe. Chloe earned three times as much as Sydney.

Solving for the variable, *s*, showed us that Sydney earned $6.25. Chloe earned three times this amount. So, she earned $3 \times \$6.25$.

$$3 \times \$6.25 = \$18.75$$

Chloe earned $18.75. Keisha earned $2.50 more than Chloe. So Keisha earned $\$18.75 + \$2.50$.

$$\$18.75 + \$2.50 = \$21.25$$

So Keisha earned $21.25, which means answer choice D is correct.

Let's Review!

In this chapter, you've reviewed vocabulary and skills that will be helpful in answering the algebra questions on the SHSAT Math test. Let's go over some of the key points that you will need to remember in order to do your best on this section of the test.

- **Variables** are letters or symbols that take the place of unknown values in math. In the equation $9x = 27$, *x* is the variable.
- **Absolute value** is the distance of a number from zero on the number line. The absolute value of a number is always positive. The absolute value of 17 is shown as |17|.

$$|-45| = 45$$

- When the value of a variable is given, the value can be substituted into an equation or expression. Substitution can also be used to check your answer when solving for the value of a variable.
- To find the value of a variable, the variable must be isolated on either side of the equation. Use opposite operations of those included in the equation, being sure to perform the same operation on both sides of the equals sign.
  - Addition and subtraction are opposite operations.
  - Multiplication and division are opposite operations.
  - Squared numbers and square roots are opposites.
- Inequalities are solved in the same way as equations. The only difference is that the inequality symbol is reversed when multiplying or dividing by a negative number. These symbols may be included in inequalities: <, >, ≤, ≥.
- A **monomial** is a single term that may contain a constant, a variable, an exponent, or the product of a constant and one or more variables. $15x^2y^3$ is an example of a monomial.
- A **polynomial** is the sum or difference of more than one term. In other words, it is a group of two or more monomials being added or subtracted. $15x^2y^3 + 32x - 14$ is an example of a polynomial. It contains three terms.

- In order to solve some word problems, they must be written as algebraic expressions. Key words in the problem indicate which operation, or operations, must be used to solve the problem.

CHAPTER 9
Geometry

Some of the questions on the Math section of the SHSAT will assess how well you understand geometry. As you know, geometry questions deal with lines, angles, and shapes. There are a number of important vocabulary words and formulas you will need to know in order to answer these questions. Pay close attention to each of these. It is important that you not only memorize the formulas, but also be comfortable applying them to solve problems.

Lines and Angles

There are quite a few important terms to understand in geometry. While you will not have to define these words on the SHSAT, it will be important to recognize and understand each of them. Let's start by reviewing the types of lines and angles that are the building blocks of many geometric figures.

Lines, Segments, and Rays

Lines are one of the basic geometrical terms. By definition, a line continues forever in both directions. For this reason, the ends of lines have arrows on them.

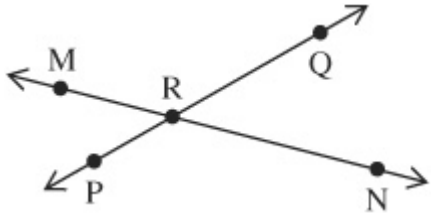
A line that passes through two points is often named for those points. In the example below, the line passes through points C and D, so it can be named *line CD*. The symbol CD $\overleftrightarrow{CD}$ represents this line.

A **point** identifies an exact location on a line. The point has no length or width. It is simply a specific location. The line above contains points C and D.

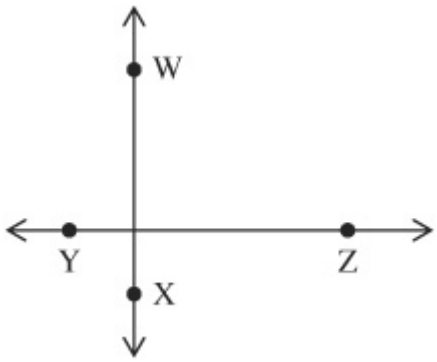
As we just discussed, lines continue indefinitely in a specific location. **Line segments**, on the other hand, have two definite end points. The end of a line segment is identified by a point. Line segment FG, also known as $\overline{FG}$, is shown below. Notice that it begins at exactly point F and ends at exactly point G.

Like line segments, **rays** also have a specific beginning point, however, they continue forever in one direction. So one end of a ray is indicated by an endpoint, and the other is indicated by an arrow, pointing in the direction in which the ray continues. The ray below would be known as *ray HJ*, or $\overrightarrow{HJ}$.

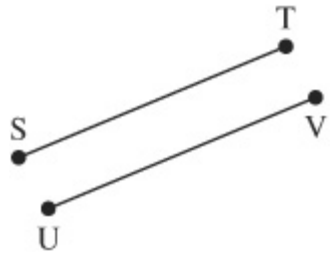
Intersecting lines, segments, and rays share a common point. In other words, the lines cross one another. For example, in the following diagram, lines MN and PQ intersect, or cross, at point R.



Lines that are **perpendicular** meet or intersect and form a 90-degree angle. In the diagram below, lines WX and YZ are perpendicular.



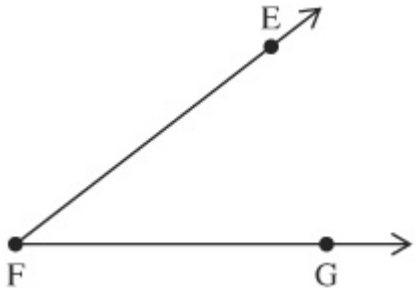
Lines, segments, or rays that are **parallel** never intersect, or cross each other. The lines can go on forever, and they will never touch. They remain the same distance apart at all points. Line segments ST and UV below are parallel. This is written as segment ST || segment UV, or $\overline{ST} \parallel \overline{UV}$, and is read *segment ST is parallel to segment UV*.



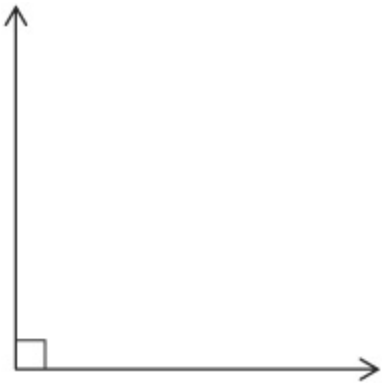
> Having trouble remembering the difference between perpendicular and parallel lines? Take a look at the two l's in parallel. They look like parallel lines. Notice that they never cross each other, and they remain the same distance apart.

Types of Angles

Two rays that share an endpoint are called an **angle**. The rays are the sides of the angle. The endpoint where the rays meet is called the **vertex**. An angle can be named by a point on each of the rays and the vertex. In the example below, points E and G are located on the rays, and point F is the vertex. This angle would be called *angle EFG* or *angle GFE*. The name of the vertex is always in the middle. Angle EFG can be named by the symbol ∠EFG.

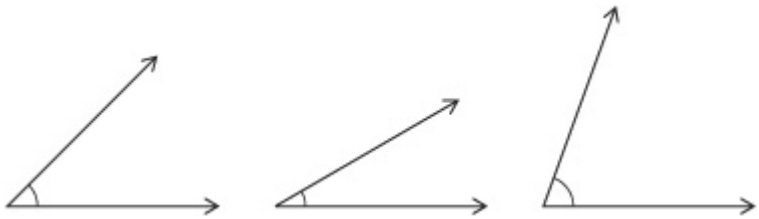


There are several types of angles, each named for the size of the space between the rays. If the rays are closer together, with a narrow space between them, the angle has a smaller measure. If the rays have a wide space between them, the angle has a larger measure. Two rays that are perpendicular to each other form a 90-degree angle, which looks like the letter L. A 90-degree angle is known as a **right angle**.

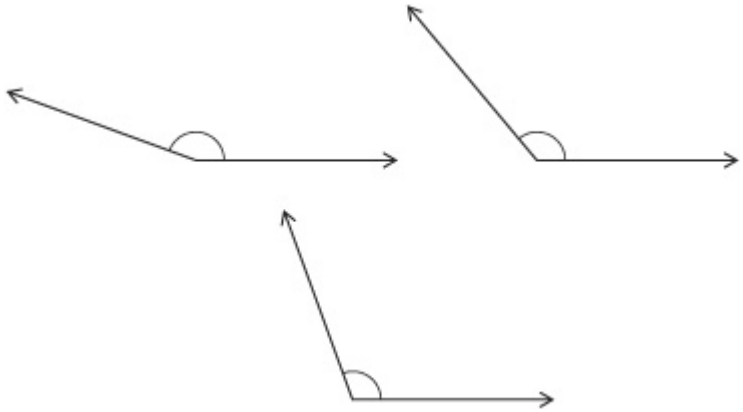


The small square symbol (□) in the corner of an angle indicates a right angle. Take a look at the symbol in the example of the right angle. When you see the little square, you know that the angle measures 90 degrees.

An angle that measures less than 90 degrees is known as an **acute** angle. Each of the following angles is an acute angle.

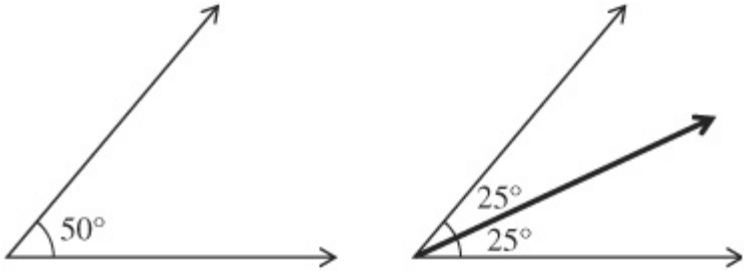


An angle that that measures greater than 90 degrees is known as an **obtuse** angle. Each of the following angles is an obtuse angle.



An angle that is completely **straight** measures 180 degrees.

Sometimes, a ray divides an angle into two equal parts. A ray that does this is called an **angle bisector**. The angle on the left below measures 50 degrees. On the right, an angle bisector divides it into two 25-degree angles.

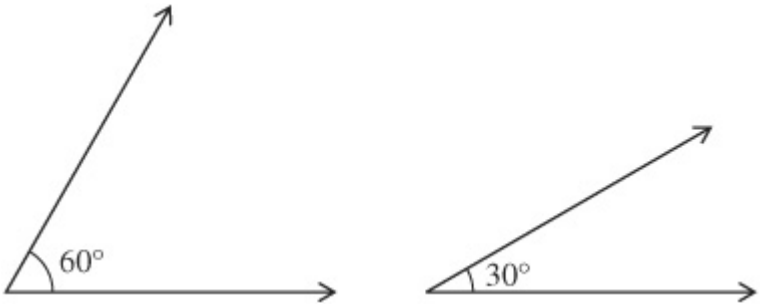


Relationships Between Angles

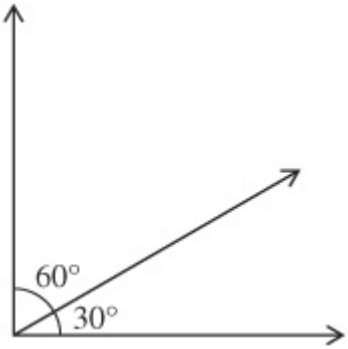
Understanding the relationship between types of angles can help in solving problems and determining the unknown measurement of an

angle.

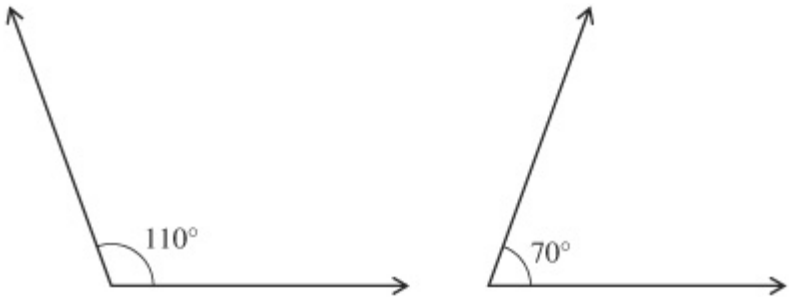
Two angles are said to be **complementary** if the sum of the angles equals 90 degrees. For example, an angle measuring 30 degrees and an angle measuring 60 degrees are complementary because their sum is 90 degrees: $30 + 60 = 90$.



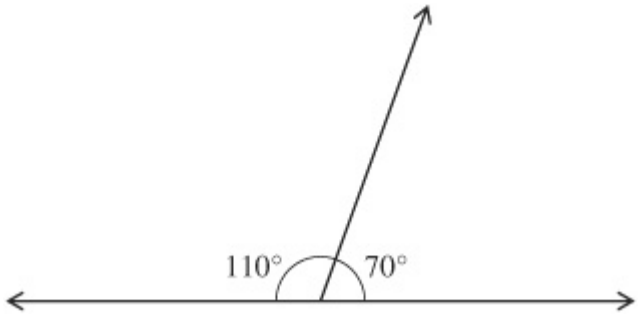
When put together, the complementary angles equal a right angle.



Two angles are said to be **supplementary** if the sum of the angles equals 180 degrees. For example, an angle measuring 70 degrees and an angle measuring 110 degrees are supplementary because their sum is 180 degrees: $70 + 110 = 180$.

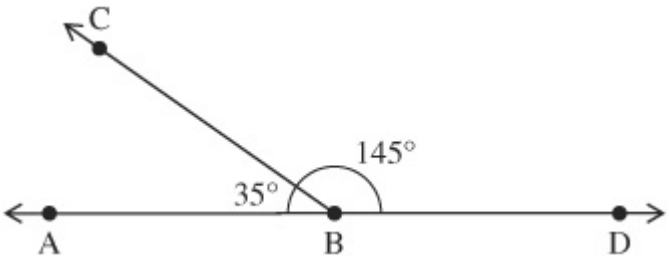


When put together, the supplementary angles form a 180-degree angle, or a straight line.



Let's use some of this information to answer a question.

In the diagram below, ∠ABC and ∠CBD are supplementary. What is the measure of ∠CBD?



A. 55°
B. 90°
C. 145°
D. 155°
E. 180°

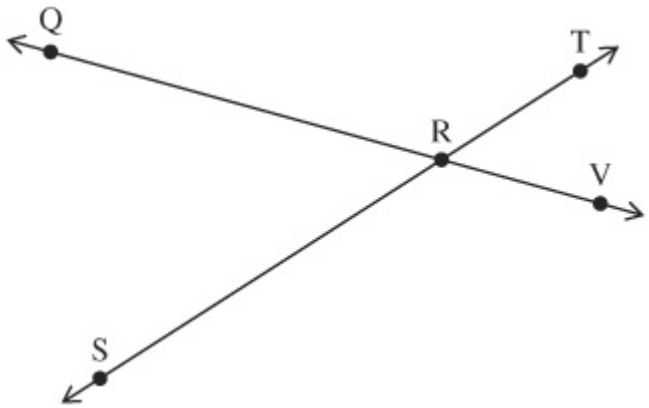
We know that when two angles are supplementary, the sum of their measures is 180 degrees. So $\angle ABC + \angle CBD = 180$. To find the missing measurement, we can subtract the measure of ∠ABC from 180.

$$180 - 35 = 145$$

The measure of ∠CBD is 145 degrees.

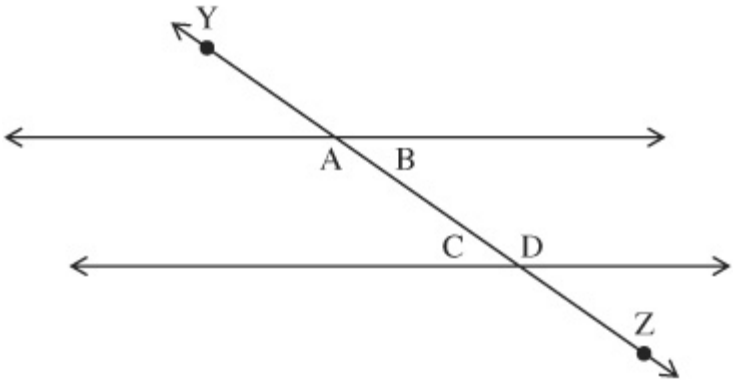
In the example above, not only are the angles supplementary, they are also adjacent. **Adjacent angles** share a vertex and a common side. Angles divided by a bisector are also adjacent since they share a vertex and a common side.

Many angles are formed by lines that meet or intersect. **Vertical angles** are opposite of each other when two lines intersect. The important thing to remember about vertical angles is that their measurements are equal. In the diagram below, ∠QRS and ∠TRV are vertical angles. So, $\angle QRS = \angle TRV$.



Suppose ∠QRS measures 48 degrees. That means ∠TRV also measures 48 degrees. Notice that in this example, ∠QRT and ∠SRV are also vertical angles because they are opposite of each other.

Other angle pairs also have equal measurements. Take a look at the following diagram.

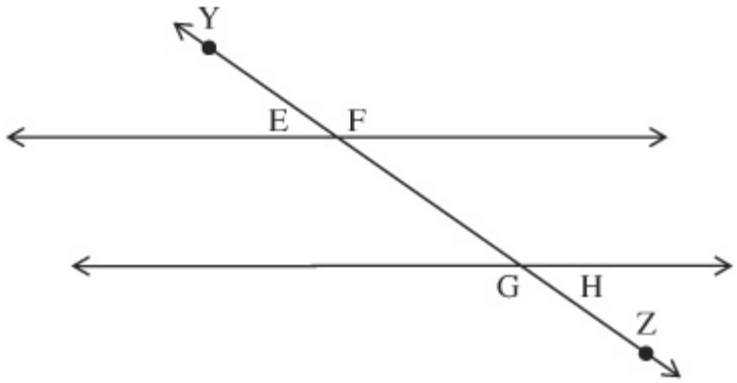


Alternate interior angles are created when two parallel lines are intersected by a third line, as shown. The line that crosses the

parallel lines is called a **transversal**. Alternate interior angles are a pair of angles on opposite sides of the transversal, and they have the same measurement. They are located inside, or between, the set of parallel lines. In the diagram, line YZ is the transversal. Angles B and C are alternate interior angles. Since angle B has a measure of 35 degrees, angle C also has a measure of 35 degrees. (Angles A and D are also alternate interior angles.)

> Alternate interior angles are on alternate, or opposite, sides of the transversal. They are on the interior, or inside, of the parallel lines.

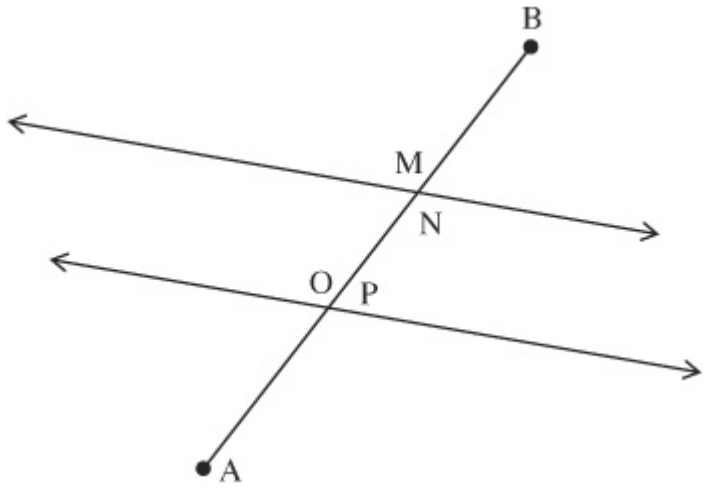
Like alternate interior angles, alternate exterior angles also have the same measurement. **Alternate exterior angles** are on opposite sides of the transversal, but they are on the outside of the parallel lines.



In the previous diagram, line YZ is the transversal. Angles E and H are on opposite sides of the transversal and are outside the set of parallel lines. So angles E and H are alternate exterior angles, and they have equal measurements. Likewise, angles F and G are alternate exterior angles.

Let's use what we've reviewed about angles to answer the following question.

Line segment AB is a transversal intersecting a pair of parallel lines. Angle M has a measurement of 118 degrees.



What is the measure of angle P?

A. 28°
B. 62°
C. 72°
D. 118°
E. 162°

We know that angle M has a measurement of 118. Since angles M and N are vertical angles, they have the same measurement. So, ∠M and ∠N are both 118 degrees. Angle N and angle O are alternate interior angles, which means they also have the same measurement. Since we know that ∠N measures 118 degrees, ∠O also measures 118 degrees. Angle O and angle P are supplementary angles, so the sum of ∠O and ∠P equals 180 degrees. To find the measurement of ∠P, subtract 118 from 180.

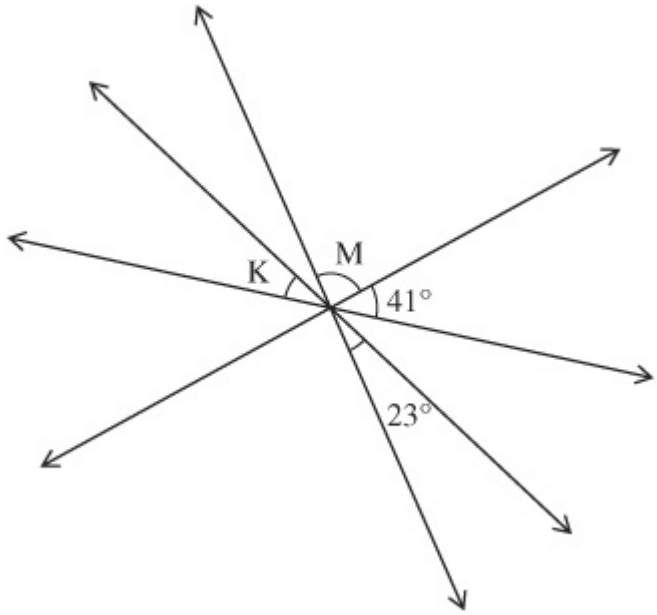
$$180 - 118 = 62$$

So the measure of angle P is 62°.

As you know, variables represent unknown values. We worked with variables in the chapter of this book that reviews algebra skills.

There will be questions on the SHSAT that require you to use skills from several areas of mathematics. For example, you will need to rely on your knowledge of both algebra and geometry to answer the following question.

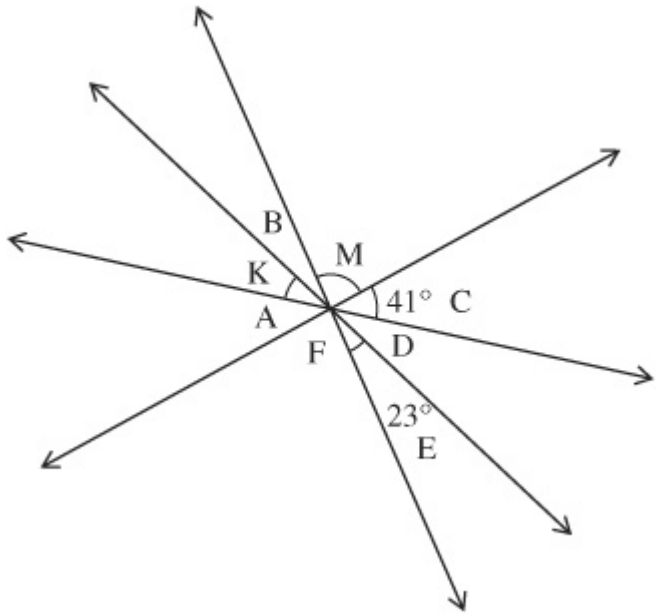
The four lines in the diagram intersect at a single point.



What is the measure of ∠K in terms of M?

A. 180 – M
B. 157 – M
C. 139 – M
D. 126 – M
E. 116 – M

In order to discuss each of the angles in the diagram, let's label them. We'll label them A through F, although you could use any letters or numbers you choose. The purpose of labeling them is simply for our discussion of how to solve the problem.



Angles A, K, B, and M are on the same straight line. That means, the sum of these angles is 180 degrees. In other words, $\angle A + \angle K + \angle B + \angle M = 180$.

Angle A and angle C are vertical angles, which means they have the same measurements. Since we know from the diagram that ∠C equals 41 degrees, we know that ∠A also equals 41 degrees. Let's substitute this value into the equation.

$$\angle A + \angle K + \angle B + \angle M = 180$$

$$41 + \angle K + \angle B + \angle M = 180$$

Next, take a look at angles B and E. These are also vertical angles, which means that since ∠E equals 23 degress, ∠B also equals 23 degrees. Let's add this information to the equation as well.

$$41 + \angle K + \angle B + \angle M = 180$$

$$41 + \angle K + 23 + \angle M = 180$$

Now we can solve for the value of ∠K in the same way we would solve for any other variable. Begin by combining like terms and isolating ∠K on one side of the equation.

$$41 + \angle K + 23 + \angle M = 180$$
$$41 + 23 + \angle K + \angle M = 180$$
$$64 + \angle K + \angle M = 180$$
$$64 - 64 + \angle K + \angle M = 180 - 64$$
$$\angle K + \angle M = 116$$
$$\angle K + \angle M - \angle M = 116 - \angle M$$
$$\angle K = 116 - \angle M$$

So in terms of M, $\angle K = 116 - M$.

Two-Dimensional Figures

When you think of geometry, you probably think of shapes, such as squares and triangles. Much of geometry does deal with polygons. A **polygon** is a two-dimensional figure made from three or more straight sides. In fact, the word *polygon* actually means "many sided." The shapes that often come to mind when you think of geometry, such as squares and triangles, are polygons. The list of polygons below tells how many sides each of the figures contains.

NUMBER OF SIDES

Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

The number of angles in any polygon is equal to the number of sides. For example, since a triangle has three sides, it also has three

angles. And since an octagon has eight sides, it also has eight angles.

The sum of the angles of a triangle is 180 degrees. So a triangle could have three 60 degree angles, because 60 + 60 + 60 = 180. Or it could have one 90 degree angle and two 45 degree angles, because $90 + 45 + 45 = 180$. We'll talk more about triangles later in this chapter.

The sum of the angles of a quadrilateral, however, is 360 degrees. As you know, a square has four sides, so it is a quadrilateral. A square has four 90 degree angles, and $90 + 90 + 90 + 90 = 360$. The sum of the interior angles of any quadrilateral equals 360 degrees.

The sum of the interior angles of a pentagon is 540 degrees, and the sum of the interior angles of a hexagon is 720 degrees. But rather than trying to remember a list of polygons and the sums of their interior angles, all you have to memorize is the formula below.

$$(n - 2) \times 180^\circ$$

In the formula, the variable *n* represents the number of sides of the polygon. So what the formula tell us is that the number of sides of a polygon, minus 2, multiplied by 180 equals the sum of the interior angle of the figure. Let's prove this using a triangle. Since we know a triangle has 3 sides, we'll substitute 3 for *n*.

$$(n - 2) \times 180^\circ$$

$$(3 - 2) \times 180^\circ$$

$$(1) \times 180^\circ$$

$$180^\circ$$

As we already reviewed, the sum of the interior angles of a triangle is 180 degrees. Let's try this formula again to find the sum of the angles of an octagon.

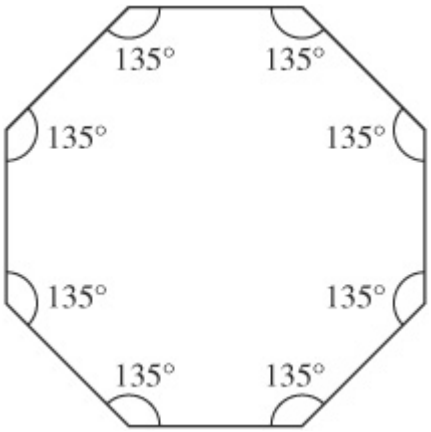
$$(n-2)\times 180^\circ$$

$$(8-2)\times 180^\circ$$

$$(6)\times 180^\circ$$

$$1{,}080^\circ$$

The sum of the interior angles of an octagon is 1,080 degrees.

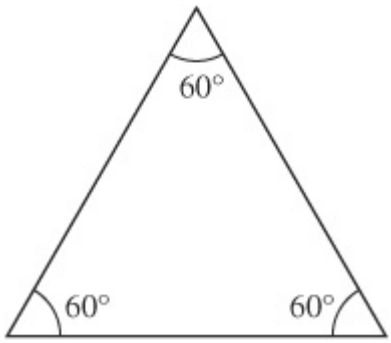


Now that we've reviewed the number of sides and the sum of the angles of polygons, let's go over the types of triangles and quadrilaterals.

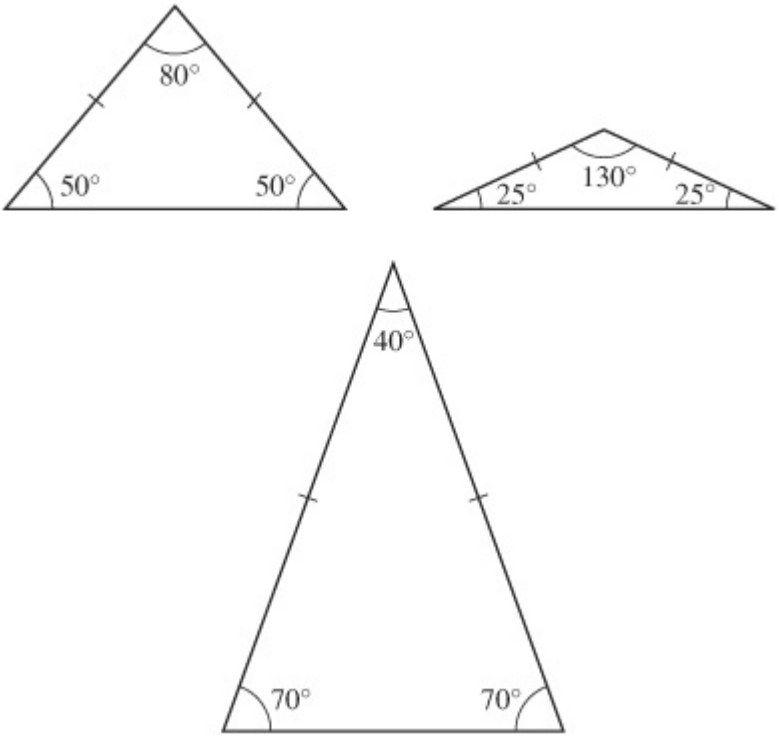
Triangles

All triangles have three sides and three angles, but the lengths of the sides and the measures of the angles can vary. The sum of all of the angles is always 180 degrees, but the individual angles can be quite different. Triangles can be classified by the length of their sides or by their angle measurements.

A triangle that has three equal sides and three equal angles is called an **equilateral triangle**. Each of the angles of an equilateral triangle measures 60 degrees.

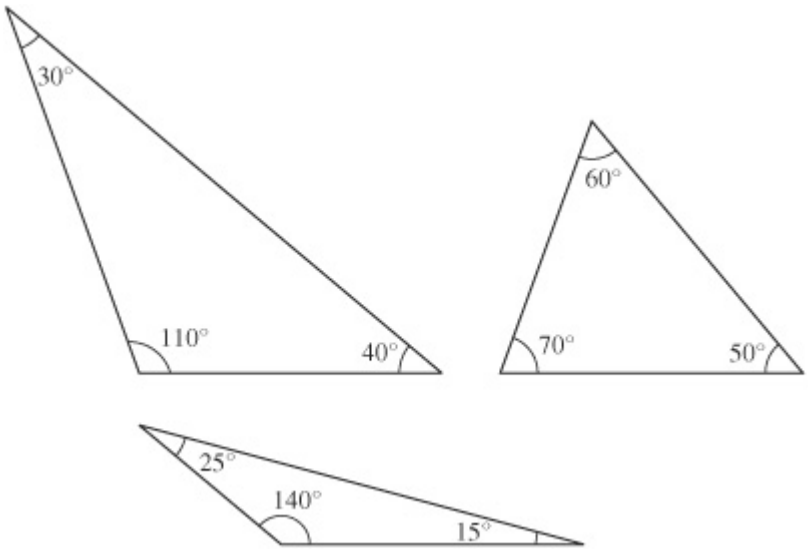


An **isosceles triangle** has two sides that are equal and two angles that are equal. Notice that in each of the isosceles triangles below, two angles have the exact same measure and two of the sides have the exact same length.

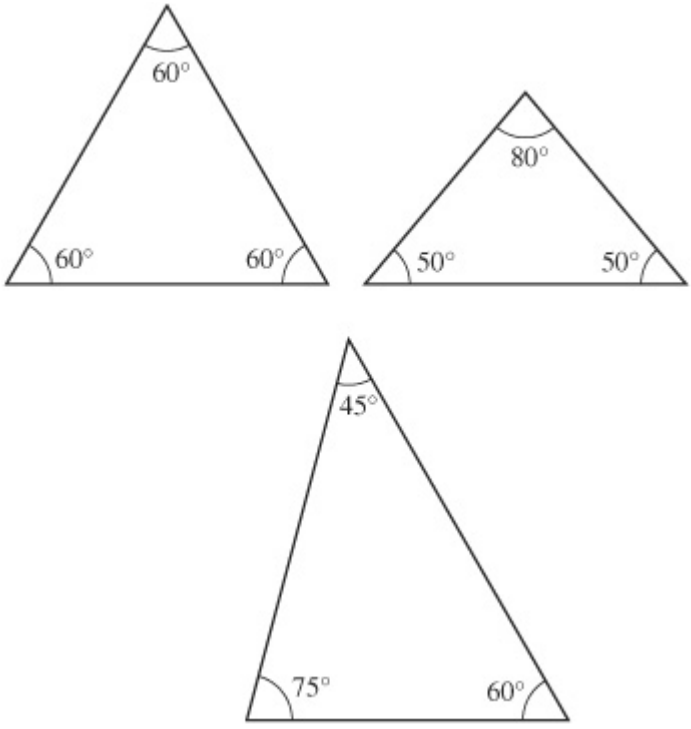


Notice the little lines marking two of the sides on each of the isosceles triangles. In geometry, these marks indicate that the lines are the same length. So, in an equilateral triangle, all three sides would be marked since all three sides are equal in length.

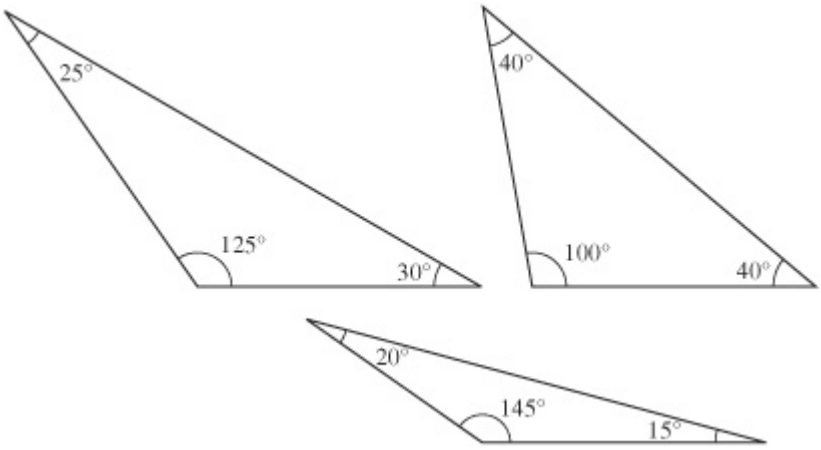
A triangle with no equal sides is called a **scalene triangle**. Since each of the sides of a scalene triangle has a different length, each of the three angles in a scalene triangle also has a different measure.



The angles of an **acute triangle** are all less than 90 degrees.

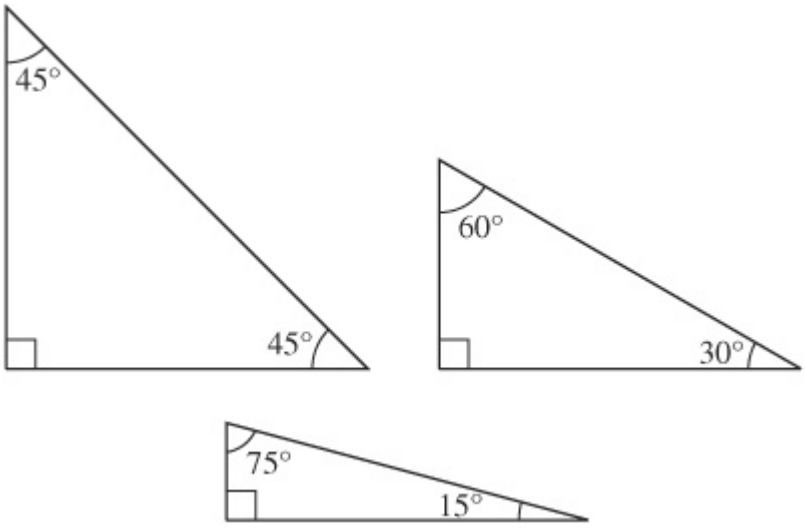


An **obtuse triangle** has one angle that measures greater than 90 degrees.

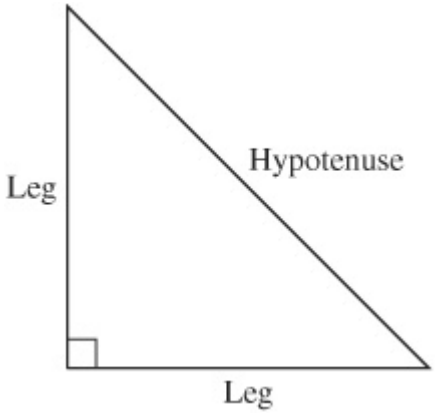


Triangles can be classified in more than one way. For example, an equilateral triangle is also an acute triangle, since all of the angles are less than 90 degrees. A triangle that is classified as a scalene obtuse triangle has one angle that is greater than 90 degrees, and each of the sides is a different length.

A **right triangle** has one right angle, which measures 90 degrees. The other angles may or may not be equal to each other, and the other sides, or legs, of the triangle may or may not be equal to each other. Notice that each of the right triangles below is different from the others.

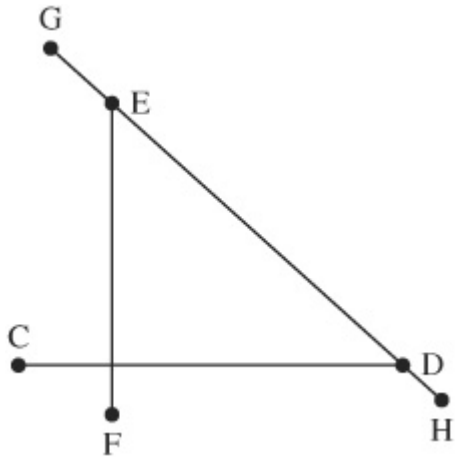


The side of the triangle that is across from the right angle has the greatest length and is called the **hypotenuse**. The other two sides of the triangle are called **legs**.



Let's use what you've learned about triangles to answer the following question.

In the diagram, line segment EF is perpendicular to line segment CD. The measure of ∠FED is 48 degrees.



What is the measure of ∠CDE?

A. 32
B. 42
C. 90
D. 138
E. 222

Perpendicular lines form a right angle, so the triangle formed by the three line segments is a right triangle. We know that the angle formed by lines EF and CD is a right angle, which measures 90 degrees, and that the measure of angle FED is 48 degrees. The sum of the interior angles must equal 180 degrees.

$$90 + 48 + \angle CDE = 180$$
$$138 + \angle CDE = 180$$
$$138 - 138 + \angle CDE = 180 - 138$$
$$\angle CDE = 42$$

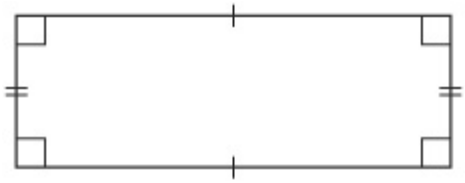
Therefore angle CDE has a measurement of 42 degrees.

Quadrilaterals

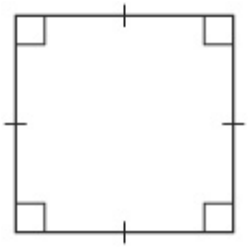
Quadrilaterals are polygons that have four sides and four angles. A square, rectangle, rhombus, parallelogram, and trapezoid are all types of quadrilaterals. The angle measures, side length, and

number of parallel sides vary between each of these figures, however, the sum of the interior angles of all quadrilaterals equals 360 degrees.

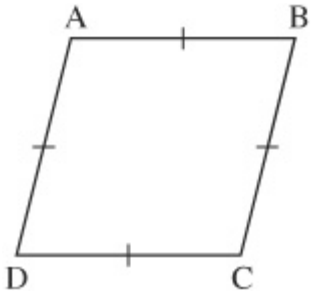
A **rectangle** is probably one of the first figures you ever learned about. As you know, rectangles have four right angles and two pairs of equal, parallel sides.



A **square** also has four right angles and is actually a type of rectangle. But the sides of a square are all the same length.



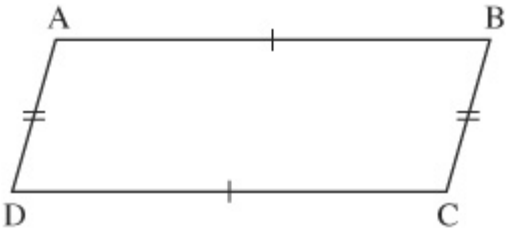
Like a square, a **rhombus** also has four sides of equal length, and the opposite sides are parallel. The corners of a rhombus can be angles of any size; however, opposite angles are equal. In the following example, corners A and C have equal measures, and corners B and D have equal measures.



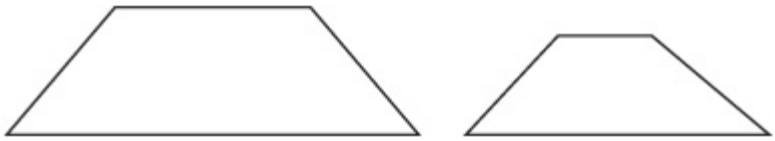
Not only does a square match the definition of a rectangle, it also matches the definition of a rhombus: it has four sides of

equal length, opposite sides are parallel, and opposite angles are equal.

Squares, rectangles, and rhombuses are all examples of another type of quadrilateral, the **parallelogram**. The opposite sides of a parallelogram are parallel and have equal lengths. Notice in the following example that the top and bottom sides are equal in length, and the left and right sides are equal in length. Again, angles A and C have the same measure, and angles B and D have the same measure.



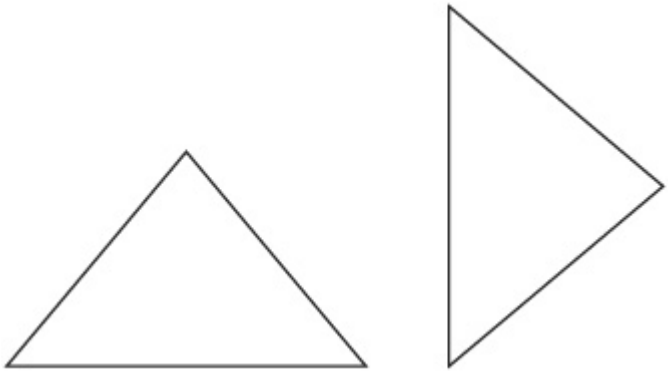
A **trapezoid** is a quadrilateral that has one pair of parallel sides. Notice that the sides do not have to be equal in length.



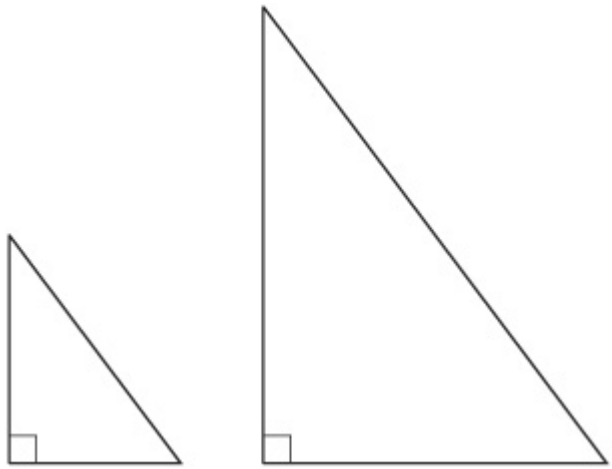
Congruence and Similarity

Figures that have exactly the same shape and size are **congruent**. The lengths of the sides of congruent figures are equal, and the angle measures of congruent figures are equal.

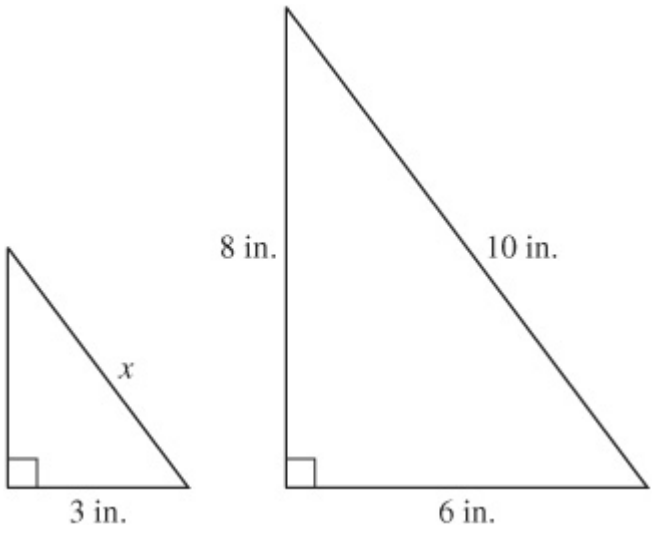
The figures can be positioned differently and still be congruent, as long as the lengths and angle measures are the same.



Sometimes a pair of figures will have exactly the same shape, but be different sizes. These figures are said to be **similar**. Take a look at the similar triangles. Both are right isosceles triangles. Both have the exact same angle measures. However, one is larger than the other. They are not congruent since the lengths of the sides are not equal; however, they are similar to one another.



Understanding similarity and congruence can help you solve problems regarding the measurements of geometric figures. Let's take another look at the similar right triangles above. Suppose we know the measurements of the larger triangle, but do not know the length of the hypotenuse of the smaller triangle.



Because the triangles are similar, we can use a ratio of the corresponding sides to find the length of the hypotenuse of the smaller triangle. We know that the ratio of the shorter leg to the hypotenuse of the large triangle is 6:10, or $\frac{6}{10}$. We know that the ratio of the shorter leg to the hypotenuse of the smaller triangle is 3:*x*, or $\frac{3}{x}$. We can use the ratios to set up a proportion and solve for *x* to find the length of the hypotenuse.

$$\frac{6}{10}=\frac{3}{x}$$

Now cross multiply and solve for the variable.

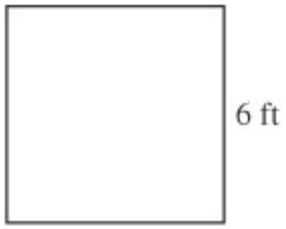
$$\frac{6}{10}=\frac{3}{x}$$
$$6(x)=10(3)$$
$$6x=30$$
$$\frac{6x}{6}=\frac{30}{6}$$
$$x=5$$

So, the hypotenuse of the smaller triangle is 5 inches.

Perimeter and Area

Measurement of figures is an important part of geometry. We've already discussed angle measurements. Now let's review what you need to know about perimeter and area of two-dimensional figures.

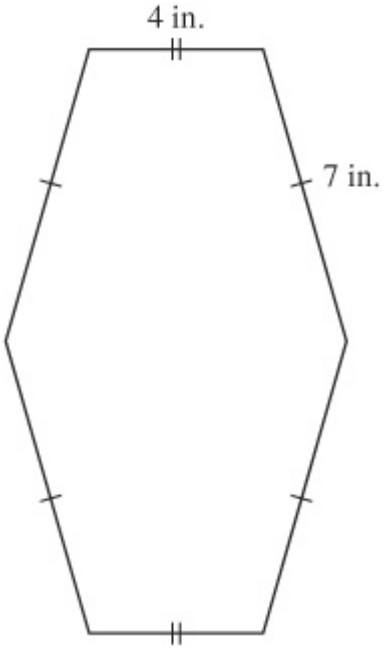
Perimeter is the distance around a figure. To find the perimeter, simply add the lengths of each of the sides. Take a look at the square below.



The length of each of the sides of the square shown is 6 feet. So the distance around the entire figure would be 24 feet, because $6 + 6 + 6 + 6 = 24$.

Since a square and a rhombus have four equal sides, we can simply multiply the length of one of the sides by 4 to find the perimeter. For other polygons, add to find the sum of the side lengths to determine the perimeter of the figure.

What is the perimeter of the hexagon?

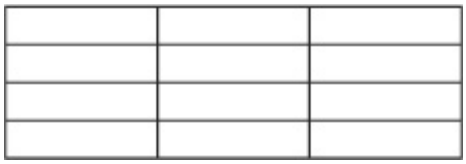


To find the perimeter of the figure, find the sum of the lengths of the sides. The symbols for equal side lengths indicate that four of the sides have a length of 7 inches each, and two of the sides have a length of four inches each. Add the side lengths to find the perimeter.

$$7 + 7 + 7 + 7 + 4 + 4 = 36$$

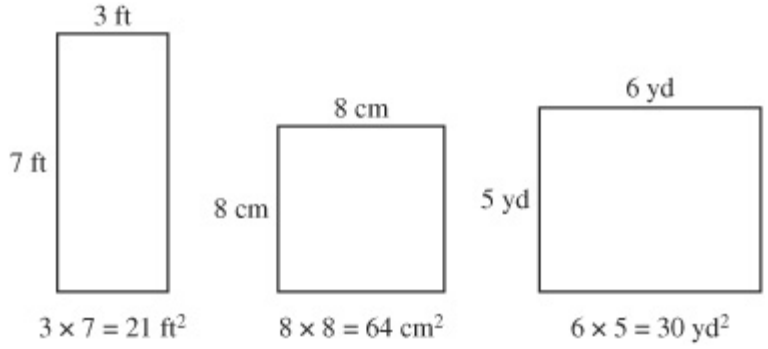
So the perimeter of the figure is 36 inches.

While perimeter measures the distance around shapes, **area** measures the space within two-dimensional figures. For example, the rectangle below contains 12 square units. So we would say that the area of the rectangle is 12 square units.



Examples of common square units often used to measure area include square inches, square feet, square yards, or square miles. If you were measuring the area of your bedroom, you would likely measure in square feet. If you were telling someone about the size of New York City, you would report the area in square miles.

To find the area of a rectangle or a square, we can simply multiply the length times the width of the figure.



The area of the first rectangle is 21 square feet. The area of the square is 64 square centimeters. The area of the final rectangle is 30 square yards.

There are different formulas for finding the areas of each type of two-dimensional figure. In the formulas for area, you will use the length, width, base, and/or height of the figures. Each measurement is represented by the letter shown.

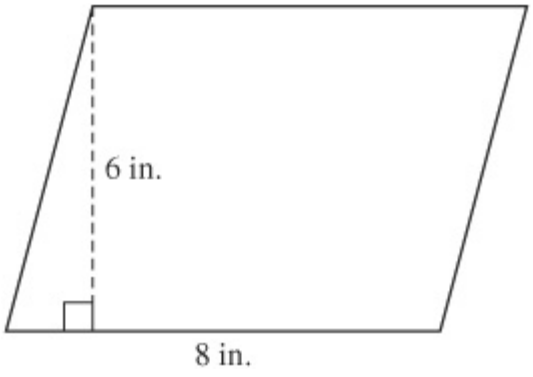
Area	A
Length	l
Width	w
Base	b
Height	h

The formulas for area are listed below. On the SHSAT, you will need to know the formulas and be able to apply them to solve problems. Make sure you memorize the following list and are comfortable using each formula.

FORMULAS FOR AREA

Rectangle and square	$A = l \times w$
Parallelogram	$A = b \times h$
Trapezoid	$A = \frac{1}{2} h(b_1 + b_2)$
Triangle	$A = \frac{1}{2} b \times h$

Now let's practice applying these formulas to find the areas of various figures. To find the area of a parallelogram, we must multiply the base by the height. The **base** is the line on the bottom of the figure. The **height** measures how tall the figure is.



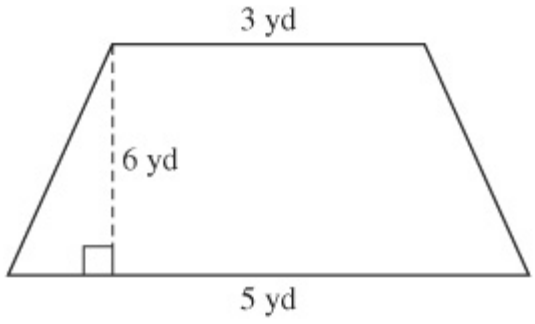
Notice that the height is different from the length of the sides. The dotted line indicating the height is called an **altitude** and is perpendicular to the base. According to the diagram, the base of the parallelogram is 8 inches, and the height is 6 inches. Multiply these lengths to find the area: $8 \times 6 = 48$. So the area of the parallelogram is 48 square inches.

> The area of a figure is always stated in square units. The words *square feet* or *square meters* may be used, for example, or the abbreviation for the unit of measure may be included, with an exponent of 2, such as ft^2 or m^2.

The formula for the area of a trapezoid included the measures of two bases.

$$A = \frac{1}{2}h(b_1 + b_2)$$

As you know, a trapezoid has two parallel lines. These are both considered to be bases, and the symbols b_1 and b_2 refer to base 1 and base 2. It does not matter which measurement you use for base 1 and which you use for base 2. The important thing is that we will be using the sum of the lengths of the two bases to find the area. So let's try it. Use the formula for the area of a trapezoid to determine the area of the figure shown.



The altitude line is labeled 6 yards, so we know this is the height of the trapezoid. One base is labeled 3 yards, and the other is labeled 5 yards, so we will put these measurements into the formula.

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$A = \frac{1}{2}6(3 + 5)$$

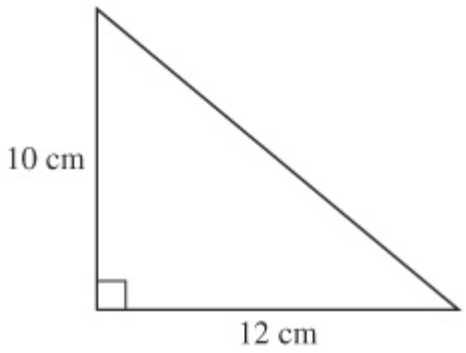
$$A = \frac{1}{2}6(8)$$

$$A = \frac{1}{2}(48)$$

$$A = 24$$

So the area of the trapezoid is 24 square yards.

In the examples of parallelograms and trapezoids we have worked with so far, the height has been indicated by a line that is not one of the sides of the figure. Sometimes, when finding the area of a triangle, the height is the same as one of the legs. Take a look at the following right triangle.



Notice that one leg of the triangle indicates the height. An altitude line is not needed. We can use the given measurements to find that the area of the right triangle is 60 cm^2.

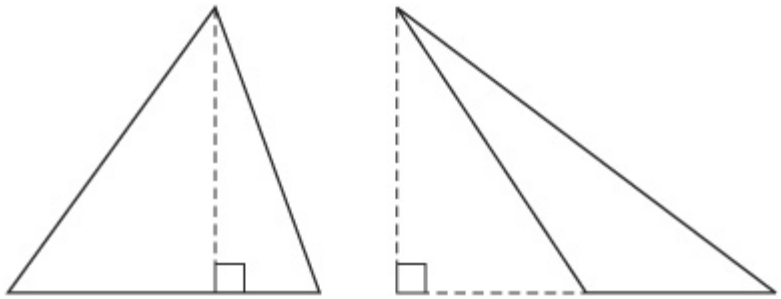
$$A = \frac{1}{2}b \times h$$

$$A = \frac{1}{2}(12 \times 10)$$

$$A = \frac{1}{2}(120)$$

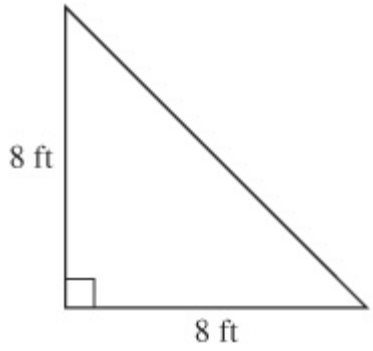
$$A = 60$$

There will be times when the altitude of a triangle is drawn within the figure, as shown in the following example on the left. There will be other times when the altitude is beside the triangle, as shown in the example on the right. Regardless of where the altitude is drawn, it still represents the height of the figure and is used in the formula to find the area.



Let's practice using what you have learned.

A rectangle has the same area as the triangle below.



The length of the rectangle is 16 feet. What is the width?

A. 2 feet

B. 4 feet

C. 18 feet
D. 32 feet
E. 36 feet

We know that the area of the rectangle is the same as that of the triangle. Let's start by finding the area of the triangle.

$$A = \frac{1}{2}b \times h$$

$$A = \frac{1}{2}(8 \times 8)$$

$$A = \frac{1}{2}(64)$$

$$A = 32$$

The area of both figures is 32 square feet. We also know that the length of the rectangle is 16 feet. Since the area of a rectangle is equal to the product of the length times the width, we can use this formula to help us find the width.

$$A = lw$$

$$32 = 16w$$

$$\frac{32}{16} = \frac{16w}{16}$$

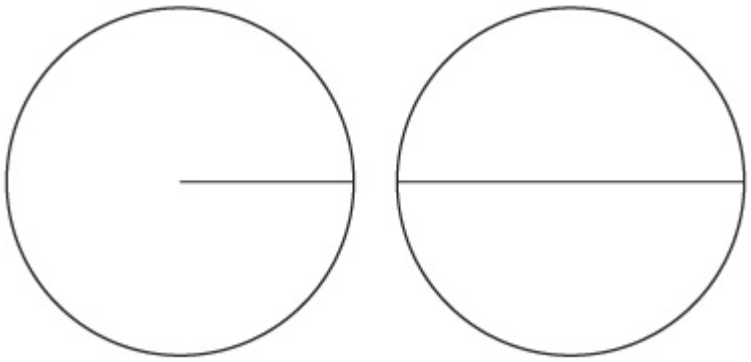
$$2 = w$$

So the width of the rectangle is 2 feet.

Circles

So far, the two-dimensional figures we have discussed in this chapter have all been polygons. Circles are also two-dimensional figures, but since they do not have any straight sides, they are not polygons. As you know, a **circle** is a geometric figure that is a curve on which every point is an equal distance from the center. A **radius**

is the distance from the center of a circle to any point on its edge. The **diameter** of a circle is the distance across the center of a circle, from one edge to the other. The length of the diameter is twice that of the radius.



The formulas to find the measurements of circles include **pi**, which is represented by the symbol π. Pi is actually an example of an irrational number, as discussed earlier in this book, because it is a nonterminating decimal. Since the decimal never ends and never repeats, we use the approximation 3.14 for pi. We use the approximation 3.14 for pi, but the actually number goes on forever, something like 3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679821480 86 … you get the idea."

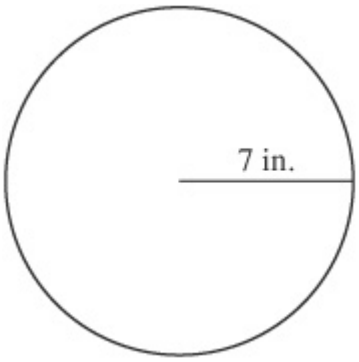
$$\pi = 3.14$$

As you know, perimeter is the distance around a polygon. **Circumference** is the distance around a circle. The following formula is used to find circumference, where *C* represents circumference and *r* represents radius.

$$C = 2\pi r$$

Since the diameter is equal to twice the radius, you can also use the formula $C = \pi d$, where d represents diameter, to find the circumference of a circle.

The circle below has a radius of 7 inches. Let's find the circumference.



Substitute the measurement of the radius into the formula. Remember to use the decimal 3.14 for pi, then find the distance around the figure.

$$C = 2\pi r$$

$$C = 2\pi(7)$$

$$C = 2(3.14)(7)$$

$$C = 43.96$$

So the circumference of the circle is 43.96 inches.

To find the area of a circle, we will also use pi and the measurement of the radius. The formula is shown below.

$$A = \pi r^2$$

Let's find the area of the same circle. Remember, the radius is 7 inches.

$A = \pi r^2$

$A = \pi(7)^2$

$A = (3.14)(7)^2$

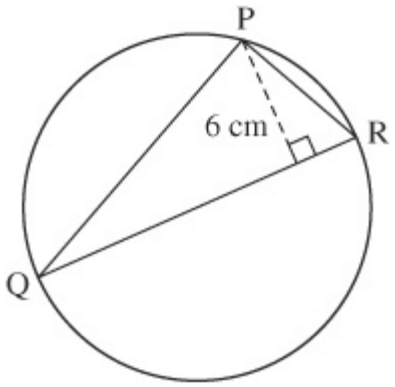
$A = (3.14)\ (49)$

$A = 153.86$

So the area of the circle is 153.86 in^2.

Some of the math questions on the SHSAT may ask you to simply find the perimeter, area, or circumference of a figure. Others will require you to apply several geometric skills in order to solve a problem. For example, in order to answer the following question, you will need to use what you know about circles and the area of right triangles in order to determine the area of the circle. Use what you have reviewed in this chapter to answer the question.

In the diagram shown, the area of triangle PQR is 48 cm^2, and line segment QR is a diameter of the circle.



What is the area of the circle?

A. 50.24 cm^2

B. 100.48 cm^2

C. 200.96 cm^2

D. 256.0 cm^2

E. 401.92 cm^2

To find the area of the circle, we need to know the length of the radius. Since line segment QR is a diameter, we can use this measurement to find the radius. But first we must determine the length of the diameter. We know that the area of the triangle is 48 square centimeters, and that the height is 6 centimeters. We can use this to determine the length of line segment QR, the base of the triangle.

$$A = \frac{1}{2}bh$$
$$48 = \frac{1}{2}b(6)$$
$$48 = \frac{1}{2}(6)(b)$$
$$48 = 3b$$
$$\frac{48}{3} = \frac{3b}{3}$$
$$16 = b$$

So the length of line segment QR, the diameter of the circle, is 16 centimeters. Since the radius is half the length of the diameter, the radius of the circle is 8 centimeters. Now we have the information needed to find the area of the circle.

$$A = \pi r^2$$
$$A = (3.14)(8)^2$$
$$A = (3.14)(64)$$
$$A = 200.96$$

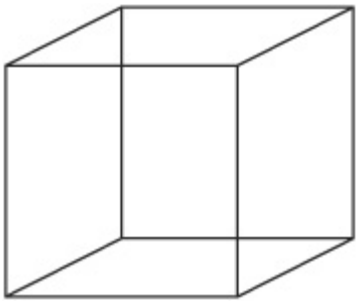
So the area of the circle is 200.96 square centimeters.

Three-Dimensional Figures

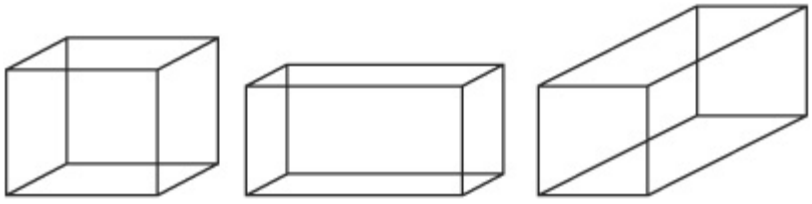
So far in this chapter, all of the geometric figures we have discussed have been two-dimensional, or flat. Now let's talk about **three-dimensional figures**, which are known as such because they have three dimensions: length, width, and height. When discussing these figures, it is important to understand a few vocabulary words.

- **Face.** The flat side of a three-dimensional figure.
- **Vertex.** The corner points of a three-dimensional figure.
- **Edge.** Where two faces meet.
- **Volume.** The capacity of a three-dimensional figure; how much the shape can hold.
- **Surface area.** The total area of the surface of a three-dimensional figure.

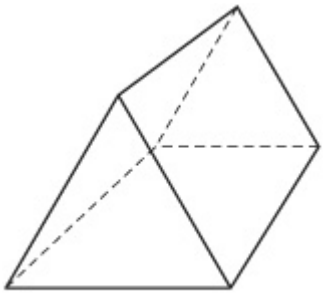
Now let's review a few common geometric solids. A **cube** is a three-dimensional figure that has six faces. Each side is an identical square. The length, width, and height of a cube are all equal.



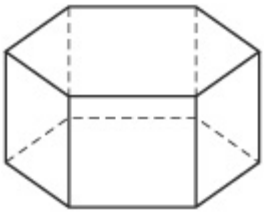
A **rectangular prism**, often called a **rectangular solid**, also has six faces, each of which is a rectangle. The length, width, and height of a cube are all equal, however, the length, width, and height of a rectangular solid may be different. Also, notice in the examples below that the opposite faces are identical.



Prisms, such as the rectangular prisms above, have two congruent parallel bases that are polygons. The number of faces of a prism depends on the shape of the bases. Each will have two bases, then a number of faces equal to the number of sides of the base. For example, since a rectangle has four sides, a rectangular prism has 4 side faces, plus the two bases, for a total of 6 faces. A triangular prism has 3 side faces, plus the two bases, for a total of 5 faces. A pyramid with hexagons as bases would have 6 side faces, plus two bases, for a total of 8 faces. Take a look at the following examples.



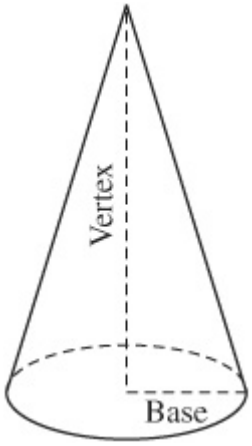
Triangular prism



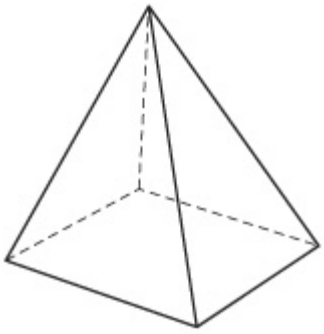
Hexagonal prism

A **cylinder** has two parallel bases that are congruent circles. The bases are joined by a curved surface.

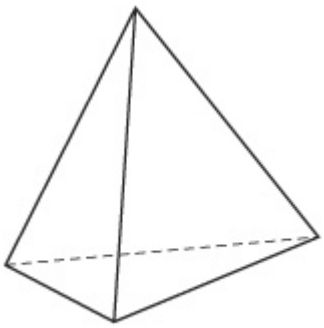
A **cone** has one circular base and a curved surface that connects the base to the vertex.



Pyramids have triangle shaped faces that connect the base and the vertex. The base of a pyramid may be any kind of polygon. The number of sides of a pyramid depends on the shape of the base. For example, a **square pyramid** has a square base. Since a square has 4 sides, a square pyramid has four triangular faces plus the square base, for a total of 5 faces. **A triangular pyramid** has a triangle shaped base. Since a triangle has 3 sides, a triangular pyramid has 3 triangle shaped faces plus the triangular base, for a total of 4 faces.



Square pyramid



Triangular pyramid

> Many polygons could serve as the base of a pyramid. Just remember that the number of sides the pyramid has depends on how many sides the polygon base has.

Each three-dimensional shape has a different formula for determining its surface area and volume. Make sure you know each of these formulas before taking the SHSAT, and be comfortable using each. Let's take a look at the formulas for determining these measurements. In these, *s* represents the length of one side of a cube, *V* represents volume, and *l*, *w*, and *h* represent length, width, and height. *SA* represents surface area.

FORMULAS FOR VOLUME

Cube	$V = s^3$
Rectangular solid	$V = lwh$
Prism	$V =$ area of base $\times$ height
Cylinder	$V = \pi r^2 h$*
Cone	$V = \frac{1}{3}\pi r^2 h$**
Pyramid	$V = \frac{1}{3} \times$ area of base $\times$ height

*Notice that this is the same as area of the base × height.

**This is the same as $\frac{1}{3}$ × area of the base × height.

FORMULAS FOR SURFACE AREA

Cube	$SA = 6s^2$
Rectangular solid	$SA = 2(lw + wh + lh)$
Cylinder	$SA = 2\pi r^2 + 2\pi rh$

Surface area is the sum of the areas of each of the faces. Remember, s^2 tells the area of a square. Since a cube is made up of six identical squares, $6s^2$ tells the area of all six of the squares, which is the surface area.

Let's try applying some of these formulas.

The height of the cone is four times the measure of the radius of the base. The diameter of the base is 6 inches. What is the volume of the cone?

To find the volume of the cone, we need to know the radius and the height. Neither of these is given in the problem; however, we have the information we need to determine both measurements. The

height is four times the measure of the radius. We don't know the radius, but we do know that it equals half the diameter. So, if the diameter is 6 inches, the radius is 3 inches. And if the height is four times the radius, the height is equal to 4 × 3.

Diameter = 6 in.
Radius = 3 in.
Height = 4 × radius = 4 × 3 = 12

Now we can insert the length of the radius and the height into the formula for determining the volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(3)^2\ (12)$$

$$V = \frac{1}{3}\pi(9)\ (12)$$

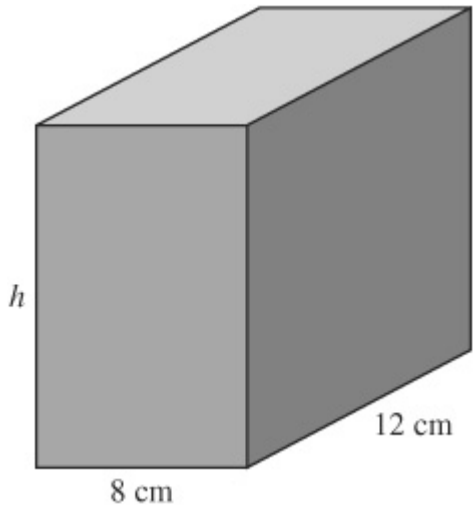
$$V = \frac{1}{3}(3.14)\ (9)\ (12)$$

$$V = \frac{1}{3}(339.12)$$

$$V = 113.04$$

Now you try it! Use the formulas reviewed in this section to answer the following question.

A rectangular solid with a surface area of 712 cubic centimeters has a length of 8 centimeters and a width of 12 cm. What is the sum of the width and the height of the solid?



A. 12 cm
B. 13 cm
C. 20 cm
D. 21 cm
E. 25 cm

Since the height is not given, we will need to represent this measurement with a variable and use algebra skills to find its value. Let's substitute the given measurements into the formula for surface area of a rectangular solid and substitute the variable *h* for the unknown height.

$$SA = 2(lw + wh + lh)$$

$$712 = 2[(8 \times 12) + 12h + 8h]$$

$$712 = 2(96 + 12h + 8h)$$

Next, combine like terms.

$$712 = 2(96 + 12h + 8h)$$

$$712 = 2(96 + 20h)$$

Now multiply the terms inside parentheses by 2, and solve for the variable.

$$712 = 2(96 + 20h)$$

$$712 = 192 + 40h$$

$$712 - 192 = 192 - 192 + 40h$$

$$520 = 40h$$

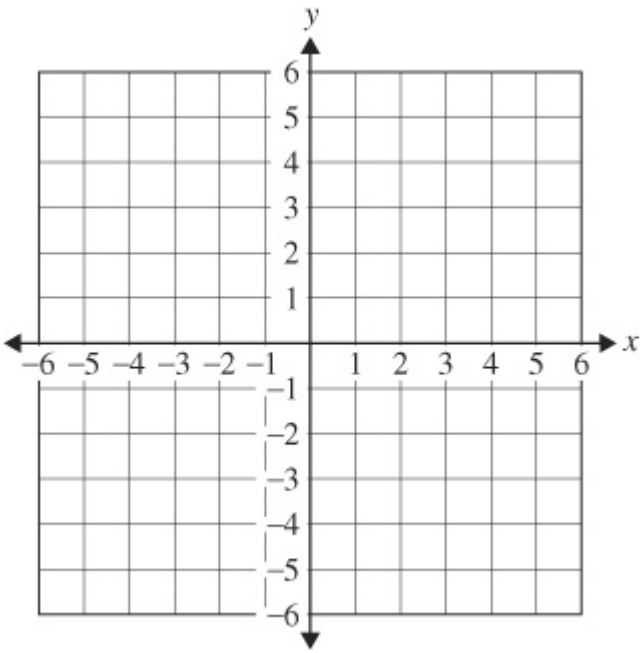
$$\frac{520}{40} = \frac{40h}{40}$$

$$13 = h$$

So, the height of the figure is 13 cm. But the question did not ask for the height; it asked for the sum of the width and the height. Since the width is 12 cm and the height is 13 cm, the sum of these measurements is 25 cm, making answer choice E correct.

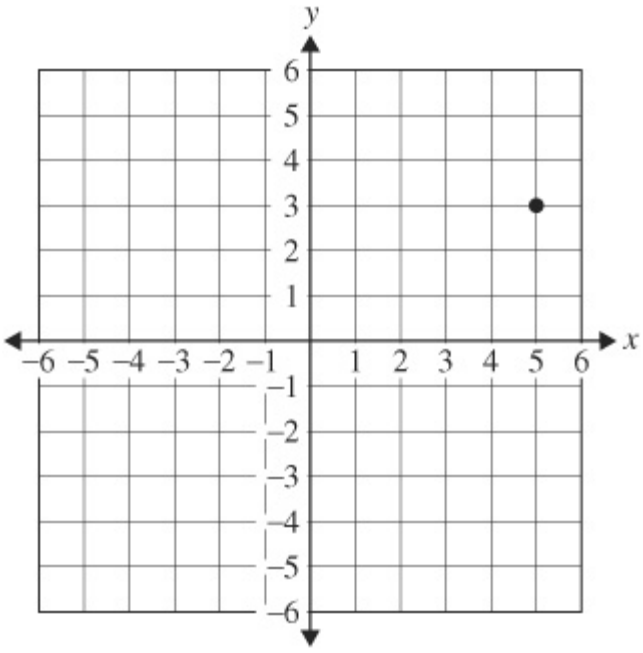
Coordinate Planes

A coordinate plane is created by a horizontal number line and a vertical number line. The point on the plane where the two lines intersect is called the **origin** and is point 0 on both number lines. The horizontal number line is called the ***x*-axis**; the vertical number line is the ***y*-axis**.



Notice that values to the right of the origin and those above the origin are positive numbers; values to the left of the origin and values below the origin are negative numbers.

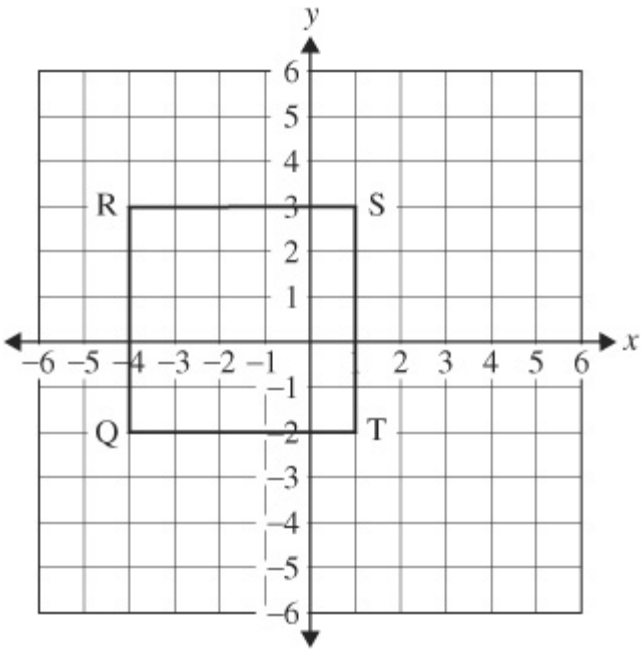
Each point on the coordinate plane can be named by its location. Take a look at the point on the plane below.



To locate this point, begin at the origin. Move to the right along the *x*-axis to the number 5, then move up to the number 3 on the *y*-axis. Since we moved over 5 spaces and up 3 spaces to reach this point, we say that it is located at (5, 3). This **ordered pair** of numbers tells the location of the point on the plane. Since the origin is located at 0 on both axes, the ordered pair that names this point is always (0, 0).

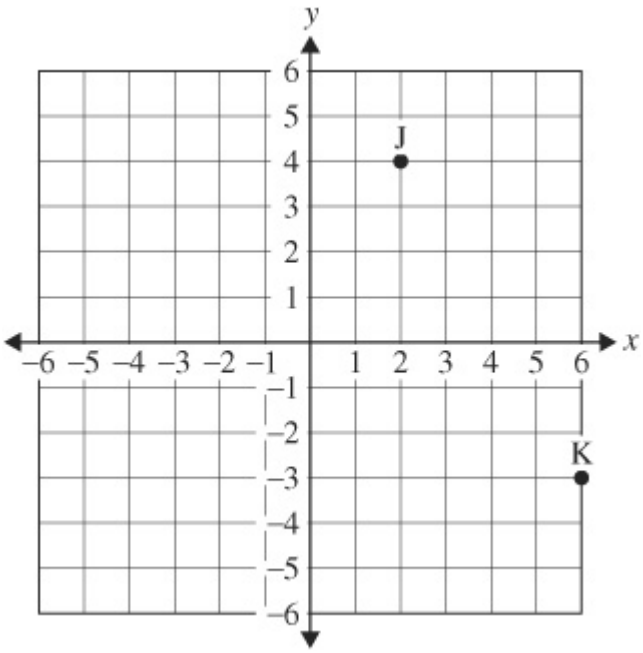
The numbers in the ordered pair are called the **coordinates**. The first coordinate in an ordered pair always names the location along the horizontal axis, or the *x*-axis. For this reason, the first number is the ***x*-coordinate**. Likewise, the second number in an ordered pair always names the location along the vertical axis, or the *y*-axis, and is known as the ***y*-coordinate**.

Two-dimensional figures may be shown on a coordinate plane. The four vertices of the following rectangle are located at (–4, –2), (–4, 3), (1, 3), and (1, –2). Notice that point Q is located to the left of and below the origin, so both coordinates are negative. Point S is to the right of and above the origin, so both coordinates are positive.



Use what you know about two-dimensional figures to determine the missing ordered pair for the following figure.

Points J, K, and L form a right triangle. Points J and K are shown.



Which could be the coordinates of point L?

A. (–6, 4)

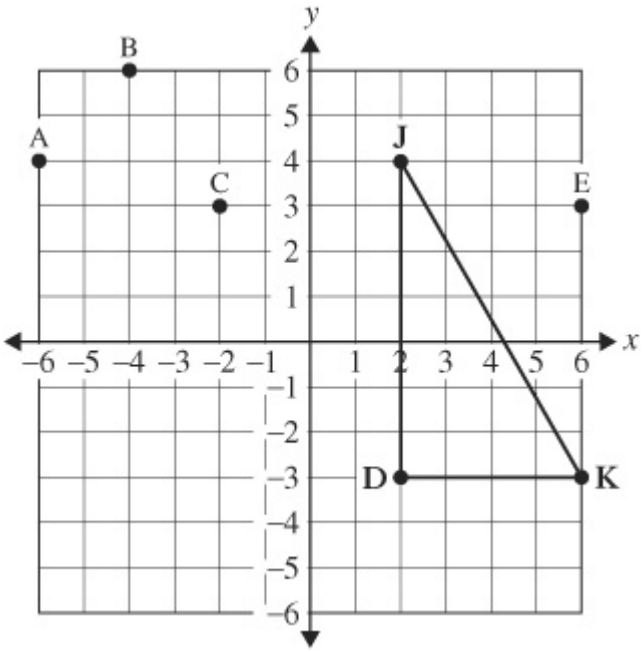
B. (–4, 6)

C. (–2, 3)

D. (2, –3)

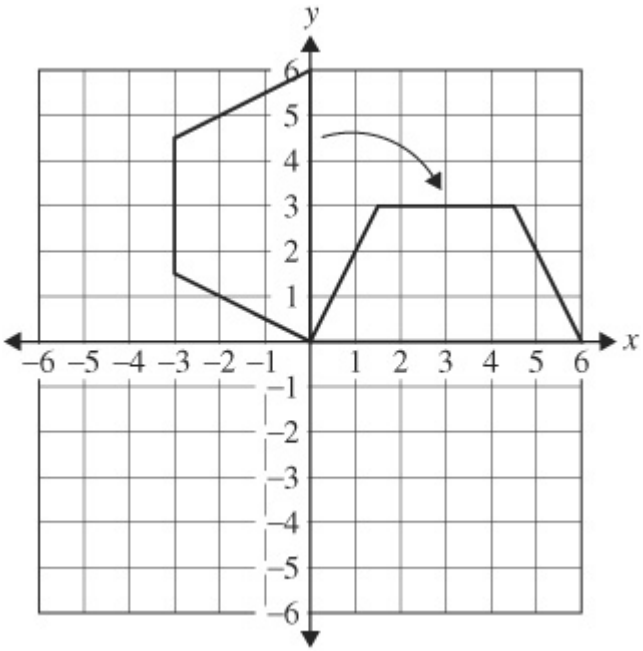
E. (6, 3)

To find the location of point L, we must determine which of the given ordered pairs will form a right angle with points J and K. Any of the points will form a triangle, but we specifically need to find the coordinates that will form a right triangle. Each of the answer choices is shown on the coordinate plane. Notice that only answer choice D (2, –3) forms a right triangle with points J and K.

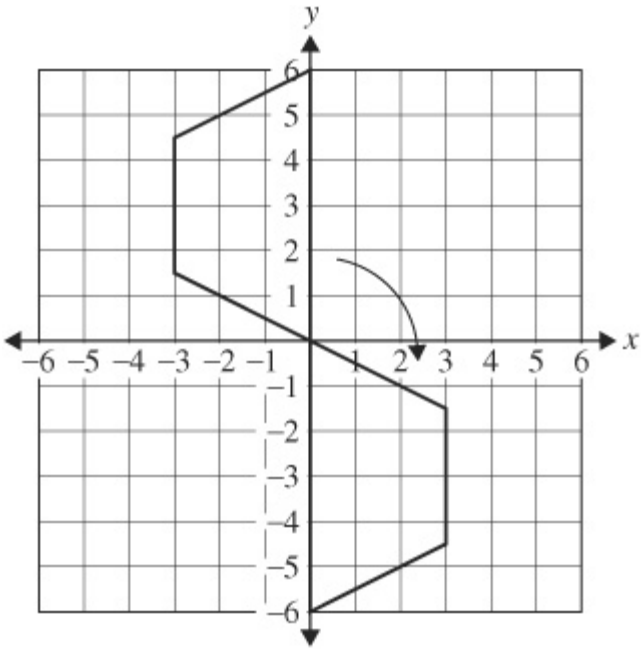


Transformations

In geometry, a **transformation** is a change that is made to a figure. There are several types of transformations that may occur. First is a **rotation**. When a shape is rotated, it turns around a fixed center point. The shape and size of the original figure remain the same. A figure can be rotated as much as 360 degrees around the center of rotation, or the fixed point around which the figure rotates.

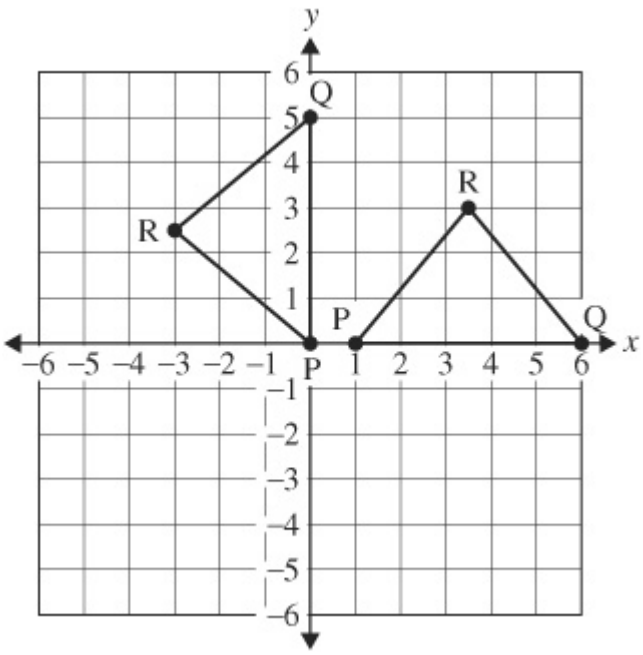


Look at the example above. Imagine there is a pin in the corner of the first figure, holding the figure in place at that point. Then imagine turning the figure clockwise, without moving the pin. The figure has rotated around the pin, and the new position is shown on the right. The trapezoid on the right has been rotated 90 degrees from its original position.



Think of a rotation as being hands on a clock. A figure that has been rotated 90 degrees has moved one-quarter of an hour; a figure that has been rotated 180 degrees has moved one-half of an hour.

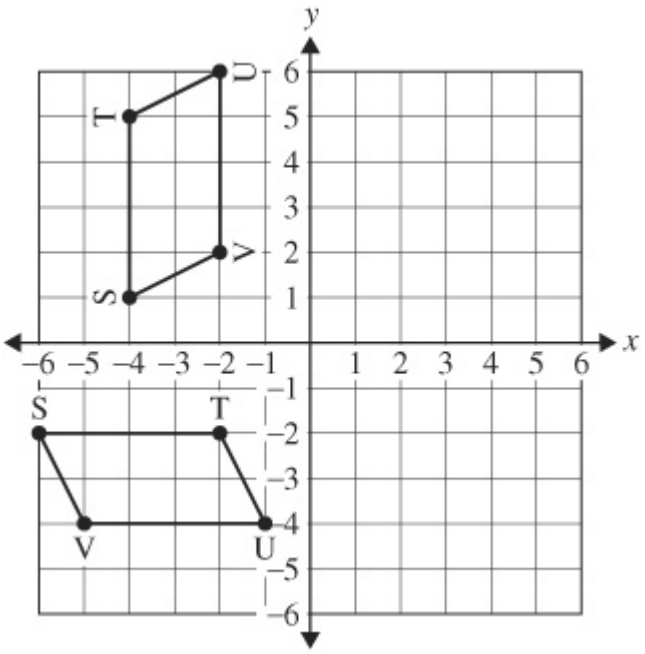
A **reflection** results when a figure is flipped over a central line. In its new position, the figure is the same distance from the central line as it was prior to being reflected. Figures may be reflected in any direction and create symmetry within the plane. The size and shape of the figures do not change.



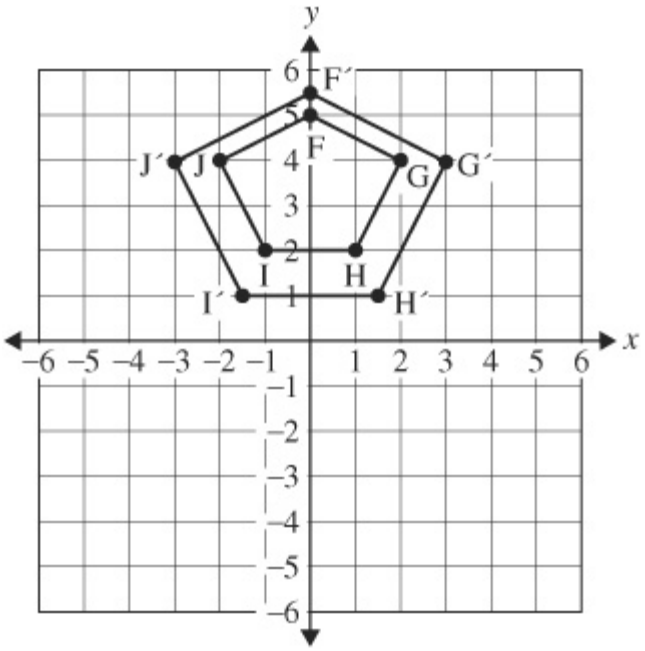
In this example, the triangle on the left was reflected across the *y*-axis. Notice that point P in both triangles is an equal distance from the *y*-axis. The same is true for points Q and R. The new triangle is a mirror image of the original. If we were to stand the edge of a mirror along the central line, this is the image we would see.

A **translation** happens when a shape is moved to another position without being turned. In elementary school, you may have called a translation a *slide*, since it shows the new position of the figure if we were to slide it across the plane. When a shape is

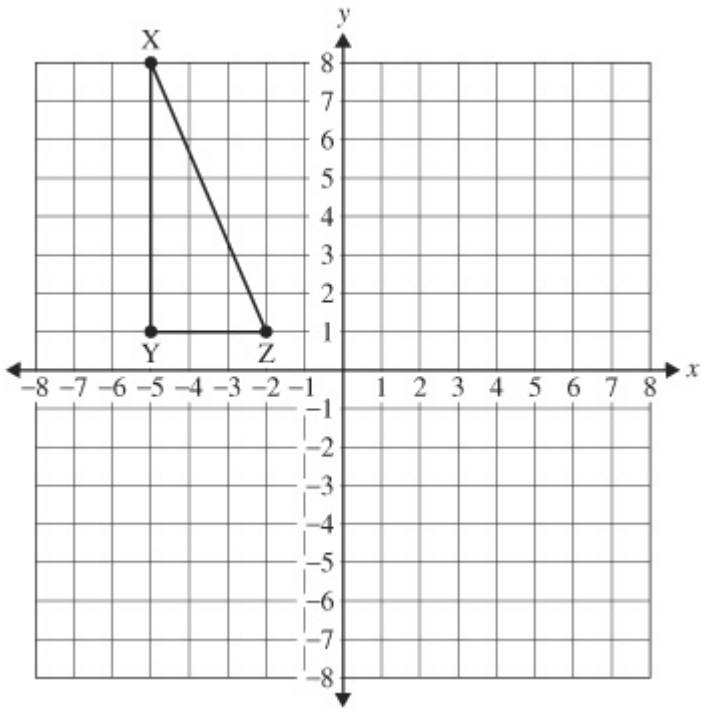
translated, every point of the figure moves the same distance in the same direction. The new image is congruent to the original.



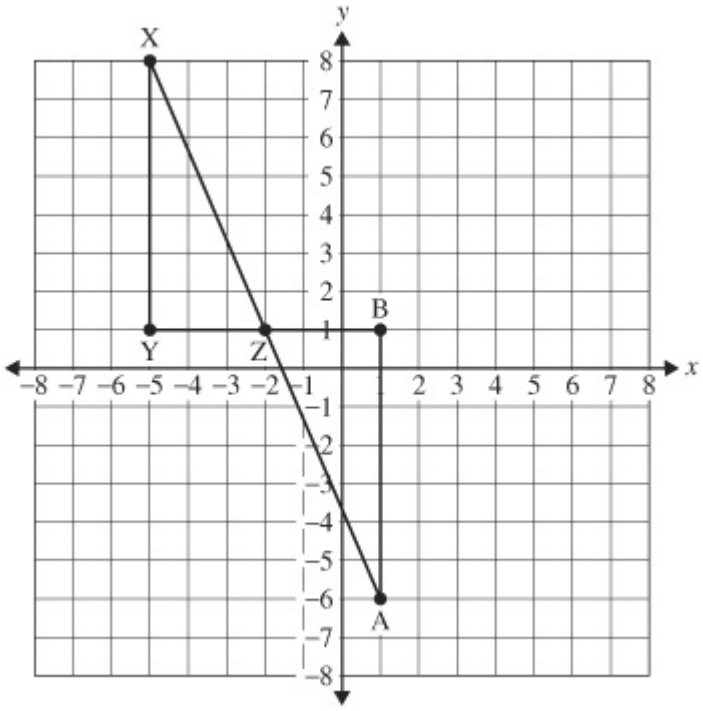
A **dilation** is a transformation that changes the size of the figure. The new figure is similar to the original. That means that the measurements of the interior angles do not change, and the corresponding sides change proportionally.



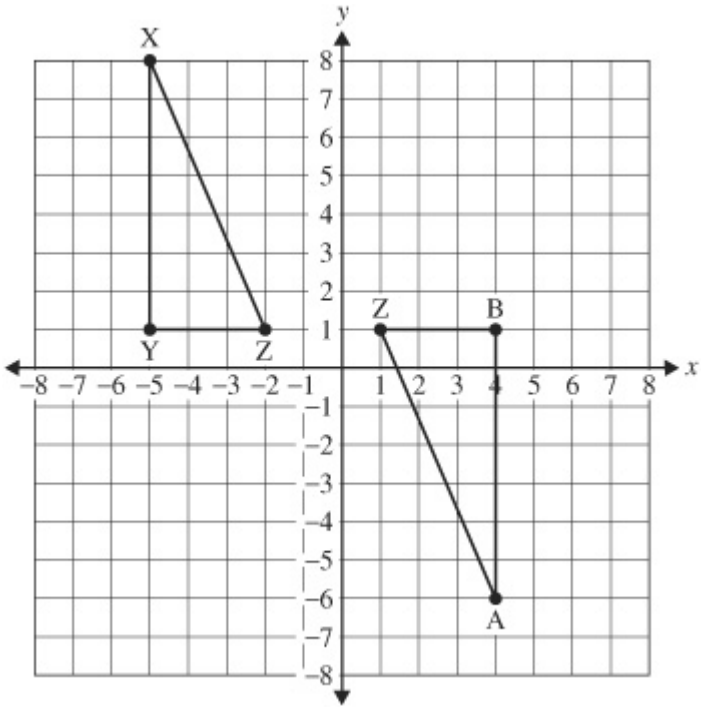
Point X in right triangle XYZ is located at (–5, 8). Rotate the figure 180 degrees. Point X becomes point A on the new triangle, and Y becomes B. Translate triangle ZAB three units to the right. What are the new coordinates of A?



Start by rotating the figure 180 degrees around point Z, and name the points of the new triangle.



Now translate triangle ZAB three units to the right. That means slide the figure three spaces along the *x*-axis. The *y*-coordinates will remain the same. In other words, the figure will not move up or down along the plane; it will only move to the right.



The new coordinates for point A are (4, –6).

You Try It!

In this chapter, you've reviewed the geometry skills you will need to know to do your best on the SHSAT. Use what you have learned to answer the following questions.

1. A decagon has a perimeter of 162 inches. One side has a length of 15 inches, and another has a length of 19 inches. Four sides each measure z inches in length, and the remaining sides each measure $3z$ inches in length. What is the value of z?

A. 8
B. 10
C. 12.25
D. 13.5
E. 32

Did you select answer choice A? Great! A decagon has 10 sides, so to find the perimeter, we must find the sum of all of the sides. We know that one side measures 15 inches, one measures 19 inches, and four have a length of z inches. So we know the measure of six of the ten sides. That means that four sides remain, and these each have a length of $3z$. Let's list the measures of the sides.

$$15 + 19 + z + z + z + z + 3z + 3z + 3z + 3z$$

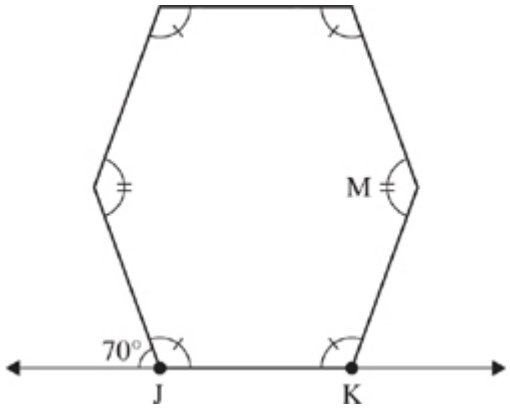
That's the same as $15 + 19 + 4z + 4(3z)$.

We know that the sum of these sides is 162 inches. Now we need to find the value of z. To do so, perform the multiplication first, then add to combine like terms. After that, isolate the variable.

$$15 + 19 + 4z + 4(3z) = 162$$
$$15 + 19 + 4z + 12z = 162$$
$$34 + 16z = 162$$
$$34 - 34 + 16z = 162 - 34$$
$$16z = 128$$
$$\frac{16z}{16} = \frac{128}{16}$$
$$z = 8$$

So the value of z is 8.

2. Points J and K are located on the same line and are located at vertices of the hexagon. What is the measure of ∠M?



A. 110
B. 120
C. 140
D. 280
E. 440

In order to determine the measure of angle M, we must first find out the sum of the interior angles of the hexagon. A hexagon has 6 sides, so we'll use this number in the formula for finding the sum of interior angles.

$(n-2)\times 180$

$(6-2)\times 180$

$4\times 180 = 720$

So the sum of the interior angles of a hexagon is 720 degrees. Now let's figure out the measures of each of the angles. Since the hexagon sits on line JK, the interior angle of the hexagon that is located at point J is supplementary to the 70 degree angle. So it measures 110 degrees (180 – 70). According to the diagram, this angle is equal to three others in the figure. That means the sum of the four equal angles is 440 degrees. Let's subtract this from the total number of degrees in the figure.

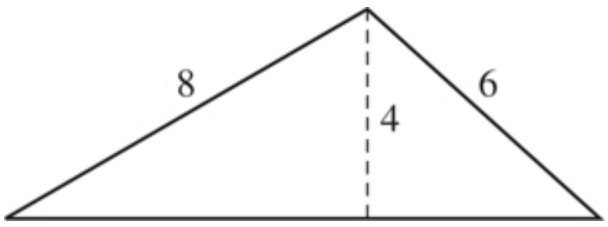
$720 - 440 = 280$

Now we know that the sum of angle M and the angle whose measure is equal to angle M is 280. We can divide this number by 2 to determine the measure of each of the angles.

$280 \div 2 = 140$

So the measure of angle M is 140 degrees.

3. The triangle has an area of 20 square inches.



If a square has the same perimeter as the triangle, what is the area of the square?

A. 10 in^2

B. 12 in^2

C. 20 in^2

D. 24 in^2

E. 36 in^2

Let's use the area of the triangle to determine the length of the base. Substitute the known values into the formula for area and determine the missing value, *b*, which is the base.

$$A = \frac{1}{2}bh$$

$$20 = \frac{1}{2}b \times 4$$

$$20 = \left(\frac{1}{2} \times 4\right) \times b$$

$$20 = 2b$$

$$\frac{20}{2} = \frac{2b}{2}$$

$$10 = b$$

Now we know that the base of the triangle is 10 inches, so we can find the sum of the side lengths, which equals the perimeter.

$$10 + 6 + 8 = 24$$

So the perimeter of the triangle is 24. The square has the same perimeter. We know that the lengths of the sides of a square are equal, so we can divide the perimeter by 4 to find the length of each side.

$$24 \div 4 = 6$$

Each side of the square is 6 inches long. The area of a square is length multiplied by width. The length and width are both 6, so we can multiply 6 × 6 to find the area: $6 \times 6 = 36$. The area of the square is 36 square inches.

Let's Review!

In this chapter, you've reviewed geometry vocabulary, formulas, and skills that will help you be successful on the SHSAT math questions. Let's take a final look at a few of the key points reviewed in this chapter.

- **Lines** continue forever in both directions and are indicated by arrows on both ends. **Line segments** have definite end points and are represented by points on both ends. **Rays** have a specific beginning point but continue forever in one direction. Rays are shown with a point on one end and an arrow on the other.
- **Intersecting** lines or segments cross each other and share a common point. Lines that are **perpendicular** intersect to form a right angle. **Parallel** lines, on the other hand, never meet and remain an equal distance apart at all points on the lines.
- An **angle** is formed by two rays that share an endpoint, called the vertex.
  - **Right angles** measure exactly 90 degrees, and form an L.
  - **Acute angles** measure less than 90 degrees, and are narrower than right angles.
  - **Obtuse angles** measure greater than 90 degrees, and are wider than right angles.
- Angle relationships can be helpful in determining information about geometric figures:
  - Two angles are **complementary** if the sum of their measures is equal to 90 degrees.
  - Two angles are **supplementary** if the sum of their measures is equal to 180 degrees.
  - **Adjacent angles** share a vertex and a common side.
  - **Vertical angles**, which are opposite each other when two lines intersect, have equal measurements.
  - **Alternate interior angles** are created when two parallel lines are cut by a transversal. The angles are on opposite sides of the transversal, are on the interior of the parallel lines, and have the same measurement.
  - **Alternate exterior angles** are on opposite sides of the

transversal, are on the outside of the parallel lines, and have the same measurement.

- **Polygons** are two-dimensional figures made from three or more straight sides. These figures are classified by the number of sides they have. The number of angles in a polygon is equal to the number of sides. The sum of the interior angles can be found using the formula shown.
  - Sum of the interior angles of a polygon = $(n-2)\times 180°$
- **Triangles** are three-sided polygons, and the sum of the interior angles of any triangle equals 180 degrees. Triangles can be classified according to the lengths of their sides.
  - **Equilateral.** Three equal sides.
  - **Isosceles.** Two equal sides.
  - **Scalene.** No equal sides. Triangles can also be classified according to the measure of their angles.
  - **Acute.** All angles measure less than 90 degrees.
  - **Obtuse.** One angle measures greater than 90 degrees.
  - **Right.** One angle measures exactly 90 degrees.
- The sides of a right triangle that form the 90 degree angle are the **legs**. The longest side, which is opposite from the right angle, is the **hypotenuse**.
- **Quadrilaterals** are four-sided polygons, and the sum of the interior angles of any quadrilateral equals 360 degrees. Quadrilaterals can be classified according to the lengths of the sides and the measures of the angles.
  - **Rectangle.** Four right angles and two pairs of parallel sides.
  - **Square.** Four right angles and four sides of equal length.
  - **Rhombus.** Four sides of equal length and opposite sides are parallel; angles can be any size; opposite angles are equal.
  - **Parallelogram.** Opposite sides are parallel and have equal lengths.
  - **Trapezoid.** Has one pair of parallel sides.
- Figures that are exactly the same shape and size are **congruent**. Figures that are exactly the same shape but different sizes are **similar**.

- **Perimeter** is the distance around a figure. It is the sum of the lengths of all of the sides of a polygon.
- **Area** is the space within a two-dimensional figure and is reported in square units. The formulas below are used to find the area of polygons:

  - Rectangle or square $A = l \times w$
  - Parallelogram $A = b \times h$
  - Trapezoid $A = \frac{1}{2}h(b_1 + b_2)$
  - Triangle $A = \frac{1}{2}b \times h$

- **Circles** are curved figures in which every point is an equal distance from the center. **Radius** is the distance from the center of a circle to any point on its edge. **Diameter** is the distance across the center of a circle, from one edge to the other. **Circumference** is the distance around a circle.

  - Circumference $C = 2pr$
  - Area $A = \pi r^2$

- **Three-dimensional figures** have length, width, and height. These figures are classified by their faces and bases.
  - **Cube.** Has six identical faces; the length, width, and height of a cube are equal.
  - **Rectangular solid.** Has six faces, each the shape of a rectangle.
  - **Prism.** Has two parallel bases that are congruent polygons; the number of faces depends on the shape of the bases.
  - **Cylinder.** Has two parallel bases that are congruent circles; the bases are joined by a curved surface.
  - **Cone.** Has one circular base and a curved surface that connects the base to the vertex.
  - **Pyramid.** Has triangle-shaped faces that connect the base and the vertex; the base may be any polygon.

- **Volume** is the capacity of a three-dimensional figure. The formula for finding volume depends on the type of figure.

  - Cube $V = s^3$
  - Rectangular solid $V = l \times w \times h$

  - Prism $V = \text{area of base} \times \text{height}$
  - Cylinder $V = \pi r^2 h$
  - Cone $V = \frac{1}{3}\pi r^2 h$
  - Pyramid $V = \frac{1}{3} \times \text{area of base} \times \text{height}$

- **Surface area** is the sum of the areas of all of the faces of a three-dimensional figure. While it is possible to determine the surface area of other three-dimensional solids, we just reviewed the formulas for cubes, rectangular solids, and cylinders.

  - Cube $6s^2$
  - Rectangular solid $SA = 2(lw + wh + lh)$
  - Cylinder $2\pi r^2 + 2\pi rh$

- On a coordinate plane, the numbers that identify the location of a point are called the **ordered pair**. The first number, or **coordinate**, in the pair is the ***x*-coordinate**, and names the location of the point along the *x*-axis. The second number is the ***y*-coordinate** and names the location of the point along the *y*-axis.
- A change that is made to a geometric figure is a **transformation**. Types of transformations include rotation, reflection, translation, and dilation.
  - **Rotation.** A figure is turned around a fixed center point; the shape and size of the figure remain constant.
  - **Reflection.** A figure is flipped over a central line; the resulting figure is a mirror image of the original, maintaining the same

shape and size.

- **Translation.** A shape is moved to another position without being turned; this slide does not change the size or shape of the figure.
- **Dilation.** Changes the size of the figure; the resulting figure is similar to the original, meaning the measurements of the interior angles remain the same and the corresponding sides increase and decrease proportionally.

CHAPTER 10
Probability and Statistics

Have you ever flipped a coin to make a decision? Maybe if it was heads, you would do the dishes, and tails your brother would wash them? A coin toss is a simple example of probability. **Probability** is the likelihood that an event will occur. Suppose that following the coin toss, you pointed out that you have washed the dishes 5 out of 7 nights this week, or that 84 percent of students do not like to wash dishes. Now you're talking statistics. **Statistics** are facts and figures that relate to a given topic.

Some of the questions on the SHSAT will assess your knowledge of these two areas of math. As is true with many of the SHSAT questions, there will be times when you will need to apply your knowledge of arithmetic and algebra to answer probability and statistics questions. In this chapter, we'll review some of the skills that will help you do your best when solving these types of problems.

Central Tendency

One of the basic elements of statistics is central tendency. Whether you realize it or not, measures of central tendency are part of your everyday life. Your report card grades are an average of the scores you've earned on a set of tests and assignments. The batting average of your school's star baseball player is based on central tendency. Even surveys conducted by your local newspaper rely on measures of central tendency.

Measures of **central tendency** are ways to report what information is in the middle of a set of numerical data. There are three ways to measure central tendency.

- Mean

- Median
- Mode

Let’s take a closer look at each of these.

Mean

The **mean** is the average of a set of data. To find the mean, add all of the values in a set, then divide the sum by the number of items. For example, suppose your scores on five math quizzes were 88, 100, 90, 95, and 87. To find the mean, or average, of the set of scores, find the sum, then divide by 5, since there were 5 quizzes.

$$88 + 100 + 90 + 95 + 87 = 460$$

$$460 \div 5 = 92$$

So the mean of the set of data is 92. Notice that 92 is not one of the scores on the list. Mean may or may not be a value that is included in the set of data. Also there is only one mean for any set of data. As you will learn later in this chapter, a set of data may have more than one mode; however, it will only have one mean.

As you know, the mean of a set of data is the average of all of the values in the set. If you determine that the mean is greater than the highest value in the set, or is less than the lowest value, double-check your work. Common mistakes when determining central tendency are leaving out one or more of the values or dividing by the incorrect number. Make sure that your answer makes sense.

At times you may be asked to simply find the mean of a given set of data. At other times, you may need to use what you know about mean to solve a problem involving more steps. Let’s use what we’ve reviewed about mean to solve a problem similar to one you might see on the SHSAT.

The table shows the number of car wash tickets sold by 10 members of the cheerleading squad.

Tickets Sold	Number of Cheerleaders
12	2
15	4
18	3
20	1

What is the mean number of tickets sold?

Look carefully at what the question is requiring you to do. At first, it may be tempting to simply find the mean of 12, 15, 18, and 20. But that is not what the question is asking. Notice that two cheerleaders each sold 12 tickets. That means that when we find the sum of the number of tickets sold, we actually have added 12 twice. According to the table, we'll need to add 15 four times, and so on. As you know, the first step in determining the mean is to find the sum of the set of data, so let's do that.

$12 + 12 + 15 + 15 + 15 + 15 + 18 + 18 + 18 + 20 = 158$

So the total number of tickets sold is 158. Next, divide this sum by 10 since there were 10 cheerleaders selling tickets.

$158 \div 10 = 15.8$

So the mean number of tickets sold is 15.8.

Median

Have you ever been on a car trip and noticed the grassy or concrete area in the middle of the street, dividing your lanes from oncoming traffic? That area in the middle is called the *median*. This same word is also used to name a measure of central tendency. The **median** in a set of data is the middle value in the set. Just like the median in the

street, the median in math is in the middle. In order to find the median of a set of data, you must first arrange the values in order from least to greatest.

Suppose there are seven homeroom classes at your grade level, and you want to find the median number of students in the homerooms. The number of students in each homeroom is shown below.

21, 25, 17, 19, 23, 20, 26

First, arrange the values from least to greatest.

17, 19, 20, 21, 23, 25, 26

Now find the number in the middle of the set.

17, 19, 20, 21, 23, 25, 26

The number in the middle of the set is 21, so 21 is the median number of students in the homeroom classes. Like the mean, a set of data has only one median. In this case, the median was a number included in the set. However, that is not always the case. When a set of data has an even number of values, there is not a single number in the set that is exactly in the middle. Suppose you take six classes each day. The number of students in each of your classes is shown below.

22, 25, 18, 24, 32, 27

To find the median, arrange the values from least to greatest.

18, 22, 24, 25, 27, 32

Now find the numbers in the middle of the set.

18, 22, 24, 25, 27, 32

Since 24 and 25 are in the middle of the set, we must add these together, then divide the sum by 2. In other words, find the average of the two middle numbers.

$$24 + 25 = 49$$

$$49 \div 2 = 24.5$$

So the median number of students in the classes is 24.5, a number that is not included in the set of values.

Mode

Mode is the value that appears most frequently in a set of data. That means mode is always a number that is included in the set. Suppose the friends in your study group earned the following scores on a math exam.

92, 100, 97, 89, 91, 92, 97, 99, 90, 97, 96

The mode is the score that is included in the list the most times. To find which is the mode, let's arrange the scores in order from least to greatest.

89, 90, 91, 92, 92, 96, 97, 97, 97, 99, 100

Notice that the following values each appear in the list a single time: 89, 90, 91, 96, 99, and 100. The number 92 appears twice; 97 appears three times. Since 97 appears the greatest number of times, the mode of the set of data is 97.

As you read previously, a set of data can have more than one mode. When this happens, two or more values must occur in the set an equal number of times, and the number of times these numbers appear must be greater than the occurrences of any other value.

So far this season, the Titusville Tigers football team is undefeated. The scores of their games are listed below.

16, 17, 15, 28, 15, 34, 22, 28, 21, 17, 23

What is the mode of the team's scores?

First, arrange the data in order from least to greatest so you can easily see which values occur more than once.

15, 15, 16, 17, 17, 21, 22, 23, 28, 28, 34

As you can see, 15, 17, and 28 each appear twice in the list. No other number appears more than twice. So this set of data has three modes: 15, 17, and 28.

> Did you notice that *mode* sounds a little bit like *most*? That could be helpful in remembering that the mode is the value that occurs most often in a set of data.

Now you try it! Use what you have learned about central tendency to solve the following problem.

A book store had 251 customers on Friday and 384 customers on Saturday. How many customers were there on Sunday if the mean number of daily customers for the three days was 319?

A. 285
B. 318
C. 322
D. 351
E. 367

Did you select answer choice C? Awesome! Let's review why this answer choice is correct.

In order to find the mean, or average, number of customers per day, we must find the sum of the visitors for each day, then divide by

the number of days.

Mean = (sum of the customers for Friday, Saturday, and Sunday) ÷ (number of days)

We know the number of customers for Friday and Saturday but not for Sunday. We also already know that the mean number of customers is 319. Let's substitute this information into the formula, using the variable *c* for the number of customers on Sunday. Then solve the equation to find the value of the variable.

$$319 = (251 + 384 + c) \div 3$$

$$319 \times 3 = (251 + 384 + c) \div 3 \times 3$$

$$319 \times 3 = (251 + 384 + c)$$

$$957 = (251 + 384 + c)$$

$$957 = 635 + c$$

$$957 - 635 = 635 - 635 + c$$

$$322 = c$$

So the number of customers on Sunday was 322.

Determining Probability

Have you ever watched a football game? Before the game begins, a coin is tossed to determine which team will get the ball first. What are the chances that the coin will land on heads? The chances of the coin landing heads up is 1 in 2, or one-half. The chances of the coin landing tails up is also 1 in 2, or one-half.

This is an example of **probability**, or the statistical chance that a given event will occur. Before we go any further discussing probability, let's review a few important key words.

- **Experiment.** A controlled, repeatable situation involving chance or probability.
- **Outcome.** A single possible result of performing an experiment one time.
- **Event.** The set of possible outcomes of an experiment.

In the example of the football game, the coin toss is the experiment. Tossing the coin is a situation of chance; there is a chance it will land on heads, and there is a chance it will land on tails. Other experiments could include rolling a number cube, spinning a spinner, or pulling a marble from a bag.

There are two possible outcomes, or results, of tossing a coin: heads or tails. When rolling a number cube, there are six possible outcomes: rolling the number 1, 2, 3, 4, 5, or 6.

The event is a single coin toss with a single outcome, such as landing on heads. If you were spinning a colored spinner and wanted to find the probability of landing on a blue section, the event would be landing on blue. If you were rolling a number cube and wanted to land on 5, the event would be landing on 5.

In order to measure the probability of an event occurring, we can use the formula shown below, where P represents probability and A represents a given event.

$$P(A) = \frac{\text{the number of ways event A can occur}}{\text{the total number of possible outcomes}}$$

Let's use this formula to prove that the probability of a coin toss resulting in heads is one-half. When a coin is tossed, there is only one way for the result to be heads. That is, it lands heads up. So the top number in the formula will be 1, because there is only one way the event (the coin landing on heads) can occur. There are a total of two possible outcomes: heads up or tails up. So the number on the bottom of the formula will be 2.

$$P(\text{heads}) = \frac{1}{2}$$

This proves that the probability of a coin toss resulting in heads is one-half.

But what about an event that can occur in more than one way? Let's consider rolling a six-sided number cube. Suppose we want to find the probability of rolling an even number. How many ways can the event occur? We could roll a 2, 4, or 6. So there are three possible ways for the event, rolling an even number, to occur. How many possible outcomes are there when rolling the cube? The possible outcomes are rolling 1, 2, 3, 4, 5, or 6. So there are six possible outcomes.

$$P(\text{rolling an even number}) = \frac{3}{6}$$

$$P(\text{rolling an even number}) = \frac{1}{2}$$

So the probability of rolling an even number on a number cube is $\frac{1}{2}$.

Let's try another one.

What is the probability of rolling a number greater than 4 on a six-sided number cube?

First, determine the number of ways the event can occur. The event is rolling a number greater than 4. That would mean rolling either a 5 or a 6, so there are two ways for the event to occur. We know that there are six possible outcomes when rolling a single number cube. Use the formula to determine the probability.

$$P(\text{rolling a number greater than 4}) = \frac{2}{6}$$

$$P(\text{rolling a number greater than 4}) = \frac{1}{3}$$

So there is a one-third chance of rolling a number greater than 4 on a single roll of the cube.

Now you try it. Use the formula for determining probability to solve the problem below.

A jar contains a total of 18 marbles. There are 5 red marbles, 3 yellow marbles, 4 blue marbles, 1 green marble, and 5 black marbles. What is the probability of randomly selecting a marble that is a primary color?

A. $\frac{1}{18}$

B. $\frac{5}{18}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

E. $\frac{1}{2}$

To solve this, we must first determine how many ways the event can occur. The event is selecting a marble that is a primary color. Since there are 5 red, 3 yellow, and 4 blue marbles, there are a total of 12 ways to select a primary color, because $5 + 3 + 4 = 12$. We know that there are a total of 18 marbles, so there are 18 possible outcomes to selecting a single marble.

$$P(A) = \frac{\text{the number of ways event A can occur}}{\text{the total number of possible outcomes}}$$

$$P(\text{primary color}) = \frac{12}{18}$$

$$P(\text{primary color}) = \frac{2}{3}$$

So answer choice D is correct.

Expressing Probability

Probability can be reported in different ways, including as a:

- Fraction
- Decimal
- Percent

When discussing the probability of tossing a coin, the probability of it landing heads up is $\frac{1}{2}$. This is the same as 0.5 or 50%.

> Remember, to convert a fraction to a decimal, divide the numerator by the denominator. To convert a decimal to a percent, move the decimal point two places to the right and add the percent sign.

Probability is never less than zero, nor is it ever greater than 1. An event with a probability of zero is said to be an **impossible** event. For example, the probability of the sun rising in the west tomorrow morning is 0 because it is impossible for that to occur.

On the other hand, an event is **certain** when it will definitely occur. The probability of a certain event is 1. Since the sun will definitely rise in the east tomorrow, the event is certain and has a probability of 1.

Events that have a good chance of happening are considered to be **likely**. Likely events generally have a probability that is between 0.5 and 1. Think about the weather in New York during the winter. It is likely that there will be snow at some point during that season.

An event is considered **unlikely** when there is a small chance of the event happening. Generally, unlikely events have a probability that is less than 0.5. Now think about the weather in September. It is

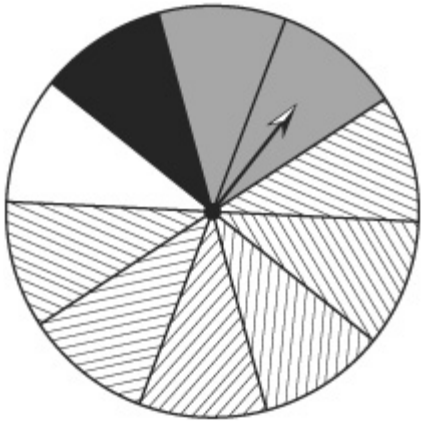
unlikely, but not impossible, that there will be snow at that time of the year.

Suppose you are playing a game that requires you to roll two regular number cubes. What events would be likely, unlikely, certain, or impossible?

- It is *likely* that you will roll a number less than 10.
- It is *unlikely* that you will roll 11 on three consecutive turns.
- It is *certain* that you will roll a number between 2 and 12.
- It is *impossible* that you will roll a 13.

What about the probability of rolling an even number? When the chances of two or more events have the same probability of occurring, they are considered to be **equally likely**. The probability of rolling an even number on a single number cube and the probability of rolling an odd number would be equally likely. When tossing a coin, the chances of it landing on heads are equally likely to the chances of it landing on tails because the chances of both outcomes are equal.

Take a look at the spinner shown below.



Based on the spinner, which event is likely?

A. landing on white
B. landing on gray
C. landing on black

D. landing on striped
E. landing on spotted

An event that is likely is the one that will probably happen. Since more than half of the sections are striped, landing on one of the striped sections is likely. So the correct answer choice is D.

Let's take a look at the other answer choices. Since one section is white and one section is black, these outcomes are equally likely. The probability of landing on white is $\frac{1}{10}$, and the probability of landing on black is also $\frac{1}{10}$.

Two of the sections are gray. Since the probability of landing on one of these sections is $\frac{2}{10}$, or $\frac{1}{5}$, it is unlikely that this will be the outcome. What about answer choice E, landing on spotted? Since there is not a spotted section on the spinner, this outcome is impossible.

Based on the spinner, what is the probability of landing on a striped section?

A. $\frac{1}{6}$
B. 0.6
C. 6%
D. 6
E. $\frac{4}{6}$

There are six striped sections, so there are 6 possible ways for the event to occur. There are a total of 10 sections, so there are 10 possible outcomes. The probability of landing on a striped section of the spinner is $\frac{6}{10}$.

$$P(\text{striped}) = \frac{6}{10}$$

To express this probability as a decimal, divide the numerator by the denominator.

$$\frac{6}{10} = 6 \div 10 = 0.6$$

This makes answer choice B correct. Let's find out how the correct answer would be expressed as a percent as well. Remember, to change a decimal to a percent, move the decimal point two places to the right and add a percent sign.

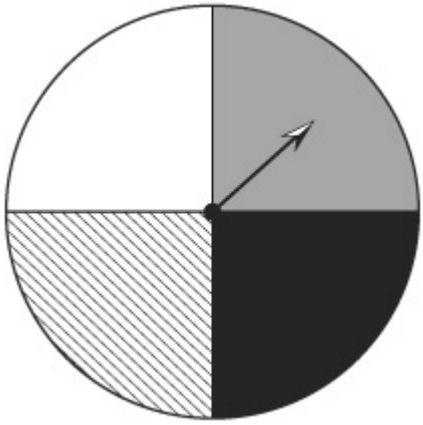
$$0.6 = 60\%$$

So, the probability of landing on a striped section is $\frac{6}{10}$, which is the same as 0.6 or 60%.

Solving Problems Involving Probability

Now that we've gone over the basics of probability, let's apply these skills to solve problems. On the SHSAT, you are likely to find questions that require you to do more than simply find the probability of a given outcome. You will need to be comfortable working with the concepts and thinking through the strategies needed to answer the questions.

Let's start by going over a few terms that you may find in the word problems. An experiment is considered to be **fair** when all of the outcomes are equally probable. Take a look at the spinner below.



Each of the sections of the spinner is the same size, making the possibility of landing on any given color or pattern equally likely. This is considered to be a fair spinner. If you were conducting a survey, you would want to include a random sample of the population in order to ensure that the results are fair and that everyone is equally represented. Let's suppose you want to survey a sample of the students in your school regarding the best way to spend fund-raising money. The choices are to purchase new football equipment, additional laptops for the computer lab, or instruments for the band. In order for the results of the survey to be fair, you would need to ask students representing all grade levels who are involved in a variety of clubs and activities.

Biased, on the other hand, is the opposite of fair and can impact the outcome of an experiment. Suppose you only ask the football players how they would like for the fund-raising money to be spent. You would probably find that they would prefer the new football equipment. So are the results fair? No, because the outcomes were not equally probable when only the football team was asked to weigh in. The results were biased since one outcome was given a better chance of success.

Sometimes an experiment may involve more than one event, such as spinning a spinner *and* rolling a number cube. These are considered to be **independent events** when they do not affect each other in any way. A few examples of independent events are shown below.

- A spinner landing on yellow *and* a coin toss landing on tails

- Selecting a red marble from a jar *and* rolling a 3 on a number cube
- A coin toss landing on tails *and* tossing the coin again and landing on heads

As you can see, the result of any one of these experiments has absolutely no effect on the outcome of the other experiment. Another way to ensure that events are independent is with **replacement**, which is exactly what it sounds like. When an object is removed, then put back, and a second object is chosen, the events do not affect each other. Consider the following example.

Rosa had a jar containing 15 red marbles and 7 blue marbles. She selected 1 red marble, then returned it to the jar. What is the probability that she will select a blue marble next?

Since Rosa put the first red marble back, this event has no effect on the outcome of the second event. By replacing the red marble back into the jar, the probability of choosing a blue marble is $\frac{7}{22}$.

What if Rosa had *not* replaced the marble? Then the events would have been **dependent**, because the outcome of the first event affected the outcome of the second. At first, the probability of selecting a blue marble was $\frac{7}{22}$, since there were 7 blue marbles and a total of 22 marbles in all. But Rosa removing one of the red marbles affected the outcome of the second event. Now there are only 21 marbles. So, the probability of selecting blue becomes $\frac{7}{21}$, or $\frac{1}{3}$. The events were dependent because the outcome of the first event changed the probability of the outcome of the second.

Not only can events be considered independent or dependent, they can also be mutually exclusive. **Mutually exclusive** events cannot occur at the same time. For example, when tossing a coin, the result cannot be both heads and tails. Landing on heads and landing on tails are mutually exclusive events.

What is the probability of rolling a 2 and an even number on a number cube?

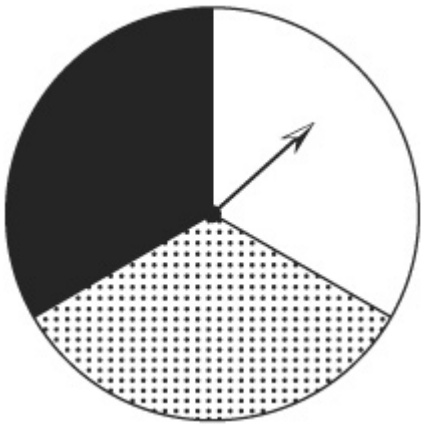
The probability of rolling a 2 is $\frac{1}{6}$. The probability of rolling an even number is $\frac{3}{6}$. Since 2 is an even number, both events can occur at the same time, so these are *not* mutually exclusive.

What is the probability of rolling a 4 and an odd number?

The probability of rolling a 4 is $\frac{1}{6}$. The probability of rolling an odd number is $\frac{3}{6}$. However, these events cannot occur at the same time. Since 4 is not an odd number, these events cannot occur at the same time. So, the events are mutually exclusive. The two events have no favorable outcomes in common.

In the previous example, there was only one favorable outcome for rolling a 4 on the number cube: landing on 4. There were three favorable outcomes for rolling an odd number: 1, 3, and 5. The set of all of the possible results or outcomes of an experiment is called the **sample space**. The sample space is listed within braces. For example, the sample space of tossing a coin is {head, tail}.

What is the sample space for the spinner shown?



There are three possible outcomes for this spinner. It could land on black, it could land on white, or it could land on spotted. So the sample space for the spinner is {black, white, spotted}.

A jar has 5 red marbles, 25 blue marbles, 7 yellow marbles, 8 green marbles, and 3 purple marbles. What is the sample space for choosing a marble from the jar?

Remember, a sample space simply lists the possible outcomes. It does not indicate the probability of any particular outcome.

{red, blue, yellow, green, purple}

Notice that the sample space includes all of the colors of marbles in the jar. It does not show that selecting blue is likely or that selecting purple is unlikely.

Now that you're familiar with the vocabulary, let's try solving a few probability word problems. As with any word problem:

- Read everything carefully.
- Think about what information you have.
- Determine what you need to find out.

A glass jar contains 28 square tiles. The probability of randomly selecting an orange tile is $\frac{2}{7}$. How many green tiles should be added to the jar in order to change the probability of selecting orange to $\frac{1}{4}$?

First, let's figure out what information is given.

- There are 28 tiles.
- The probability of selecting orange is $\frac{2}{7}$.

There are 28 tiles, and the probability of selecting orange is $\frac{2}{7}$. Since the numerator in the probability tells the total number of possible outcomes, we know that this probability has been reduced. Let's find an equivalent fraction that shows the total number of tiles in the numerator.

$$\text{P(orange)} = \frac{\text{number of orange}}{\text{number of tiles}} = \frac{2}{7} \times \frac{4}{4} = \frac{8}{28}$$

Multiplying the probability $\frac{2}{7}$ by $\frac{4}{4}$ tells that the number of orange tiles is 8.

Now we need to find out how many green tiles should be added to change the probability of orange to $\frac{1}{4}$. By adding green tiles, we are not changing the number of orange tiles in the jar. We will still have 8 orange tiles, but we need for 8 to be $\frac{1}{4}$ of the total number. We can use a proportion to determine what the total number of tiles will need to be in order for the probability of orange to be $\frac{1}{4}$. Use a variable to represent the total number of tiles that are needed in the jar.

$$\frac{8}{t} = \frac{1}{4}$$

Cross multiply to solve the proportion.

$$\frac{8}{t} \times 4 = 1 \times t$$

$$32 = t$$

So the total number of tiles should be 32. We started with 28. Subtract to find how many more tiles are needed.

$$32 - 28 = 4$$

We know that 4 green tiles should be added to the jar.

Now try another one. Remember to read carefully, look for given information, and determine what it is you need to find out to solve the problem.

A gumball machine contains 8 grape flavor gumballs, 12 cherry flavor, 4 watermelon, and 6 mint. Elliot buys 5 gumballs, one of which is mint. If he buys one more gumball, what is the probability that it will be mint?

Start by determining what you know:

- There were 8 grape, 12 cherry, 4 watermelon, and 6 mint gumballs.
- Elliot removed 5 gumballs.
- One of the gumballs Elliot bought was mint.

Based on this information, we can determine that there were 30 gumballs in the machine before Elliot bought any because $8 + 12 + 4 + 6 = 30$.

Now determine what it is we need to find out to solve the problem: What is the probability that Elliot will buy a mint gumball next?

To determine the probability of selecting mint, we need to find how many of the remaining gumballs are mint and how many gumballs remain in all.

We know that there were 6 mint gumballs in the machine, and that Elliot bought 1 of them. That means there are now 5 mint gumballs remaining because $6 - 1 = 5$.

We also know that there were a total of 30 gumballs and Elliot bought 5 of them. That means that now the total number of gumballs is 25 because $30 - 5 = 25$.

Let's use this to find the probability of selecting mint next.

$$P(\text{mint}) = \frac{\text{number of mint gumballs}}{\text{total number of gumballs}} = \frac{5}{25}$$

So the probability of selecting mint is $\frac{5}{25}$, or $\frac{1}{5}$. This can also be expressed as 0.2 or 20%.

Let's work through one more word problem.

A bag contains exactly 7 marbles. Of those, 6 are green and 1 is yellow. Madison randomly selects 6 marbles from the bag without replacement. What is the probability that all of the marbles she chose are green?

As always, begin by thinking about what information was given.

- There are 7 marbles.
- Six of the marbles are green.
- One of the marbles is yellow.
- Madison chose 6 marbles.

Now determine what information you need to find: What is the probability that all of the marbles she chose are green?

To solve this problem, we need to find all of the possible results of selecting 6 marbles. Let's use G to represent a green marble, and Y to represent the yellow.

YGGGGG
GYGGGG
GGYGGG
GGGYGG
GGGGYG
GGGGGY
GGGGGG

Take a look at the list of possibilities. The first shows that she drew the yellow marble first, followed by all greens. The second shows that she chose a green, then the yellow, followed by 4 more greens. This pattern continues to the final possibility listed, in which she chose only greens.

By looking at the list, we can determine that there were 7 possibilities for how Madison selected the marbles. We want to know the possibility of selecting all green.

$$P(\text{all green}) = \frac{\text{outcomes including green only}}{\text{total possible outcomes}} = \frac{1}{7}$$

So the probability of selecting all green is $\frac{1}{7}$.

You Try It!

In this chapter, you've reviewed skills and definitions that will be important in answering the probability and statistics questions on the SHSAT. Now use what we have reviewed to solve the problems below.

1. The mean age of the 12 guests invited to Abby's birthday party was 23. Her cousin was unable to attend, so his age was dropped. This changed the mean of the ages to 24. What age was dropped?

A. 1
B. 11
C. 12
D. 22
E. 25

The original mean of the ages, 23, is the result of dividing the sum of the ages by 12, the total number of guests. Let's start by finding the original sum of the ages. Let *a* represent this sum.

$$\frac{a}{12}=23$$

$$12\times\frac{a}{12}=23\times 12$$

$$a=276$$

So original sum of the ages was 276. If one of the ages was dropped, 11 ages remain. The mean of these 11 ages is 24. Let's find the sum of the 11 remaining ages.

$$\frac{a}{11}=24$$

$$11\times\frac{a}{11}=24\times 11$$

$$a=264$$

The age that was dropped is the difference between the original sum of the ages and the sum of the remaining ages.

$$276 - 264 = 12$$

So the age that was dropped from the list is 12.

2. The number of library books checked out by each student in Ms. Callahan's literature class is shown in the table.

Number of Books	Students
0	3
1	4
2	5
3	6
4	2
5	1
6	0
7	2
8	1

What is the median number of books the students checked out?

A. 1
B. 2.5
C. 2.8
D. 3
E. 4

The median is the middle number in the set. So, begin by listing the number of books checked out, in order from least to greatest. Remember to include each number as many times as it appears. For example, since three students checked out zero books, 0 will be included three times.

0, 0, 0, 1, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3,
4, 4, 5, 7, 7, 8

Now find the number in the middle of the set. Since there are a total of 24 numbers in the set, we need to find the twelfth and thirteenth numbers, since these are the values in the middle.

0, 0, 0, 1, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3,
4, 4, 5, 7, 7, 8

Next, average the two values in the middle of the set.

$$2 + 3 = 5$$

$$\frac{5}{2} = 2.5$$

So the median number of books checked out by the students is 2.5.

3. Each month, Ivan records the total number of miles he has hiked with his family. The miles for each month this year are shown below.

25, 28, 26, 46, 30, 28, 52, 43, 26, 32, 34, 26

What is the mode of the miles hiked each month?

A. 26
B. 28
C. 29
D. 33
E. 52

To find the mode, or the number appearing most often in the set, arrange the values from least to greatest. Then look for any value(s) that appear more than once.

25, 26, 26, 26, 28, 28, 30, 32, 24, 43, 46, 52

As you can see, 26 appears three times and 28 appears twice. All other values only appear once. Since 26 appears most often in the set, it is the mode of the data.

4. Lamar had 41 coins in a can. Of those, 16 were quarters, 3 were nickels, 5 were dimes, and the rest were pennies. He took 7 quarters and one additional coin from the can, at random. What is the probability that the coin he removed was a penny?

A. $\frac{17}{41}$

B. $\frac{17}{33}$

C. $\frac{12}{17}$

D. $\frac{1}{8}$

E. $\frac{1}{2}$

The first step in solving this problem is to determine what information is given.

- Lamar began with a total of 41 coins.
- He had 16 quarters, 3 nickels, and 5 dimes.
- The remaining coins in the can were pennies.
- He removed 7 quarters and one other coin.

Now determine what you need to find out: What is the probability that the additional coin he removed was a penny?

To determine the probability that he removed a penny, we must figure out how many pennies there were in the jar originally. We know how many quarters, nickels, and dimes were in the can, and we know that the total number of coins was 41. So the sum of all of the coins equals 41. Let p represent the number of pennies.

$$\begin{aligned} 16 + 3 + 5 + p &= 41 \\ 24 + p &= 41 \\ 24 - 24 + p &= 41 - 24 \\ p &= 17 \end{aligned}$$

There were 17 pennies in the can. To find the probability that he removed a penny, we need to determine how many coins were in the

can after removing the 7 quarters.

$$41 - 7 = 34$$

When he removed the final coin, there were a total of 34 coins remaining in the can. The probability that he removed a penny can be determined using the following formula.

$$P(\text{penny}) = \frac{\text{number of pennies}}{\text{total number of coins remaining}} = \frac{17}{34} = \frac{1}{2}$$

The probability that the final coin Lamar removed was a penny is $\frac{1}{2}$.

5. Amy, Jason, Hannah, Emma, and Katie all want to be on the student council; however, there is only enough room for four of the students. The principal places each of the students' names in a jar and randomly draws four names to determine who will be on the council. What is the probability that all of the students chosen are girls?

A. $\frac{1}{5}$

B. $\frac{4}{5}$

C. $\frac{1}{4}$

D. $\frac{5}{4}$

E. $\frac{5}{1}$

To solve this problem, list all of the possible outcomes of randomly drawing four names. Let G represent the girls' names, and B represent the boy's name.

BGGG
GBGG
GGBG
GGGB
GGGG

There are five possible outcomes. Only one of the outcomes is all girls.

$$P(\text{all girls}) = \frac{1}{5}$$

The probability that all of the students chosen will be girls is $\frac{1}{5}$.

Let's Review!

In this chapter, you've reviewed the probability and statistics skills and vocabulary that will help you correctly answer these questions on the SHSAT. Let's take one more look at some of the information discussed in this chapter.

- **Statistics** are facts and figures that relate to a given topic.
- Measures of **central tendency** are often used to report statistical information. These are ways of indicating what information is in the middle of a set of numerical data, and they include mean, median, and mode.
- **Mean** is the average of a set of data and is found by adding the values in a set, then dividing the sum by the number of items. Any set of data will have only one mean, which may or may not be one of the values included in the set.
- The **median** of a set of data is the value in the middle of the set. If the set includes an even number of items, the median is determined by finding the average of the two numbers in the middle. A set of data will have a single median, which may or may not be one of the values that is included in the set.

- The **mode** is the value that appears most frequently in the set of data. Therefore, the mode must always be a number that is included in the set. There may be more than one mode.
- **Probability** is the statistical chance, or likelihood, that an event will occur. It can be expressed in the form of a fraction, decimal, or percent.

$$P(A) = \frac{\text{the number of ways event A can occur}}{\text{the total number of possible outcomes}}$$

The probability of any event is always between 0 and 1.

The following terms are often included in discussions of probability.

  - **Experiment.** A controlled, repeatable situation involving chance or probability.
  - **Outcome.** A single possible result of performing an experiment one time.
  - **Event.** The set of possible outcomes of an experiment.
- The probability of an event can be classified as being likely, unlikely, certain, or impossible.
  - A **likely** event has a good chance of occurring and generally has a probability that is between 0.5 and 1.
  - An **unlikely** event has a small chance of occurring and generally has a probability that is between 0 and 0.5.
  - A **certain** event will definitely occur and has a probability of 1.
  - An **impossible** event cannot occur and has a probability of 0.
  - Two events that have the same probability of occurring are **equally likely**.
- An event is **fair** when all outcomes are equally probable. An event that is **biased** is the opposite of fair.
- Two or more events that do not affect each other in any way are **independent** events. When the first of two events involves **replacement**, in which an object is put back before the second event occurs, the events are independent. Events are **dependent** when the outcome of the first affects the outcome of the second.

- Events that are **mutually exclusive** cannot occur at the same time.
- A **sample space** is the set of all possible results or outcomes of an experiment.

PART 4
SHSAT Practice Tests

CHAPTER 11
SHSAT Practice Test 1

On the following page you'll find a form similar to that of the actual SHSAT. Carefully tear it out of the book and use it as you take the diagnostic test, for both the Verbal and Mathematics sections.

PART 1 VERBAL

SCRAMBLED PARAGRAPHS

Paragraph 1

The second sentence is	Q	R	S	T	U
The third sentence is	Q	R	S	T	U
The fourth sentence is	Q	R	S	T	U
The fifth sentence is	Q	R	S	T	U
The sixth sentence is	Q	R	S	T	U

Paragraph 2

The second sentence is	Q	R	S	T	U
The third sentence is	Q	R	S	T	U
The fourth sentence is	Q	R	S	T	U
The fifth sentence is	Q	R	S	T	U
The sixth sentence is	Q	R	S	T	U

Paragraph 3

The second sentence is	Q	R	S	T	U
The third sentence is	Q	R	S	T	U
The fourth sentence is	Q	R	S	T	U
The fifth sentence is	Q	R	S	T	U
The sixth sentence is	Q	R	S	T	U

Paragraph 4

The second sentence is	Q	R	S	T	U
The third sentence is	Q	R	S	T	U
The fourth sentence is	Q	R	S	T	U
The fifth sentence is	Q	R	S	T	U
The sixth sentence is	Q	R	S	T	U

Paragraph 5

The second sentence is	Q	R	S	T	U
The third sentence is	Q	R	S	T	U
The fourth sentence is	Q	R	S	T	U
The fifth sentence is	Q	R	S	T	U
The sixth sentence is	Q	R	S	T	U

LOGICAL REASONING

11	A	B	C	D	E
12	F	G	H	J	K
13	A	B	C	D	E
14	F	G	H	J	K
15	A	B	C	D	E
16	F	G	H	J	K
17	A	B	C	D	E
18	F	G	H	J	K
19	A	B	C	D	E
20	F	G	H	J	K

READING

21	A	B	C	D	E
22	F	G	H	J	K
23	A	B	C	D	E
24	F	G	H	J	K
25	A	B	C	D	E
26	F	G	H	J	K
27	A	B	C	D	E
28	F	G	H	J	K
29	A	B	C	D	E
30	F	G	H	J	K
31	A	B	C	D	E
32	F	G	H	J	K
33	A	B	C	D	E
34	F	G	H	J	K
35	A	B	C	D	E
36	F	G	H	J	K
37	A	B	C	D	E
38	F	G	H	J	K
39	A	B	C	D	E
40	F	G	H	J	K
41	A	B	C	D	E
42	F	G	H	J	K
43	A	B	C	D	E
44	F	G	H	J	K
45	A	B	C	D	E
46	F	G	H	J	K
47	A	B	C	D	E
48	F	G	H	J	K
49	A	B	C	D	E
50	F	G	H	J	K

PART 2 MATHEMATICS

MATHEMATICS PROBLEMS

| | | |

One hundred and fifty million years ago, dinosaurs roamed the Western United States in the area that is now the states of Colorado, Montana, Utah, and Wyoming.

_____ **Q.** In addition to the many dinosaurs who roamed the land, the skies were crowded with flying reptiles who were predators and scavengers.

_____ **R.** One that was prominent was the large apatosaurus, a plant eater that had a long neck enabling it to reach the top of vegetation.

_____ **S.** The allosaurus was a successful predator with short front limbs for grasping prey and strong back legs for speed.

_____ **T.** A second notable dinosaur was the allosaurus, a fierce carnivore with sharp teeth and a large head.

_____ **U.** At that time the West was very different because it was hot and moist and dotted with many lakes and swamps that provided a habitat for the many different types of dinosaurs that existed.

Paragraph 2

Prior to the invention of commercially satisfactory methods of refrigeration, the preservation of fresh meat was a difficult task.

_____ **Q.** Cities were famous for the particular blend of spices that they used to preserve the meat they made into sausages.

_____ **R.** The ancient city of Salamis was well known for the spices it used, and the meat named for that city, the salami sausage, has been famous for more than 2,000 years.

_____ **S.** In the Middle Ages, meat was mixed with different combinations of seasoning with each city using its own special blend of additives.

_____ **T.** One solution to this problem of how to preserve meat was to season it with salt and spices, a method that worked very well and was widely used even in ancient times.

_____ **U.** The frankfurter, named after the great German city of Frankfurt, and the bologna sausage, from the Italian city of that name, are the most famous examples of these preserved sausages.

Paragraph 3

The Underground Railroad played a significant part in American history from 1830 until the end of the American Civil War.

_____ **Q.** These safe houses on the Underground Railroad often would put out some sign for the fugitive—perhaps a certain color quilt on the clothesline.

_____ **R.** Of course there was no actual railroad, but the name is given to a secret network of people who helped slaves escape from the South throughout that period.

_____ **S.** After a slave spent the night, he or she would be directed to the next safe house along the route that ran from Maryland and Pennsylvania to New York State and across Lake Ontario to Ontario, Canada.

_____ **T.** These people helped escaping African Americans by providing them with a route to travel to the North, means and money for travel, and safe houses to hide in.

_____ **U.** As a result of these people's efforts, somewhere between 30,000 and 100,000 enslaved people of all ages, some in family groups, managed to find safety after traveling that route to Canada.

Paragraph 4

Few people know that Henry M. Stanley, the African explorer who found the missing Dr. David Livingstone in Zanzibar, was a runaway from Wales who grew to manhood in America.

_____ **Q.** Young Rowlands so loved his benefactor that he took the man's name as his own prior to joining the Confederate army, an act that was meant to honor his adopted father's memory.

_____ **R.** There the man, whose name was Henry Morton Stanley, cared for the boy and finally adopted him just before he died when the boy was 20 years old.

_____ **S.** His real name was John Rowlands, and he was brought up by a wealthy trader who rescued him from the docks of New Orleans and brought him to his home on the Mississippi River.

_____ **T.** After his discovery of Livingstone, he continued exploring the remotest parts of Africa as well as writing books about his adventures that brought lasting fame to the name of Henry Morton Stanley.

_____ **U.** Later he became a renowned journalist for the *New York Herald*, which commissioned him to find Livingstone, who had been missing and was presumed dead by some.

Paragraph 5

In developed countries, a large proportion of all deaths are due to a disease that is called arteriosclerosis.

_____ **Q.** Some of this hardening is caused by high-cholesterol diets and by insufficient exercise as well as having a family history of this disease.

_____ **R.** This is because arteriosclerosis is caused by the wear and tear resulting from the constant flow of blood through the arteries over the course of a lifetime.

_____ **S.** Medical researchers believe that every man or woman, if he or she lives long enough, will inevitably suffer from arteriosclerosis.

_____ **T.** Arteriosclerosis is a condition in which fatty material collects on the walls of the arteries, causing hardening of the arteries and possible blockage.

_____ **U.** But exercise and a low-fat diet can help control the onset of the disease, and there are many procedures and medicines to help keep it under control should it develop.

Logical Reasoning

QUESTIONS 11–20

DIRECTIONS: Read the scenarios below, then choose the **best** answer to each question. You must base your response **only on the information given.**

Read the scenarios with careful attention to the precise meaning of the words used. For example, “The red shirt hanging **between** the yellow and blue shirts” does not necessarily mean “The red shirt is **between and next to** the yellow and blue shirts”; one or more other articles of clothing may separate the red shirt from the yellow shirt or from the blue shirt. This precaution also applies to location words such as **above, below, before, after, ahead of,** and **behind.**

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11. At a jewelry store, there are five different kinds of gems placed in trays one on top of the other. The gems consist of diamonds, emeralds, rubies, sapphires, and pearls.

1) The diamond tray is higher in the stack than the pearl tray is.
2) Exactly two gem trays are between the pearls and the rubies.
3) The emerald tray is third from the top.

If the bottom tray contains pearls, which gems are on the top?

A. sapphires
B. diamonds
C. rubies
D. Either sapphires or diamonds, but it is not possible to determine which one.
E. Either sapphires or rubies, but it is not possible to determine which one.

12. There are two bicycles in front of the school.

1) One bicycle is a 10-speed; the other is a 3-speed.
2) The 10-speed bicycle is used only on Saturdays.
3) One bicycle is red.
4) One bicycle is used only on Fridays and is scratched.
5) The bicycle that is not scratched needs new tires.

Based only on the information above, which of the following **must** be true?

F. The 10-speed bicycle is used on Fridays.
G. The 10-speed bicycle needs new tires.
H. The 10-speed bicycle is scratched.
J. The 3-speed bicycle is not red.
K. The 3-speed bicycle is not scratched.

13. Five weather symbols are displayed next to each other on a wall. They are numbered consecutively from left to right.

1) One symbol is a sunny symbol.
2) A snow symbol is in the second position.
3) There are exactly three symbols between the sunny and the cloudy symbols.
4) There is exactly one symbol to the right of the rain symbol.

In which position is the sleet symbol?

A. second
B. third
C. fourth
D. fifth
E. Cannot be determined from the information given.

14. Penny will not go to the concert if Betty does. Betty will go to the concert if Diane is not late getting home.

Based only on the information above, which of the following **must** be true?

F. If Diane is late getting home, Penny will go to the concert.
G. If Betty does not go to the concert, Penny will go to it.
H. Betty will not go to the concert if Diane is late getting home.
J. If Diane goes to the concert, Betty will go also.
K. If Diane is on time getting home, Betty will go to the concert.

15. People who become members of the Harwich Community Garden are expected to help in the garden two hours a week with weeding and other maintenance. Fred volunteers one hour a week for the Harwich Community Garden.

Based only on the information above, which of the following **must** be true?

A. Fred is a volunteer for other organizations, so he does not have enough time to work two hours for the Harwich

Community Garden.

B. If Fred had more free time, he would volunteer more hours to the community garden.

C. If all members of the community garden helped in the garden only one hour a week, the garden would be overrun with weeds.

D. Fred is not a member of the Harwich Community Garden.

E. If Fred is a member of the Harwich Community Garden, he is not doing what is expected of him.

16. Four people were rowing canoes across a lake. They got across the lake one at a time. Their canoes were made of different materials. One was made out of wood, another was made out of fiberglass, another was made of polyethylene, and the other one was made of aluminum.

1) Devin arrived right after Marina.
2) Marina was in the canoe made of wood.
3) Charles was not in the canoe made of aluminum.
4) The canoe made of wood got across the lake before the canoe made of polyethylene.
5) Hannah, who was in the canoe made of fiberglass, arrived after Devin.

Based only on the information above, which of the following **must** be true?

F. The canoe made of aluminum got across the lake before the canoe made of polyethylene.

G. Charles arrived across the lake third.

H. Devin was in the canoe made of polyethylene.

J. Hannah arrived third.

K. The canoe made of polyethylene arrived second.

17. Five pots, each one a different color, are placed next to each other according to their size, from smallest to largest, left to right.

1) The green pot is smaller than the purple pot but bigger than the blue pot.
2) The purple pot is smaller than the red pot.
3) The yellow pot is in the third spot.

Which pot is in the largest?

A. the blue pot
B. the purple pot
C. the red pot
D. the green pot
E. Cannot be determined from the information given.

Questions 18 and 19 refer to the following information.

In the code below, (1) each letter always represents the same word, (2) each word is represented by only one letter, and (3) in any given sentence, the position of a letter is **never** the same as that of the word it represents.

B	F	H	E	D	means
"The	horse	gallops	four	miles."	

E	J	F	B	H	means
"The	horse	gallops	two	miles."	

H	K	E	F	J	means
"The	horse	trots	two	miles."	

18. Which letter represents the word "gallops"?

F. J
G. F
H. B
J. D
K. E

19. Which word is represented by the letter H?

A. the
B. horse
C. two
D. Either “the” or “horse,” but it is not possible to determine which one.
E. Either “horse” or “miles,” but it is not possible to determine which one.

20. Most of the students attending the conference are science majors. However, some of them are not very good students.

F. All of the students who are attending the conference are science majors.
G. Some students attending the conference are excellent in science.
H. Being a good student is necessary to attending the conference.
J. Most students attending the conference are good students.
K. Students who are not science majors can attend the conference.

Reading

QUESTIONS 21–50

DIRECTIONS: Read the passages below, then answer the questions that appear after each passage. Select the **best** answer for each question. Reread the passages if necessary. You must base your response **only on information contained in the passage.**

One of the greatest discoveries of all time took place in the People's Republic of China. It happened in March 1974 when a farmer who was digging a well stumbled across a clay soldier buried in the dirt. The site of the discovery was in a field 18 miles east of Xi'an, the capital of Shaanxi province. After the farmer's discovery, archaeologists continued to dig. They were amazed to discover what are now called the Qin Terra-cotta Warriors and Horses.

Xi'an is one of the oldest cities in China. It dates back more than 3,000 years. The clay figures were crafted over 2,200 years ago during the reign of the Emperor Qin Shihuang (259–210 BCE). They were found adjacent to Emperor Qin's tomb. Qin is considered by historians to be the first emperor of China. He is credited as the person who unified China. His reign began the Qin Dynasty. He was also responsible for starting construction on the Great Wall of China.

The clay figures are remarkable! They are life-sized but vary in height. The warriors all have different facial expressions and hairstyles. Their uniforms differ according to rank. Their gestures are all different as well. It is as though they were alive. There are horsemen, longbow bearers, archers, officers, and generals. There are even officials, acrobats, and musicians. Included with the soldiers are chariots and horses. And these amazing warriors are all positioned in battle formation as dictated by the *Art of War*. This treatise was written by Sun Tzu some 500 to 200 years earlier. Altogether, 6,000 figures were discovered. According to historians, it took 720,000 workers 37 years to craft the terra-cotta warriors, chariots, and horses.

The terra-cotta warriors and horses were originally painted with a colored lacquer. This made them look even more realistic

45 and lifelike. Over the years, the paint wore off. The warriors carried iron swords and spears that were coated with chromium oxide. This kept them from rusting over that many years.

Remarkably, the preservation technique was not known in the West until the eighteenth century.

Some believe this terra-cotta army was meant to defend the tomb of Qin Shihuang from thieves and marauders. Others think

55 the warriors and horses were there to allow Qin to rule another empire in the afterlife. Whatever the reason, the terra-cotta army is a magnificent spectacle to view. It is a testament to the power the

60 emperor possessed to commission such a monumental undertaking.

21. Which of the following best tells what this passage is about?

A. how archaeologists dug up the terra-cotta army
B. why the terra-cotta army was created
C. what Emperor Qin achieved
D. what the terra-cotta army was like
E. how the terra-cotta soldiers were created

22. The passage suggests that Emperor Qin

F. had a problem getting the terra-cotta soldiers made.
G. was a man of many accomplishments.
H. believed in an afterlife.
J. died when he was rather young.
K. wanted the terra-cotta army discovered.

23. Which of these statements is true about the Qin Terra-cotta Warriors and Horses?

A. They were made by famous artists of the time.
B. They were the only terra-cotta statues made in China.

C. They were the only items that Qin wanted buried with him.

D. They show what an army was like during Qin's reign.

E. They are proof that the Chinese were great philosophers.

24. Which of the following best describes what is suggested by the statement that "this preservation technique was not known in the West until the eighteenth century" (lines 49–51)?

F. At that time the Chinese were more advanced technologically than the Western world was.

G. At that time the Chinese were better educated than the people in the Western world.

H. During this period, the Chinese were in touch with many other cultures in order to learn how to preserve items.

J. During this period, the Chinese were intent upon being more capable than people in the Western world.

K. At that time, scientists in China had done more research than scientists in the Western world.

25. What does the existence of the terra-cotta army suggest about this period in China?

A. It was a time when science flourished.

B. It was a time when the country was unified.

C. It was a time when armies had been disbanded.

D. It was a time when most people were at peace.

E. It was a time when armies were needed.

26. What was **not** a concern of the archaeologists who studied the terra-cotta army?

F. why the terra-cotta army was created

G. how the terra-cotta army was discovered

H. how many soldiers there were in the terra-cotta army

J. figuring out how long it took to create the terra-cotta army

K. the cost of creating the terra-cotta army

Three hundred feet beneath the surface of the Earth, below rolling hills of farmland and picturesque villages near Geneva, Switzerland, physicists are looking for the building blocks of the universe. What they are seeking are subatomic particles that are not possible to find in a normal laboratory setting. The scientists are using a device called an atom smasher to simulate the conditions that existed when the universe started. The beginning of the universe is called the big bang by scientists. The big bang theory postulates that around 15 billion years ago the universe suddenly expanded. It acted like a balloon that has been popped. And as it expanded, it cooled. It continues to expand and cool to this day.

The Large Hadron Collider (LHC) is the world's largest atom smasher. It started operating in 2009. It is a large ring that is 17 miles in circumference. There are 9,300 magnets in the ring. The collider shoots two beams of proton particles in opposite directions. The beams are guided by the magnets. The more protons are accelerated, the greater the chance that two protons will smash into each other. The bigger the smasher, the more force at collision. The particles travel around the ring 11,245 times a second at 99.99 percent of the speed of light (about 186,282 miles per second.) They produce 600 million collisions every second. When they collide, they produce debris that very quickly disintegrates. Computers analyze the results.

The scientists are smashing particles together to see what results. Atoms can be broken down into other, smaller particles.

The physicists are looking for rare particles that can't be broken down any further. They are called fundamental or elementary particles and include quarks, leptons, and bosons. So far, 57 elementary particles have been found. Some have an electrical charge and a spin. Scientists believe that other hypothetical particles exist, but their

presence has never been proven. One such particle is the Higgs boson. It is proposed by the physicist Peter Higgs as an answer to why some particles possess mass and others don't. It is often referred to as the "God particle." Researchers are also looking to prove the existence of what is called dark matter. This is invisible matter that is thought to be most the prevalent material in the universe. It is theorized that it was what was left over after the big bang. The Large Hadron Collider was created with the hope that it would provide answers to scientists' questions. It will probably revolutionize particle physics. And it may also lead to more questions.

27. Which of the following best tells what this passage is about?

A. why scientists believe in the big bang theory
B. which fundamental particles have been identified
C. that dark matter is invisible
D. how the Large Hadron Collider is constructed and operates
E. what scientists are trying to achieve with the Large Hadron Collider

28. The author implies that some fundamental particles

F. can be shot from the Large Hadron Collider.
G. have been improperly identified.
H. may never be identified positively.
J. are easily tested.
K. stay charged for a long time.

29. Which of these statements is true about the Large Hadron Collider?

A. It is the first atom smasher ever used.
B. It can work continuously for 24 hours.

C. It was designed by astronomers.
D. It is difficult to operate.
E. It provides new information for physicists.

30. Which best describes the relationship between an atom and a quark?

F. A quark is a part of an atom.
G. A quark lasts longer than an atom.
H. They are the same thing.
J. They both are fundamental particles.
K. Their relationship is unknown.

31. Which is **not** true of the Higgs boson?

A. It would explain why some particles possess mass and others do not.
B. It may be found by using the Large Hadron Collider.
C. It is also called the “God particle.”
D. It was named for Peter Higgs.
E. It has been verified.

32. What might be inferred about dark matter?

F. It is part of an atom.
G. It is a very small particle.
H. It exists only in smaller galaxies.
J. There is very little known about it.
K. It can be verified more easily than other matter.

Haller Park, also known as the Bamburi Nature Trail, located along the Indian Ocean in Mombasa, Kenya, is a lush paradise of forests, ponds, savannahs, and wetlands. Animals thrive in this wildlife sanctuary. Visitors flock to this nature preserve today, but they find it hard to believe that Haller Park is, indeed, an ecological experiment.

When Rene Haller, a Swiss agronomist, a person who studies soil and crop management, first visited the area in 1971, it was an uninhabitable wasteland. The Bamburi Cement Company had mined limestone there for years. The exhausted quarry that Dr. Haller first observed had no trees or plants. There were pools of brackish water everywhere, but no living creatures. With the backing of the cement plant, he set out to transform this industrial scar.

At first Haller found that every tree he planted withered and died. Then one day he discovered casuarina trees. Native to Australia, the casuarina tree is an evergreen that can survive in barren, arid locations. He began planting young trees. Conocarpus, a tree native to Somalia that can also withstand harsh environments, were also planted. As the trees began to thrive and grow, the trees dropped needles to the ground. Millipedes fed on the needles. Their droppings contained bacteria that fertilized the soil. Termites ate the dead vegetation. Their droppings also enriched the ground. Mushrooms began to grow. Slowly the land began to transform. Haller dug out ponds with a bulldozer. He stocked the ponds with guppies, tilapia, and black mollies. Bulrushes were planted along the shores to provide shelter for birds. Ferns were planted. Eucalyptus, coconut palm, banana, and mango trees were all introduced. The baobab tree, native to Kenya, and the neem tree, native to India, which provide excellent shade, also thrived. Today, there are miles of nature trails where storks, cranes, and weaver birds abound. Antelopes, buffalo, giraffes, and crocodiles also live there. Most of the animals were rescued, such as the hippos, Sally and Potty. The sanctuary is interactive. Each afternoon, visitors can hand-feed the giraffes. There is a 100-year-old tortoise name Mzee that likes to have his throat tickled. In addition, oryx antelopes and elands are domesticated for meat at the Nguuni Wildlife Sanctuary. There is a crocodile farm and an aquaculture fish farm as well. Visitors to the park can even try a taste of crocodile steak, antelope meat, or tilapia filet at a restaurant in the park.

In 1987, Rene Haller was presented with the United Nations Environment Programme “Global 500 Roll of Honor for Environmental Achievement” Award. It is a fitting tribute to a man who had a vision and persevered to see it come true.

33. Which of the following best tells what this passage is about?

A. why Haller Park was a wasteland
B. what kinds of animals are living at Haller Park
C. which trees were used to change Haller Park into a wildlife sanctuary
D. how a wasteland was changed into a wildlife sanctuary
E. what awards Rene Haller has received

34. What is **not** mentioned as being present at Haller Park?

F. rescued animals
G. ponds with fish
H. crocodiles
J. sheep
K. savannahs

35. According to the passage, which of the following statements about Haller Park is accurate?

A. Haller Park is the only nature sanctuary that was once a wasteland.
B. Haller Park has acted as a model for other ecologically devoid areas.
C. Only rescued animals are permitted at Haller Park.
D. Only animals that are native to the Haller Park region are allowed there.
E. Haller Park was supported by the company that caused it to be a wasteland.

36. The information about the crocodile farm was included in order to illustrate

F. that crocodile meat is a popular food.
G. that Haller Park is an experiment.
H. that Rene Haller enjoys entertaining people.
J. that some people travel long distances to dine at Haller Park.
K. that Haller Park produces its own food.

37. Which of the following is Rene Haller most likely to choose for another project?

A. starting fish farms in areas where there is a lot of unemployment
B. creating a cooperative garden
C. organizing a community park
D. cleaning up a landfill so it can be used as a storage facility
E. changing a desert into a lush area where many fruit trees grow

38. What is probably true of Rene Haller?

F. He is an animal lover.
G. He is a humble person.
H. He enjoys publicity.
J. He likes to make others happy.
K. He has great determination.

Dolphins are well known to many people. They entertain children and adults in venues like Sea World. There are facilities where people can swim with dolphins, an

experience that many call unforgettable. Dolphins are also used in research labs. And they are kept by the U.S. Navy in San Diego for military studies. In all of these circumstances, the dolphins are kept in captivity.

Do animals have rights? The question is a difficult one, with animal rights activists on one side and animal research proponents on the other. As scientists learn more about the mental life of animals, a number of astounding facts have begun to emerge. Scientists and researchers now conclude that dolphins are the second most intelligent creatures after humans. What is important is brain size in relation to overall size.

Dr. Lori Marino, a zoologist with Emory University in Atlanta, Georgia, has conducted extensive MRI (magnetic resonance imaging) scans on the brains of dolphins. She has concluded that the mind of a dolphin works in a manner very similar to the brain of a human. "The tests suggest a psychological continuity between humans and dolphins," said Marino.

In another experiment, researchers marked two dolphins with triangles and circles on their foreheads, backs, and flippers, places they could not see. They then placed a mirror in the tank. Both dolphins immediately swam to the mirror and examined themselves. They exhibited a level of self-awareness only seen in humans and chimpanzees. This showed that dolphins have a concept of themselves.

In Australia, a rescued dolphin was taught by trainers to tail-walk, the feat of moving backward through the water on its tail. After recuperating, the dolphin was released back into the wild. Amazingly, that dolphin taught dozens of other wild dolphins how to tail-walk. Learned behavior in a community of animals is mainly for getting food. Tail-walking has no practical function; it is only done for fun.

The question then becomes, can the same quality of research be done in the wild as in captivity? Pro-captivity supporters contend that the dolphins are better off and reproduce well in captivity. They point out that more than 50 percent of dolphins in the wild have scars from shark attacks. Naval researchers claim that even though their dolphins are penned in San Diego Bay, they could easily jump out and swim away, but they don't. Those who advocate releasing dolphins into the wild feel that dolphins are so much like people that they shouldn't be kept in captivity. They believe

65 that dolphins should be granted a special ethical status and should be treated as nonhuman persons. The debate will no doubt continue for many years.

39. Which of the following best tells what this passage is about?

A. how dolphins are taught to tail-walk
B. why sharks attack dolphins
C. whether dolphins should be kept in captivity or not
D. what research has discovered about dolphins' brains
E. the fact that dolphins in captivity do not try to escape

40. The best reason that the author began the passage by describing how people can swim with dolphins was to

E. remind the readers that dolphins are in captivity.
G. impress the reader with how gentle dolphins are.
H. suggest that dolphins are able to learn tricks easily.
J. show how easy it is to keep dolphins in captivity.
K. suggest that people abuse dolphins.

41. Which reason is **not** given for letting dolphins go free?

A. They are extremely intelligent.
B. They have a concept of themselves.
C. Their brains work in the same way as human brains.
D. They are often attacked by predators.
E. They should be given the status of nonhuman persons.

42. Which is true of dolphins?

F. They like to be alone.
G. They try to protect people.

H. They are emotional beings.
J. They are second to chimpanzees in intelligence.
K. They do not try to escape when in captivity.

43. The author includes information about Dr. Marino to

A. lend the passage greater authority.
B. show that the author is knowledgeable about the subject.
C. suggest that some research is more accurate than other research.
D. emphasize how much research has been done on dolphins.
E. make sure the passage tells both sides of the debate.

44. What is likely to happen if dolphins are no longer kept in captivity?

F. People will swim with them in the ocean.
G. Dolphins will grow in number.
H. People will not bother with them.
J. Research on them would end.
K. More dolphins are likely to die.

Beryl Markham was a unique and brave woman. Born Beryl Clutterback in Leicester, England, in 1902, she moved with her family to Kenya when she was 5 three. Her father, Charles, bought a coffee plantation. After a short time, Charles discovered that he had a knack for training horses, and he used the land to breed and train horses for racing. Beryl grew up on the farm. She learned the native language, and she also learned to ride and train horses. Her horses won many races. That was quite unusual because at that time horse racing was a sport dominated by 10 15 men

Then one day something happened that would change her life. She met Tom Campbell Black, a world-famous aviator. Once she had experienced flying, Beryl became obsessed with the idea of being a pilot. She learned to fly and received her commercial pilot's license. She was the first woman pilot in Kenya, becoming a "bush pilot," transporting passengers, supplies, and mail to remote parts of Kenya. There were no airports; she landed and took off on fields or clearings in the forest. At that time, in the infancy of the aviation age, air racing was popular, and Tom tried to convince her to enter a race from Nairobi to Johannesburg, South Africa. Instead Beryl determined to fly from Kenya to England. Hindered by persistent engine problems and consistent bad weather, it took her 35 days, but she became the first person to fly solo from Nairobi to London.

In 1936 Beryl decided she wanted to be the first woman to fly alone across the Atlantic. Charles Lindbergh had already soloed across the ocean in 1927 from the United States to England. Beryl wanted to go the opposite way, from east to west. That was much more difficult because of the prevailing winds that would blow against her. On September 4, 1936, Beryl left London in a Vera Gull airplane, a single-engine monoplane crafted from wood and fabric. The plane was outfitted with extra fuel tanks, but Beryl had no radio and no navigational equipment when she took off into the darkness.

Twenty hours later, her fuel line froze, and the plane crash-landed in a peat bog on Cape Breton Island, Nova Scotia. She emerged from the plane unharmed. Two fishermen approached her, and she said calmly, "I'm Mrs. Markham. I've just flown from England." Beryl was taken to New York City and honored with a ticker tape parade. Her feat was reported in newspapers around the world. In 1942 she wrote *West with the Night* about her life and accomplishments. After living in America for a number of years, Beryl returned to Kenya to be with her beloved horses. She won many more races. She died in 1986.

45. Which of the following best tells what this passage is about?

A. why flying to the United States was difficult
B. what Beryl Markham did as a young person
C. how Beryl Markham became a pilot
D. how Beryl Markham made her mark on history
E. what Beryl Markham did when she landed in Nova Scotia

46. How did the experience of meeting Tom Campbell Black affect Beryl Markham?

F. She became a better person.
G. She decided to write a book.
H. She realized what she wanted to do with her life.
J. She left Kenya for the United States.
K. She decided to be a career woman.

47. Why does the author call Beryl Markham "a unique and brave woman" (lines 1–2)?

A. She was good at training horses.
B. She did something that was dangerous and hard to do.
C. She returned to Kenya after many years had passed.
D. She learned to fly a plane.
E. She received a commercial pilot's license.

48. How did what Charles Lindbergh achieved differ from what Beryl Markham did in her flight across the Atlantic Ocean?

F. Her plane was lighter than his.
G. She was newer to aviation than he was.
H. Her flight lasted a long time.

J. The winds were against her.
K. She was alone.

49. Why do you suppose Beryl Markham returned to Kenya?

A. She always considered it her home.
B. She could not get an American visa.
C. She had run out of money.
D. She was asked to by the Kenyan government.
E. She wanted to stop flying.

50. What can be inferred about Beryl Markham?

F. She was concerned about how she was perceived by the public.
G. She was difficult to get along with.
H. She was not practical.
J. She thought mainly about herself.
K. She did not let obstacles stand in her way.

PART 2
MATHEMATICS
75 Minutes ▪ 50 Questions

General Instructions

Find the solution to each problem, then choose the best answer from the options given. Mark the letter of your answer choice on the answer sheet. You may use your test booklet or paper provided by the proctor as scrap paper for your calculations. However, DO NOT make calculations or put any stray marks on your answer sheet.

Important Notes

(1) No formulas or definitions of mathematical terms and symbols are provided.

(2) Diagrams are not necessarily drawn to scale. Be careful not to make assumptions about relationships in diagrams.

(3) Diagrams can be assumed to be in one plane unless otherwise stated.

(4) Graphs are drawn to scale. Therefore, you may assume relationships according to appearance. This means lines that look parallel can be assumed to be parallel, angles that look like right angles can be assumed to be right angles, and so forth.

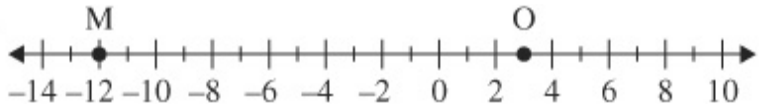
(5) All fractions must be reduced to lowest terms.

---

51. *X* is 30% of *Y* and *Y* is 45% of 600. What is the value of *X*?

A. 54
B. 81
C. 99
D. 180
E. 270

52.



N (not shown) is a point on the number line such that $\overline{MN}$ is $\frac{2}{5}$ the length of $\overline{MO}$. What is the location of point N?

F. –15
G. –6

H. 6
J. 9
K. 15

53. Convert $\frac{3}{40}$ to a decimal form.

A. 0.00075
B. 0.0075
C. 0.075
D. 7.50
E. 75.0

54. If *s* is positive and $s^2 = 75$, between which two numbers does *s* lie?

F. 4 and 5
G. 5 and 6
H. 6 and 7
J. 7 and 8
K. 8 and 9

55. NUMBER OF BOOKS READ DURING THE SUMMER

Books Read	15	20	12	18
Number of Students	4	3	3	5

The table above shows the number of books read by 15 students during the summer. What is the mean number of books read?

A. 10.4

B. 12.4
C. 15.5
D. 16.4
E. 17.5

56. What is the greatest prime number less than 23?

F. 17
G. 19
H. 20
J. 21
K. 22

57. $7p(3-5q) =$
A. $21-5q$
B. $21-35q$
C. $21p-35q$
D. $21p-35pq$
E. $21pq-35pq$

58. What is the least integer greater than $\frac{31}{3}$?

F. 10
G. 11
H. 12
J. 13
K. 14

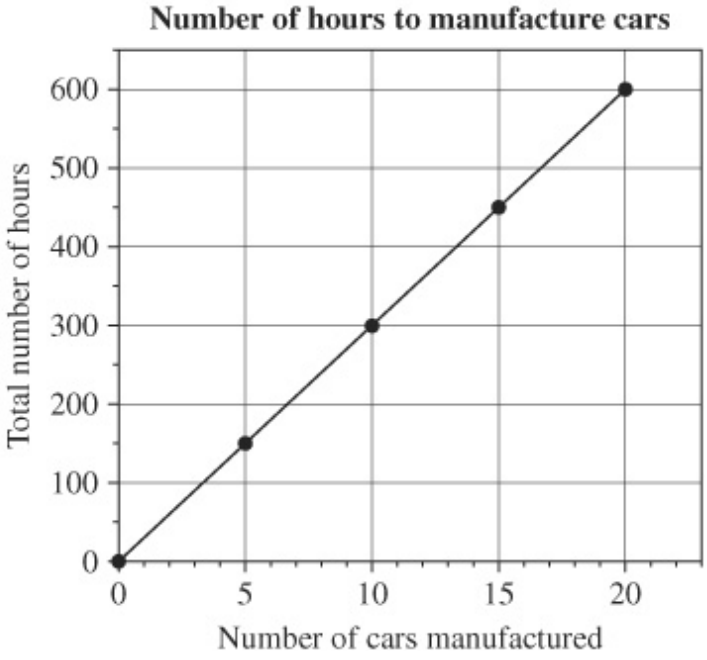
59. W is a point that is not on line *n*. How many lines can be drawn through W that are perpendicular to line *n*?

A. 1
B. 2
C. 3
D. 4
E. 5

60. At 6:00 a.m., the temperature was 7 degrees below zero Fahrenheit. At noon, the temperature was 10 degrees higher. At 6:00 p.m., the temperature was 5 degrees cooler than it was at noon. What was the temperature at 6:00 p.m.?

F. –2°
G. –1°
H. 3°
J. 5°
K. 8°

61.



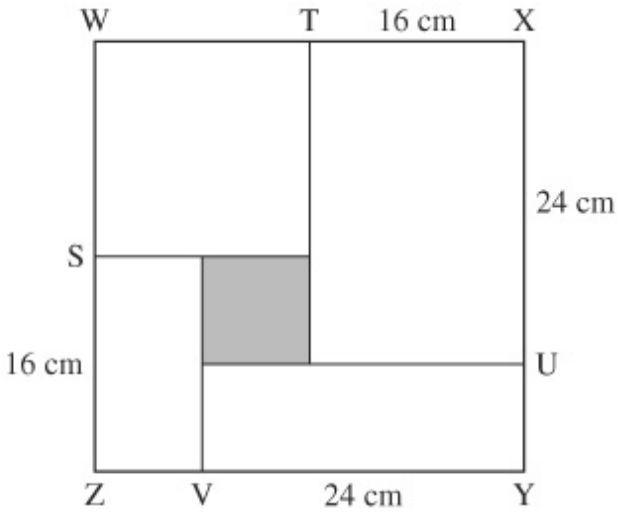
The graph above shows the relationship between the number of cars manufactured and the number of hours it took to manufacture them. How many hours did it take to manufacture one car?

A. 20 hours
B. 30 hours
C. 40 hours
D. 50 hours
E. 60 hours

62. The product of a nonzero integer and 12 has the same value as the product of 3 and the square of the integer. What is the integer?

F. 2
G. 3
H. 4
J. 6
K. 9

63.



WXYZ is a square that is made up of rectangles and squares. The side length of WXYZ is 32 centimeters. SZ and TX are each 16 cm. UX and VY are 24 cm. What is the area of the shaded region?

A. 16 sq cm
B. 24 sq cm
C. 36 sq cm
D. 48 sq cm
E. 64 sq cm

64. Which of the following is equivalent to the inequality $17 > x + 3$?

F. $x < 14$
G. $x > 14$
H. $x < -14$
J. $x < 20$
K. $x > 20$

65. $(-5x + 8) - (2 + x) =$

A. $-6x + 6$
B. $-6x + 10$
C. $-4x + 10$
D. $4x + 6$
E. $6x + 6$

66. What is the sum of 30% of 3 and 3% of 30?

F. 0.6
G. 0.9
H. 1.2
J. 1.8
K. 9.9

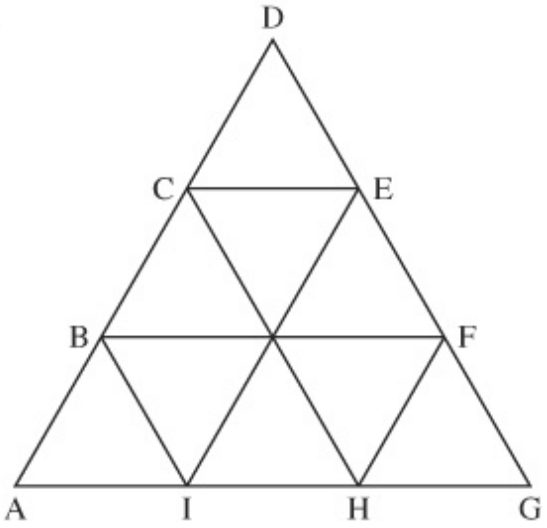
67. If $42{,}592 = 2^x \times 11^y$, what is xy?

A. 8
B. 9
C. 15
D. 22
E. 25

68. Stuart and Brandy live 5 miles apart on the same street. Stuart starts jogging at 3 miles per hour from his house to Brandy's house. At the same time, Brandy rides her bicycle at 12 miles per hour from her house to Stuart's house. How far from Stuart's house will they meet?

F. 1 mile
G. 2.5 miles
H. 3 miles
J. 4 miles
K. 4.5 miles

69.



The figure above is made up of 9 equilateral triangles. How many triangles in the figure are similar to triangle ABI? (Do not

include ABI itself.)

A. 1
B. 3
C. 8
D. 10
E. 12

70. One-fourth the product of two numbers is 4. If one of the numbers is 8, what is the sum of the two numbers?

F. 2
G. 8
H. 9
J. 10
K. 16

71. There were 5,000 beads in a container.
Out of 25 beads, 13 were gold and 12 were silver. Which is the best estimate of the total number of gold beads in the container?

A. 400
B. 600
C. 650
D. 2,400
E. 2,600

72. The area of a rectangle is 165 square feet. If the width is 11 feet, what is the perimeter?

F. 15 feet
G. 26 feet
H. 37 feet
J. 41 feet
K. 52 feet

73. The bakery sells 7 different types of bagels. Melissa orders one type of bagel. Then Julio orders a different type of bagel. How many different ways can Melissa and Julio order?

A. 13
B. 14
C. 27
D. 42
E. 49

74. If $x' = x - 2$, what is the value of $\frac{6'}{3'}$?

F. 1
G. 2
H. 3
J. 4
K. 6

75. What is the value of $xy + \frac{x}{y}$ when $x = -1.5$ and $y = -0.3$?

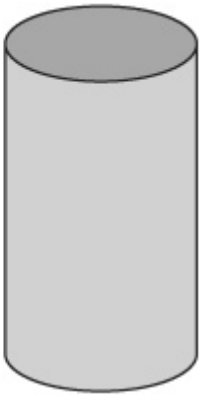
A. –4.55
B. –3.20

C. 0.95
D. 5.45
E. 6.80

76. Jerri was x years old 5 years ago. Serena is half as old as Jerri is now. In terms of x, how old is Serena?

F. $\frac{1}{2}(5x)$
G. $\frac{1}{2}(x-5)$
H. $\frac{1}{2}(x+5)$
J. $\frac{1}{2}x-5$
K. $\frac{1}{2}x+5$

77.



The diagram above shows a cylindrical pipe that is painted gray on the side. The pipe is 9 cm tall and has a diameter of 10 cm. What is the area of the painted portion of the pipe?

A. 20π sq cm
B. 60π sq cm
C. 90π sq cm
D. 140π sq cm
E. 180π sq cm

78. What is the least common multiple of X and Y?

$X = 2 \times 2 \times 2 \times 2$
$Y = 2 \times 2 \times 2 \times 3$

F. 2×3
G. $2^3 \times 3$
H. $2^4 \times 3$
J. $2^6 \times 3$
K. $2^7 \times 3$

79. In a scale drawing, 1 centimeter represents 200 meters. How many square centimeters on the drawing represent 10 square meters?

A. 0.0025 sq cm
B. 0.025 sq cm
C. 0.25 sq cm
D. 2.50 sq cm
E. 25.0 sq cm

80. Matthew has 16 coins. Together, the coins have a value of $2.19. He spends 7 quarters, 3 dimes, and 1 nickel on a bottle of water. What coins does he have left?

F. 1 quarter and 1 penny
G. 2 dimes and 1 penny
H. 1 nickel and 2 dimes
J. 1 nickel and 4 pennies
K. 2 dimes and 3 pennies

81. x, y, and z are consecutive multiples of 4, counting from smallest to largest. What is $x + z$ in terms of y?

A. $2y$
B. $4y$
C. $2y + 8$
D. $2y - 8$
E. $4y + 4$

82. A rectangle has a width of 24 inches and a length of 72 inches. It is made up of squares of the same size. What is the largest possible size of the squares?

F. 6 in. by 6 in.
G. 8 in. by 8 in.
H. 9 in. by 9 in.
J. 12 in. by 12 in.
K. 24 in. by 24 in.

83. The volume of a pyramid with a square base is 80 cubic centimeters. The side length of the square base is 4 centimeters. What is the height of the pyramid?

A. 5 cm
B. 10 cm
C. 15 cm
D. 20 cm
E. 25 cm

84. What fraction, reduced to its lowest terms, is halfway between $\frac{2}{3}$ and 0.2?

F. $\frac{2}{13}$

G. $\frac{2}{30}$

H. $\frac{7}{15}$

J. $\frac{13}{15}$

K. $\frac{13}{30}$

85. Which of the following indicates that x is greater than 12 and that x is less than or equal to 3 times the value of y?

A. $4 \leq x < y$

B. $4 < 3x \leq y$

C. $4 < x \leq \frac{y}{3}$

D. $4 < \frac{x}{3} \leq y$

E. $4 \leq x < 3y$

86. If $x = 1$ and $y = 2$, what is the value of $\frac{2x^y}{y^{2x}}$?

F. $\frac{1}{4}$

G. $\frac{1}{2}$

H. 1

J. 4

K. 6

87. The following table shows the amount of trash put into landfills in the United States. What is the median weight of trash for the years shown?

AMOUNT OF TRASH PUT INTO LANDFILLS
(IN MILLIONS OF TONS)

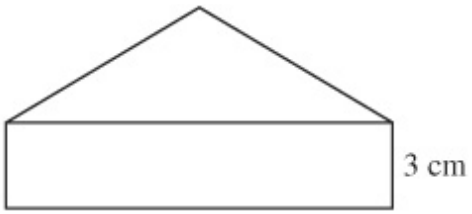
Year	1960	1970	1980	1990	2000	2003	2005	2007	2009
Amount of Trash	83	113	134	142	136	134	139	138	135

A. 83
B. 113
C. 134
D. 135
E. 142

88. Selena collected six times as many signatures as Liza on a petition. Clare collected 5 times as many signatures as Liza. If Clare collected 150 signatures, how many signatures did Selena collect?

F. 25
G. 30
H. 125
J. 150
K. 180

89.



The base of the triangle has the same measure as the length of the rectangle. The width of the rectangle is 3 cm. The area of the triangle is twice the area of the rectangle. What is the height of the triangle?

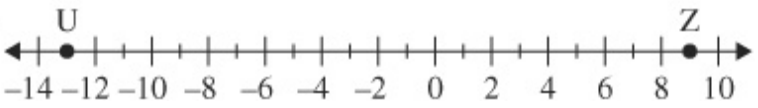
A. 4 cm
B. 6 cm
C. 8 cm
D. 10 cm

E. 12 cm

90. Merle has 500 eggs that he needs to pack into egg cartons. Each egg carton can hold 12 eggs. How many more eggs does Merle need to completely fill the last egg carton?

F. 4
G. 5
H. 6
J. 7
K. 8

91.



If $\overline{UZ}$ is divided into 4 equal parts by points W, X, and Y (not shown), what position will point X fall on? (Assume $W < X < Y$).

A. –7.5
B. –5.5
C. –2
D. 2
E. 3.5

92. Marylou bought a used car that has been driven 95,200 miles. Saundra bought a used car that has been driven 88,000 miles. If Marylou drives 3,600 miles a year and Saundra drives 5,400 miles a year, after how many years will Marylou and Saundra have the same miles on their car?

F. 1

G. 2
H. 3
J. 4
K. 5

93. When *n* is divided by 7, the remainder is 4. What is the remainder when $n+3$ is divided by 7?

A. 0
B. 2
C. 4
D. 5
E. 7

94. Julio has a bag containing 7 red marbles, 8 blue marbles, 3 green marbles, and 9 yellow marbles. He randomly draws 3 red marbles, 1 blue marble, and 2 yellow marbles from the bag. What is the probability that the next marble he draws will be green?

F. $\frac{1}{5}$
G. $\frac{1}{6}$
H. $\frac{1}{7}$
J. $\frac{1}{8}$
K. $\frac{1}{9}$

95. For how many values of *x* is $\frac{2x}{3x} < 0$? (Assume $x \neq 0$)

A. 0

B. 1
C. 2
D. 3
E. 4

96. _____, 5, 7, _____, _____, _____, _____

In the list above, each number is one-third the sum of the number that comes before it and the number that comes after it. What is the first number in the list?

F. 1
G. 3
H. 4
J. 6
K. 8

97. A rectangle is drawn on a coordinate plane. The coordinates of one corner of the rectangle are (0, 0). The coordinates of the opposite corner are (6, 5). What are the coordinates of the third corner?

A. (0, 6)
B. (0, –6)
C. (6, 0)
D. (–6, 6)
E. (6, 6)

98. Jerri needs $\frac{3}{4}$ cup of oil and 1 cup of vinegar to make salad dressing. If he only has $\frac{2}{3}$ cup of vinegar, how much oil should he

use?

F. $\frac{1}{2}$ cup

G. $\frac{1}{3}$ cup

H. $\frac{1}{4}$ cup

J. $\frac{1}{6}$ cup

K. $\frac{1}{8}$ cup

99. This week, Roxie exercised $\frac{5}{8}$ as many hours as she did last week. If she exercised $6\frac{2}{3}$ hours this week, how many hours did she exercise last week?

A. $3\frac{3}{4}$

B. $4\frac{1}{6}$

C. $9\frac{3}{5}$

D. $10\frac{2}{3}$

E. $11\frac{1}{5}$

100. Allie buys 8 DVDs. The mean price of all 8 DVDs was $9.25. If Allie returns one of the DVDs, the mean price of the remaining DVDs drops to $8. What was the price of the DVD Allie returned?

F. $10.00

G. $10.25

H. $18.00

J. $18.50

K. $20.00

THIS IS THE END OF THE TEST. IF TIME REMAINS, YOU MAY CHECK YOUR ANSWERS TO PART 2 AND PART 1. BE SURE THAT THERE ARE NO STRAY MARKS, PARTIALLY FILLED ANSWER CIRCLES, OR INCOMPLETE ERASURES ON YOUR ANSWER SHEET.

Test 1 Answer Key

Paragraph 1

U R Q S T

Paragraph 2

T R S Q U

Paragraph 3

R T Q S U

Paragraph 4

S R Q U T

Paragraph 5

T Q U S R

11. D

12. G

13. B

14. K

15. E

16. F

17. C

18. H

19. B

20. K

21. D

22. G

23. D

24. F

25. E

26. K

27. E

28. H

29. E

30. F

31. E

32. J

33. D

34. J

35. E

36. K

37. E

38. K

39. C

40. G

41. D

42. K

43. A

44. K

45. D

46. H

47. B

48. J

49. A

50. K

51. B

52. G

53. C

54. K

55. D

56. G

57. D

58. G

59. A

60. F

61. B

62. H

63. E

64. F

65. A

66. J

67. C

68. F

69. E

70. J

71. E

72. K

73. D

74. J

75. D

76. H

77. C

78. H

79. A

80. J

81. A

82. K

83. C

84. K

85. D

86. G

87. D

88. K

89. E

90. F

91. C

92. J

93. A

94. H

95. A

96. K

97. C

98. F

99. D

100. H

Explanation of Answers—Verbal Section

Scrambled Paragraphs

Paragraph 1: U R T S Q

Sentence U follows the given sentence because it is an example of the statement about what the area of the West was like during the time that dinosaurs existed. R follows next since it is about a prominent dinosaur that lived in the area. T follows since it is about another “notable” dinosaur. S describes the allosaurus, so it comes next. Q comes next as another detail about that period in the West.

Paragraph 2: T R S Q U

T follows the given sentence since it introduces a solution to the problem mentioned in the first sentence. R ties into what is said about the process being used in ancient times since it talks about the ancient city of Salamis. S comes next as it is in chronological order and expands on how each city had its own blend of additives. Q follows logically because it talks more about how cities were famous for a particular blend of spices. U has examples of two cities famous for their sausages, so it completes the paragraph.

Paragraph 3: R T Q S U

The given sentence talks about the significant part that the Underground Railroad played in American history. R follows the given sentence because it talks about the fact that there was no actual railroad, but a network of people who helped the slaves escape. T is next because it tells how these people helped the slaves. Q comes next since it continues to talk about the people who helped the slaves escape and the safe houses they provided. S comes next since it is

chronologically correct. U is the final sentence since it sums up what the people and railroad were able to accomplish.

Paragraph 4: S R Q U T

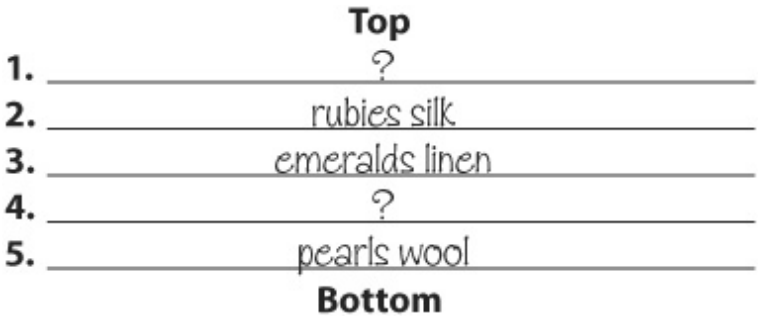
Sentence S comes after the given sentence since it tells more about the runaway from Wales. R tells about the man who adopted Rowlands, so it follows next. Q talks about what Rowlands did after his adopted father died, so it would naturally follow R. U refers to what he did after he was in the army. T comes next as the logical conclusion of the paragraph and what Rowlands continued to do after discovering Livingstone.

Paragraph 5: T Q U S R

The given sentence talks about how many deaths are caused by arteriosclerosis. Logically the next sentence should be T, which explains what arteriosclerosis is. Q should follow next since it describes how the condition is caused. U would follow because it discusses the way the disease can be controlled. The “But” at the beginning of the sentence is an indication that it would offer different information than the sentence before it. That leaves sentences S and R. Since R opens with “This is because,” it cannot follow U. So S is next and R is a further explanation of S.

Logical Reasoning

11. **(D)** Use a diagram to help answer this question. Write “top” and “bottom” on your scrap paper. Then record what the question says about each gem tray. The question says that the bottom tray contains pearls, and statement three says that the emeralds are third from the top. Statement two tells you that exactly two gem trays are between the pearls and the rubies, which means the rubies tray is in position 2.



That leaves the sapphire and diamond trays for the remaining open positions, which are positions 1 and 4. According to statement one, the diamond tray is higher than the pearls. This condition will be fulfilled whether the diamond tray is in position 1 or 4, so we cannot determine which position it is in. There ultimately is no information at all that can be used to determine the location of the sapphire tray. Thus, the correct answer is Option D: “Either sapphires or diamonds, but it is not possible to determine which one.”

12. (G) Statement one tells us that there are two bicycles—a 10-speed and a 3-speed. Make a diagram with two headings and list the characteristics that can be assigned to one bicycle or the other.

10-speed bicycle	**3-speed bicycle**
used only on Saturdays	used on Fridays
needs new tires	scratched

Statement two says that the 10-speed bicycle is only used on Saturdays. So place that information under 10-speed bicycle. Statement three says one bicycle is red. We do not know which one, so we cannot place it in the diagram. Statement four says that one bicycle is used on Fridays and is scratched. Based on statement one, we can put this information in the diagram under the 3-speed bicycle. Statement five says that the bicycle that is not scratched needs new tires. We can figure out that this must be the 10-speed bicycle, so we should

add that to the diagram. Based on this information, we can rule out all of the options except G.

13. (B) Use a diagram to help solve this question. Write “left” and “right” on your scrap paper and the numbers 1 through 5, beginning on the left with 1. Then, from the given statements, look for specific locations of each symbol. Statement two says that the snow symbol is in the second position; write “snow” above 2. Statement four tells you that there is exactly one symbol to the right of the rain symbol, so the rain symbol must be in the fourth position. Statement two says that there are exactly three symbols between the sunny symbol and the cloudy symbol, so the sunny symbol and the cloudy symbol must be in the first and last positions, even though you cannot tell which one goes in which position. The question asks you in what position the sleet symbol is. Since you know that positions 1 and 5 are taken and that the snow symbol is in the second position and the rain symbol is in the fourth position, you are left with the third position.

14. (K) This question contains two conditional statements. It is important that you read each statement carefully. The first sentence may be restated as, “If Betty goes to the concert, Penny will not go.” This is really just a restatement of the two clauses in the first condition. Make sure to observe that the sentence only tells us what will happen *if* Betty goes to the concert. *It says nothing about what will happen if Betty does* ***not*** *go to the concert.* Therefore, it is incorrect to conclude that Penny will go to the concert if Betty does not. We simply do not know what will happen. Likewise, the second sentence can be rearranged as “If Diane is **not** late getting home, Betty will go to the concert.” But the sentence does not tell us what will happen if Diane *is* late getting home, or if Diane goes to the concert.

Option K is correct because its meaning is the same as the second statement.

15. (E) We need to check each option against the facts to figure out which one is valid. We know nothing about Fred except how many hours he helps in the garden. So Option A, although it *might* be true, cannot be verified. Options B and C also *might* be true, but we don't know that from the information that is given. The same is true for Option D. Option E is a valid conclusion, since we are told that members are expected to help out more hours than Fred does.

16. (F) This question asks you to match each canoe rider with his or her canoe and place the canoes in the order of arriving across the lake. The first thing to do is to look for statements that provide information about the person in each canoe. Statement two says that Marina was in the canoe made of wood; statement five says that Hannah was in the canoe made of fiberglass, and statement three says that Charles was not in the canoe made of aluminum. Therefore the person in the canoe made of aluminum must be Devin, which means that Charles was in the canoe made of polyethylene. You can put the canoes and the people in them in order. Based on statement one, draw a diagram showing what we know of the order in which the canoes arrived at the lake.

First

Marina in canoe made of wood
Devin in canoe made of aluminum

Last

Based on statements four and five, Charles, who was in the canoe made of polyethylene, and Hannah, who was in the canoe made of fiberglass, finished behind Devin. Either of the following orders could be correct, based on the information provided.

First	**First**
Marina	Marina
Devin	Devin
Charles	Hannah
Hannah	Charles
Last	**Last**

In either order, the aluminum canoe, which Devin was in, arrived before the polyethylene canoe that Charles was in. So the correct answer is F.

17. **(C)** It can be very helpful to draw a diagram for this kind of problem. Let an initial stand for each pot. Write “smallest” on the left-hand side of your scrap paper and “largest” on the right-hand side of the paper. Then diagram the first statement.

Smallest B G P **Largest**

After reading the second statement, you can put the red pot into the diagram.

Smallest B G P R **Largest**

After reading the third statement, you can place the yellow pot in the diagram.

Smallest B G Y P R **Largest**

Based on the statements, we can figure out that the red pot is the largest.

18. and 19. These directions state that the position of a letter is **never** the same as that of the word it represents. For example, in the first sentence, B cannot represent “the.” To answer these questions, you do not need to find out what every letter represents.

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18. (H) The word “gallops” appears in the first two sentences, but not the third. H can be ruled out because it is in the same position as “gallops.” J cannot be correct; it does not appear in the first sentence. E and F are ruled out because they appear in the third sentence, which does not include “gallops.” That leaves B, which is the correct answer.

19. (B) H appears in all three sentences, as do E and F and the words “the,” “horse,” and “miles.” H cannot represent “the” because it appears in the same position as that word; it cannot represent “miles” for the same reason. Therefore, H must represent “horse.”

20. (K) Option F is not a valid conclusion because the statement contradicts this; there may be some students who are not science majors attending the conference. Option G appeals, but there is nothing in the statement about there being some students who are excellent in science. This could be true, but we cannot assume it is. Option H is a value judgment that is not supported by the information given. While Option J might be true, there is no basis for concluding that “most” students are good students. Option K is a valid conclusion. Students who are not science majors can attend the conference.

Reading

21. (D) This question asks you to determine the general theme of the passage. Option A is mentioned but not in any detail. Option B, though this topic is discussed, is not the theme of the passage. Option C is a detail that is mentioned but not the focus

of the entire passage. Option E is not really discussed, so it cannot be the main idea. Thus, the best answer is Option D.

22. (G) This question asks you to make an inference based on the information in the passage. Option F may be true, but there is not enough information in the passage to draw this conclusion. Option H may or may not be true, but there is little in the passage to support this idea. There is no information on when Emperor Qin died, so Option J can be eliminated. Option K might appeal, but the passage does not really support this conclusion. Clearly Emperor Qin was a man of many accomplishments. Option G is the best answer.

23. (D) The question asks specifically about the Qin Terra-cotta Warriors and Horses. In order to answer the question, you must look at the information in the passage and use it to infer the answer. Option A may be true, but there is nothing to support this idea in the passage. Option C could be true or false, but the passage does not discuss this possibility. Option D is supported by the passage, which tells in great detail what the soldiers looked like and how they were dressed. While Option E may be true, it is not the answer to this question, since there is no mention of philosophers. So Option D is correct.

24. (F) The phrase that "this preservation technique was not known in the West until the eighteenth century" implies that the Chinese were more advanced technologically than the Western world was. There is no evidence for Option G. The concept of being well educated probably did not exist at that time. There is no evidence to support either Option H or J. There is nothing in the passage about Chinese scientists, so Option K can be eliminated as well.

25. (E) This is an inference question. It is necessary to look for information that supports one of the options. Option A could be true, but that is not the suggestion of the terra-cotta army. While Option B is somewhat supported by the fact that it is believed that Emperor Qin unified China, this is not what the existence of

the terra-cotta army suggests. There is nothing to suggest that Option C is supported by the passage. There is no evidence in the passage that China was at peace, although it could be true. Choice E is the most logical suggestion.

26. (K) To answer this question, you have to go through the passage and try to find the one topic that is not discussed. Options F and G are discussed in the paragraph. So is Option H. The topic of figuring out how long it took to create the terra-cotta army is also discussed. That leaves Option K. There is no mention of the cost of creating the terra-cotta army.

27. (E) This question asks you to determine the general theme of the passage. While Option A is mentioned, it is not in the context of why scientists believe in the big bang theory. It is also not the main focus of the passage. Options B and C are mentioned, but they are details, not the theme of the passage. Option D is discussed in the passage, but it is not the main theme of the passage. The passage describes what scientists are trying to achieve with the Large Hadron Collider. Thus, the correct answer is Option E.

28. (H) This is an inference question. You need to make an inference based on the information in the passage. Option F is not supported by the passage, which says that the fundamental particles are created when proton particles smash into each other. Option G is not suggested by the passage; some of the fundamental particles have been identified, but there is no mention of some being improperly identified. Option J seems to be refuted by the passage since it seems difficult to test for them. Option K is not supported by the passage, although it does say some particles have a “charge.” Option H is the one that is implied by the passage; some fundamental particles may never be identified positively.

29. (E) To answer this question, you need to read the passage carefully to see which option is correct. It is impossible to tell whether Option A is correct or not, since this subject is not

mentioned. The same holds true for Option B. No discussion of how long the collider can work is included in the passage. Option C is very unlikely since astronomy and physics are two very distinct sciences. Option D could be true, but there is not verification for it. This leaves Option E as the best choice.

30. **(F)** To answer this question, you need to concentrate on information about atoms and quarks that is present in the passage. Option G is not mentioned in the passage, so it cannot be the correct choice. Option H is clearly incorrect since a careful reading of the passage will reveal they are different from one another. Option J is not correct either; a careful reading will show that an atom is not a fundamental particle. Option K is not the correct answer; there is a known relationship between the two. Thus F is the correct choice.

31. **(E)** To answer this question, you need to review the passage and find what is true of the Higgs boson. You will also need to use the information to figure out the correct answer. Based on the information in the passage, Option A seems to be true, so that cannot be the answer. Option B is something that the passage says indirectly, so it can be eliminated. Option C is stated in the passage. Option D can be inferred. This leaves Option E. The passage says that it was not verified.

32. **(J)** To make an inference, you need to weigh all the information in a passage and come to the most logical conclusion. Option F doesn't seem to be correct. The way the passage talks about dark matter, it would seem to be a separate entity from an atom. Option G seems incorrect for the very same reason. There is nothing in the passage to suggest that Option H is correct. And there is no reason to believe it can be verified more easily than other matter (Option K). This leaves Option J. This is the best answer.

33. **(D)** This question asks you to determine what the main idea or general theme is of the passage. Option A is mentioned in the passage, but it is only a detail, not a general theme. Options B

and C are details in the passage as well, not the main focus of it. Option E is not really discussed; the passage mentions that Haller received one award only. The passage describes how a wasteland was changed into a wildlife sanctuary. Therefore, the correct answer is Option D.

34. (J) Check back through the passage to answer this question. Review what is mentioned as being at Haller Park. Options F, H, and K can be eliminated. They are all directly mentioned. The passage also names fish that are in the ponds, so Option G is not correct either. This leaves Option J. There is no mention of there being sheep in Haller Park.

35. (E) Option A cannot be verified; it could be true, but this is not discussed in the passage. The same is true of Option B. The passage says that most animals are rescued, not that all animals must be rescued animals, so Option C is not correct. There is nothing to suggest that Option D is correct; maybe it is, but there is no evidence of this. This leaves Option E; the passage says that the cement company that created the wasteland backed Haller's efforts.

36. (K) You have to figure out why the author used this information in the passage. Option F doesn't seem to be the reason that the crocodile farm was mentioned, even though it is true in some areas. Haller Park is an experiment, but that is not the reason for the crocodile farm being mentioned. There is no evidence that Option H or J is correct. Option K is the best answer. This is the most likely reason the crocodile farm was mentioned.

37. (E) You need to use your best judgment to answer this question. Option A is appealing, but what Haller did was not so much about creating jobs as transforming a wasteland. Option B and C are possibilities, but again there is nothing that would be transformed. This leaves Options D and E. Cleaning up a landfill so it can be used as a storage area seems more mundane than what Haller did. Option E, however, is very similar to what he achieved. This is the best answer.

38. (K) To answer this question, you have to examine each option carefully. Many of the options are appealing, but it is important to pick the one option that seems most likely based on the information at hand. Option F is probably true, but other than the fact that the park has animals, there is no specific evidence that Haller was an animal lover. Again, Options G or H may be true, but this side of Haller's personality is not mentioned. Option J is probably correct based on what Haller achieved, but Option K is a better answer. It is clear that Haller had great determination based on how he strove to create the park, so this is the best answer.

39. (C) This question asks you to determine what the main idea or general theme of the passage is. Option A is mentioned in the passage, but it is only a detail, not a general theme. Option B is discussed in passing, but it is clearly not the theme. Options D and E are also only details from the passage. The passage talks about whether dolphins should be free or not. This is the main idea. Therefore, the correct answer is Option C.

40. (G) The example of how people can swim with dolphins suggests that they are gentle. Options E and J are not the reasons for starting with this example, since many animals kept in captivity do not interact directly with humans because they are dangerous. Polar bears, for example, are successfully kept in captivity, but people do not enter polar bear enclosures for an "unforgettable" experience. Option H focuses on dolphins learning tricks, but this also does not have much to do with why the author stresses that people find it "unforgettable" to swim with them. Option K is not suggested at all.

41. (D) You will need to review the passage to answer this question. Scan through the passage to find what reasons are mentioned for allowing dolphins to go free. Options A, B, C, and E are definitely mentioned. Option D is mentioned as a reason for keeping dolphins in captivity, so Option D is the correct answer.

42. (K) Option F and G are not included in the passage, so it is impossible to know if either one is true or not. Option H is also not mentioned. Option J is incorrect; they are second in intelligence after humans, not chimpanzees. Option K is the correct answer.

43. (A) You will need to analyze the author's intent from what the passage says. It talks about how some people think that dolphins should not be in captivity. The information about Dr. Marino helps to back up those who argue that dolphins are very much like humans, but the intent probably was to lend the passage more authority. That is why the author cites Dr. Marino's credentials. Option A is the correct one. Options B and C are not supported by the passage, nor is Option D. While the author certainly does cover both sides of the debate, that is not the main reason the author included Dr. Marino's findings.

44. (K) Option F is possible, but not the most likely probability based on the information in the passage. Option G seems unlikely. The passage says that dolphins reproduce well in captivity. Options H and J seem very remote. This leaves Option K. It is likely more dolphins will be attacked by sharks and more will probably die.

45. (D) This question asks you to determine what the main idea or general theme of the passage is. Option A is mentioned in the passage, but it is only a detail, not a general theme. Options B and C are details in the passage, not the main focus of it. Option E is discussed, but it is not the main theme. The passage describes how Beryl Markham made her mark on history; that is the main theme. Thus, the correct answer is Option D.

46. (H) Use the information in the passage to find the answer to this question. The passage doesn't say whether Beryl became a better person after meeting Tom Campbell Black, so Option F can be eliminated. She did write a book, but that was much later on, so Option G cannot be the effect that Tom Campbell Black had on Beryl Markham. She did not leave Kenya after meeting him; she became a bush pilot there. Whether or not she decided

to be a career woman was never really a topic. Consequently, Option H is the correct answer.

47. **(B)** Option C can be eliminated; it wasn't particularly brave for her to return to Kenya. And while Options A, D, and E are all achievements, the most impressive action that Beryl Markham took was to do something dangerous and hard to do. Option B is the correct answer.

48. **(J)** Option F cannot be the answer. We are not told whether Beryl Markham's plane was lighter than Charles Lindbergh's. Nor do we learn whether she was newer to aviation or not. Option H is true, but it's likely that Charles Lindbergh's flight was also long. Option K is not correct. We learn that Charles Lindbergh "soloed" over the Atlantic, meaning that he was alone as well. We are left with Option J. This information is in the passage. It is the correct answer.

49. **(A)** You have to use the information and your own prior knowledge to answer this question. Option A is appealing. She grew up there and probably considered it her home. Option B and C are not mentioned in the passage, so we cannot determine if either is correct. Option D and E are also not mentioned in the passage, so we need to turn back to Option A. This is the best answer.

50. **(K)** You need to use the information in the passage to make an inference about Beryl Markham. She certainly had a mind of her own, but there is no sign that Option F was true. Option G is another trait that doesn't seem to apply to Beryl Markham. Option H is a possibility, but in fact, she was practical in that she knew the dangers she faced. There is no information in the passage to support Option J. Option K is the best answer.

Explanation of Answers—Mathematics Section

51. **(B)** Translate the verbal descriptions into algebraic equations:

X is 30% of *Y* means $X = 0.30Y$

Y is 45% of 600 means $Y = (0.45)(600)$

Solve for *Y*:

$$Y = (0.45)(600) = 270$$

Substitute the value of *Y* into the first equation. Then solve for *X*:

$$\begin{aligned} X &= 0.30Y \\ &= (0.30)(270) \\ &= 81 \end{aligned}$$

52. (G) The formula for the distance between two points on a number line is $b - a$ Use the formula to find the length of $\overline{\text{MO}}$.

$$\begin{aligned} \text{MO} &= \text{O} - \text{M} \\ &= 3 - (-12) \\ &= 3 + 12 \\ &= 15 \end{aligned}$$

$\overline{\text{MN}}$ is $\frac{2}{5}$ the length of $\overline{\text{MO}}$, so its measure is $\frac{2}{5} \times 15 = 6$ units.

To find the location of point N, start at point M (–12) and count 6 units to the right to –6.

53. (C) Divide 3 by 40:

$$
\begin{array}{r}
0.075 \\
40\overline{)3.000} \\
-280 \\
\hline
200 \\
-200 \\
\hline
0
\end{array}
$$

54. (K) Since 75 is not a perfect square, find the two perfect squares that lie on either side of 75: $64 < 75 < 81$.

Since $64 = 8^2$ and $81 = 9^2$:

$$8^2 < 75 < 9^2$$
$$8^2 < s^2 < 9^2$$
$$8 < s < 9$$

55. (D) The mean is the total number of books read divided by the number of students. The table shows that 4 students each read 15 books, 3 students each read 20 books, 3 students each read 12 books, and 5 students each read 18 books. Therefore, the total number of books read is the sum of $(4\times15)+(3\times20)+(3\times12)+(5\times18)$. The mean number of books read is:

$$\frac{(4\times15)+(3\times20)+(3\times12)+(5\times18)}{15} = \frac{60+60+36+90}{15}$$
$$= \frac{246}{15}$$
$$= 16.4$$

56. (G) A prime number has only two factors, 1 and itself. Count backward from 23 to find the greatest prime number that is less than 23:

23, 22, 21, 20, 19, …

The factors of 22 are 1, 2, 11, and 22, so 22 is not a prime number.

The factors of 21 are 1, 3, 7, and 21, so 21 is not a prime number.

The factors of 20 are 1, 2, 4, 5, 10, and 20, so 20 is not a prime number.

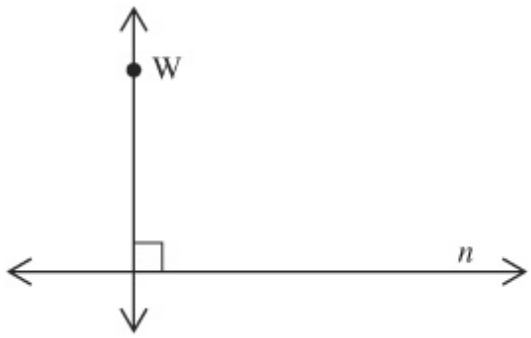
The factors of 19 are 1 and 19, so 19 is a prime number and it is the greatest prime number that is less than 23.

57. (D) Use the distributive property to write $7p(3-5q)$ as an equivalent expression. The distributive property states that for all numbers *a, b,* and *c*, $a(b+c)=a(b)+a(c)$.

$$\begin{aligned} 7p(3-5q) &= (7p)(3)-(7p)(5q) \\ &= 21p-35pq \end{aligned}$$

58. (G) $\frac{31}{3}=10\frac{1}{3}$, so the least integer that is greater than $10\frac{1}{3}$ is 11.

59. (A) A perpendicular line forms a 90° angle with line *n*. You can only draw one perpendicular line through point W.



60. (F) At 6:00 a.m., the temperature was –7°. The temperature at noon was $-7° + 10° = 3°$. The temperature at 6:00 p.m. was $3° - 5° = -2°$.

61. (B) According to the graph, it took 150 hours to manufacture 5 cars, 300 hours to manufacture 10 cars, and so on. The graph is a straight line, so the rate of change is constant.

Let *x* represent the number of hours to manufacture one car:

$$\frac{x}{1} = \frac{150}{5}$$

$$5x = 150$$

$$x = 30$$

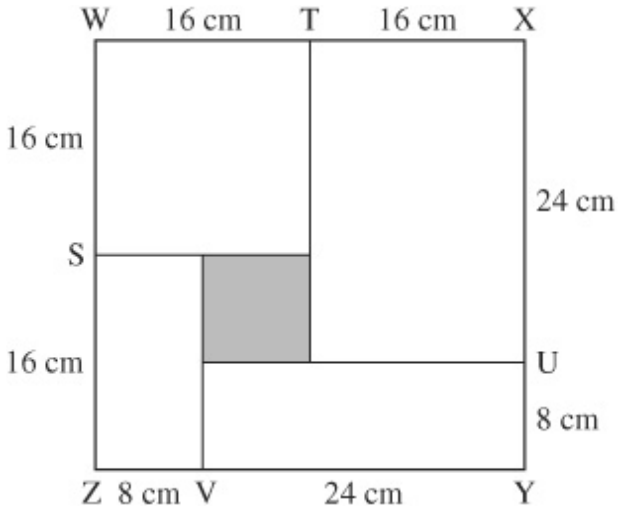
62. (H) Let *x* represent the nonzero integer. Translate the verbal description into an algebraic equation:

$$12x = 3x^2$$

$$4x = x^2$$

$$4 = x$$

63. (E) Since the side length of square WXYZ is 32 cm, $\mathrm{WT} = 16\ \mathrm{cm}$, $\mathrm{UY} = 8\ \mathrm{cm}$, $\mathrm{VZ} = 8\ \mathrm{cm}$, and SW = 16 cm. The side lengths of the shaded region can be calculated by subtracting $\mathrm{WT} - \mathrm{VZ} = 16 - 8 = 8$ and $\mathrm{XU} - \mathrm{SW} = 24 - 16 = 8$. The shaded region is a square with side lengths of 8 cm. Therefore, the area of the shaded region is 64 sq cm.



64. (F)

$$17 > x + 3$$
$$17 - 3 > x + 3 - 3$$
$$14 > x$$

That is the same as $x < 14$.

65. (A)

$$(-5x + 8) - (2 + x) = -5x + 8 - 2 - x$$
$$= -5x - x + 8 - 2$$
$$= -6x + 6$$

66. (J)

$$30\% \text{ of } 3 = (0.30)(3) = 0.90$$
$$3\% \text{ of } 30 = (0.03)(30) = 0.90$$
$$0.90 + 0.90 = 1.80$$

That is the same as 1.8.

67. (C) $42{,}592 = 2^x \times 11^y$

The equation tells us that 42,592 can be factored into powers of 2 and 11:

$$42,592 =$$
$$2 \times 21,296 =$$
$$2 \times 2 \times 10,648 =$$
$$2 \times 2 \times 2 \times 5,324 =$$
$$2 \times 2 \times 2 \times 2 \times 2,662 =$$
$$2 \times 2 \times 2 \times 2 \times 2 \times 1,331 =$$
$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 \times 121 =$$
$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 \times 121 =$$
$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 \times 11 \times 11 =$$
$$2^5 \times 11^3$$

Therefore, $x = 5$ and $y = 3$, so $xy = 5 \times 3 = 15$.

68. (F) First, use the formula *distance* = *rate* × *time* to find the time when Stuart and Brandy will meet. The distance that Stuart travels is 3*t* (3 miles per hour times "time"). The distance that Brandy travels is 12*t* (12 miles per hour times "time"). The total distance they travel is 5 miles, the distance between the two houses:

$$3t + 12t = 5$$
$$15t = 5$$
$$t = \frac{1}{3}$$

Stuart and Brandy will meet at $t = \frac{1}{3}$. At $t = \frac{1}{3}$, the distance from Stuart's house is $3t = 3 \times \frac{1}{3} = 1$.

69. (E) There are 8 triangles that are congruent to triangle ABI.

Triangles ACH, EGI, and BDF are similar to triangle ABI. Triangle ADG is also similar to triangle ABI.

The number of triangles that are similar to triangle ABI is $8+3+1=12$.

70. (J) Let *x* and *y* represent the two numbers. Translate the verbal description into an algebraic equation:

$$\frac{1}{4}xy=4$$
$$\frac{1}{4}(8y)=4$$
$$8y=16$$
$$y=2$$

The sum of *x* and *y* is $8+2=10$.

71. (E) Let *x* represent the total number of gold beads in the container. We know 13 out of every 25 beads are gold. Set up a proportion to solve for *x*:

$$\frac{13}{25}=\frac{x}{5{,}000}$$
$$13(5{,}000)=25x$$
$$\frac{13(5{,}000)}{25}=x$$
$$2{,}600=x$$

72. (K) Let *x* represent the length of the rectangle:

$$11x = 165$$
$$x = 15$$

The width is 11 feet and the length is 15 feet, so the perimeter of the rectangle is $2(11)+2(15)=52$.

73. (D) Melissa has 7 types of bagels to choose from. Since Julio orders a different type of bagel, there are 6 different ways he can choose. Use the Fundamental Counting Principle to find the number of different ways Melissa and Julio can order. The Fundamental Counting Principle is likely already familiar to you, but—put simply—it is a way of accounting for the number of possible outcomes, given a variety of choices. For example, if you have two skirts and three shirts, what are the number of possible outfits you could make? Using the Fundamental Counting Principle, you would simply multiply 2 by 3 and find that 6 is the answer. In this problem, we must find out how many possible order combinations there are, so we take the number of Melissa's possible choices and multiply them with the number of Julio's possible choices: $7 \times 6 = 42$.

74. (J)

$$\frac{6'}{3'} = \frac{6-2}{3-2} = \frac{4}{1} = 4$$

75. (D) Substitute the values of *x* and *y* into the expression and solve:

$$xy + \frac{x}{y} = (-1.5)(-0.3) + \frac{-1.5}{-0.3}$$
$$= 0.45 + (5)$$
$$= 5.45$$

76. (H) Since Jerri was *x* years old 5 years ago, her current age is $x+5$. This means Serena's age is $\frac{1}{2}(x+5)$.

77. (C) The area of the painted portion of the pipe is called the lateral area. The formula for the lateral area *L* of a cylinder is $L = 2\pi rh$, where *r* is the radius and *h* is the height. Substitute the values of *r* and *h* into the formula to find the lateral area:

$$\begin{aligned} L &= 2\pi rh \\ &= 2\pi(5)(9) \\ &= 90\pi \end{aligned}$$

78. (H) The least common multiple (LCM) of *X* and *Y* is the product of the greatest power of each of its prime factors:

$$X = 2 \times 2 \times 2 \times 2 = 2^4$$
$$Y = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

The LCM is $2^4 \times 3$.

79. (A) If 1 cm is equivalent to 200 m, then 1 sq cm is equivalent to 40,000 sq m:

$$(1 \text{ cm})^2 = (200 \text{ m})^2$$
$$1 \text{ sq cm} = 40{,}000 \text{ sq m}$$

Set up a proportion to find how many square centimeters is equivalent to 10 square meters:

$$\frac{1}{40{,}000} = \frac{x}{10}$$
$$40{,}000x = 100$$
$$x = \frac{100}{40{,}000}$$
$$= 0.0025$$

80. (J) The value of 7 quarters, 3 dimes, and 1 nickel is $\$1.75+\$0.30+\$0.05=\$2.10$. If he spends \$2.10 on a bottle of water, he has 5 coins left that have a total value of \$0.09. The only possible combination for the 5 coins is 1 nickel and 4 pennies.

81. (A) If *x* is the smallest of the 3 consecutive multiples of 4, then $y=x+4$ and $z=y+4$. In terms of *y*, $x=y-4$. Therefore, $x+z=(y-4)+(y+4)=2y$.

82. (K) Since the rectangle is made up of squares of the same size, the side length of the square must be a factor of both 24 and 72. The largest possible size square will have a side length that is the greatest common factor (GCF) of 24 and 72. The GCF of 24 and 72 is 24. Therefore, the largest possible size is 24 in. by 24 in.

83. (C) The formula for the volume of a pyramid is $V=\frac{1}{3}bh$, where *b* is the area of the base and *h* is the height. Substitute the given values into the formula and solve for *h*:

$$V=\frac{1}{3}bh$$
$$80=\frac{1}{3}(4\times 4)h$$
$$80=\frac{1}{3}(16)h$$
$$240=16h$$
$$15=h$$

84. (K) To find the value that is halfway between $\frac{2}{3}$ and 0.2, add and divide by 2:

$$\frac{\frac{2}{3}+0.2}{2}=\frac{\frac{2}{3}+\frac{2}{10}}{2}$$
$$=\frac{\frac{20}{30}+\frac{6}{30}}{2}$$
$$=\frac{\frac{26}{30}}{2}$$
$$=\frac{26}{30}\div 2$$
$$=\frac{26}{30}\times\frac{1}{2}$$
$$=\frac{13}{30}$$

85. (D) Translate the verbal description into algebraic inequalities:

$$12<x\leq 3y$$

Divide the equation by three. Of the answer choices, only option D is equivalent to $12<x\leq 3y$:

$$4<\frac{x}{3}\leq y=4(3)<\frac{x}{3}(3)\leq y(3)=12<x\leq 3y$$

86. (G) Substitute the values of *x* and *y* into the expression and simplify:

$$\frac{2x^{y}}{y^{2x}}=\frac{2\times 1^{2}}{2^{2\times 1}}=\frac{2\times 1}{2^{2}}=\frac{2}{4}=\frac{1}{2}$$

87. (D) List the numbers in order from least to greatest. The median is the middle value.

83	113	134	134	135
136	138	139	142	

88. (K) Let *C* represent the number of signatures Clare collected. Let *L* represent the number of signatures Liza collected. Let *S* represent the number of signatures Selena collected.

First, solve for *L*:

$$\begin{aligned} C &= 5L \\ 150 &= 5L \\ 30 &= L \end{aligned}$$

Next, solve for *S*:

$$\begin{aligned} S &= 6L \\ &= 6 \times 30 \\ &= 180 \end{aligned}$$

89. (E) The formula for the area of a triangle is $A = \frac{1}{2}bh$, where *b* is the base of the triangle and *h* is the height. The formula for the area of a rectangle is $A = lw$, where *l* is the length of the rectangle and *w* is the width. Since the base of the triangle has the same measure as the length of the rectangle, we can write the area of the rectangle as $A = bw$.

The area of the triangle is twice the area of the rectangle, so $\frac{1}{2}bh = 2bw$:

$$\frac{1}{2}bh = 2bw$$
$$bh = 4bw$$
$$h = 4w$$
$$= 4 \times 3$$
$$= 12$$

90. (F) Divide 12 into 500 to find the number of egg cartons Merle can completely fill and the number of eggs that will be left over:

$$500 \div 12 = 41 \text{ R}8$$

Merle can completely fill 41 egg cartons. The last egg carton will have only 8 eggs. Therefore, Merle needs 4 more eggs to completely fill it.

91. (C) The distance between points U and Z on the number line is $|U - Z| = |-13 - 9| = 22$.

If $\overline{UZ}$ is divided into 4 equal parts, each part is $5\frac{1}{2}$ units long:

$$UW = WX = XY = YZ = 5\frac{1}{2}$$

Point X is $5\frac{1}{2} + 5\frac{1}{2} = 11$ units to the right of U, so point X will fall at –2.

92. (J) Let *x* represent number of years:

$$95{,}200 + 3{,}600x = 88{,}000 + 5{,}400x$$
$$7{,}200 = 1{,}800x$$
$$4 = x$$

93. (A) Let *q* represent the quotient when *n* is divided by 7:

$$n = 7q + 4$$
$$n + 3 = 7q + 4 + 3$$
$$= 7q + 7$$
$$= 7(q + 1) + 0$$

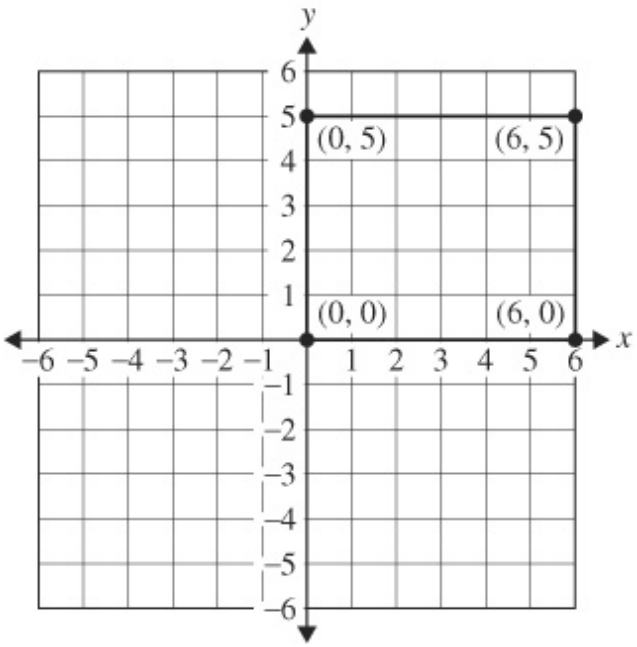
94. (H) Julio had $7 + 8 + 3 + 9 = 27$ marbles in the bag. He randomly drew $3 + 1 + 2 = 6$ marbles, none of which were green. He has $27 - 6 = 21$ marbles left in the bag. He still has 3 green marbles. So the probability that the next marble will be green is $\frac{3}{21} = \frac{1}{7}$.

95. (A) In order for $\frac{2x}{3x}$ to be less than 0, the denominator and numerator must have different signs. Either the numerator is positive and the denominator is negative, or the numerator is negative and the denominator is positive. There is no single number that can satisfy this condition.

96. (K) Let *x* represent the first number in the list:

$$\frac{1}{3}(x + 7) = 5$$
$$x + 7 = 15$$
$$x = 8$$

97. (C)



98. (F) Let x represent the amount of oil Jerri should use. Set up a proportion to solve for x:

$$\frac{x}{\frac{2}{3}\text{ cup of vinegar}} = \frac{\frac{3}{4}\text{ cup of oil}}{1\text{ cup of vinegar}}$$

$$x = \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{1}{2}$$

99. (D) Let x represent the number of hours Roxie exercised last week:

$$\frac{5}{8}x = 6\frac{2}{3}$$
$$\frac{5}{8}x = \frac{20}{3}$$
$$x = \frac{20}{3} \div \frac{5}{8}$$
$$= \frac{20}{3} \times \frac{8}{5}$$
$$= \frac{32}{3}$$
$$= 10\frac{2}{3}$$

100. (H) Let *x* represent the total price of the 8 DVDs:

$$\frac{x}{8} = 9.25$$
$$x = 74$$

Let *y* represent the price of the DVD that Allie returned:

$$\frac{74 - y}{7} = 8$$
$$74 - y = 56$$
$$74 - 56 = y$$
$$18 = y$$

CHAPTER 12
SHSAT Practice Test 2

On the following page you'll find a form similar to that of the actual SHSAT. Carefully tear it out of the book and use it as you take the diagnostic test, for both the verbal and mathematics sections.

PART 1 VERBAL

SCRAMBLED PARAGRAPHS

Paragraph 1

The second sentence is	Q	R	S	T	U
The third sentence is	Q	R	S	T	U
The fourth sentence is	Q	R	S	T	U
The fifth sentence is	Q	R	S	T	U
The sixth sentence is	Q	R	S	T	U

Paragraph 2

The second sentence is	Q	R	S	T	U
The third sentence is	Q	R	S	T	U
The fourth sentence is	Q	R	S	T	U
The fifth sentence is	Q	R	S	T	U
The sixth sentence is	Q	R	S	T	U

Paragraph 3

The second sentence is	Q	R	S	T	U
The third sentence is	Q	R	S	T	U
The fourth sentence is	Q	R	S	T	U
The fifth sentence is	Q	R	S	T	U
The sixth sentence is	Q	R	S	T	U

Paragraph 4

The second sentence is	Q	R	S	T	U
The third sentence is	Q	R	S	T	U
The fourth sentence is	Q	R	S	T	U
The fifth sentence is	Q	R	S	T	U
The sixth sentence is	Q	R	S	T	U

Paragraph 5

The second sentence is	Q	R	S	T	U
The third sentence is	Q	R	S	T	U
The fourth sentence is	Q	R	S	T	U
The fifth sentence is	Q	R	S	T	U
The sixth sentence is	Q	R	S	T	U

LOGICAL REASONING

11	A	B	C	D	E
12	F	G	H	J	K
13	A	B	C	D	E
14	F	G	H	J	K
15	A	B	C	D	E
16	F	G	H	J	K
17	A	B	C	D	E
18	F	G	H	J	K
19	A	B	C	D	E
20	F	G	H	J	K

READING

21	A	B	C	D	E
22	F	G	H	J	K
23	A	B	C	D	E
24	F	G	H	J	K
25	A	B	C	D	E
26	F	G	H	J	K
27	A	B	C	D	E
28	F	G	H	J	K
29	A	B	C	D	E
30	F	G	H	J	K
31	A	B	C	D	E
32	F	G	H	J	K
33	A	B	C	D	E
34	F	G	H	J	K
35	A	B	C	D	E
36	F	G	H	J	K
37	A	B	C	D	E
38	F	G	H	J	K
39	A	B	C	D	E
40	F	G	H	J	K
41	A	B	C	D	E
42	F	G	H	J	K
43	A	B	C	D	E
44	F	G	H	J	K
45	A	B	C	D	E
46	F	G	H	J	K
47	A	B	C	D	E
48	F	G	H	J	K
49	A	B	C	D	E
50	F	G	H	J	K

PART 2 MATHEMATICS

MATHEMATICS PROBLEMS

| | | |

_____ **Q.** Maintenance, however, of these "prisoners of the hunt" has always been a problem since it is hard to determine the right foods for captive animals, even today.

_____ **R.** Shortly thereafter, the Chinese Emperor Wen Wang also amused himself by collecting many strange animals from all parts of his empire.

_____ **S.** Even if the keepers knew their newly captured animals ate special food (such as young bamboo plants), they may have found it impossible to meet such specific dietary needs.

_____ **T.** The king wanted to keep lions in his park to amuse himself and his fellow rulers, who would come to visit him often.

_____ **U.** Early zoo keepers, who probably had little or no training, were often unsure of even basic facts such as whether the animals they must feed were carnivores or herbivores.

Paragraph 2

The journey of the salmon from salt water to freshwater in order to breed is one of the wonders of nature.

_____ **Q.** When it is time for them to breed, an extremely strong instinct causes the salmon to leave the ocean and head back to the freshwater streams where they were born.

_____ **R.** For several years, perhaps as long as nine years, they remain in the ocean eating and growing until they are adult fish.

_____ **S.** This journey, called the salmon run, is extremely difficult because the fish must swim upriver against the current, past strong rapids.

_____ **T.** Some even leap up waterfalls as they make their way back to their original breeding grounds, which may be hundreds of miles away, to fulfill their innate instinct.

_____ **U.** The fish begin their life cycle in freshwater lakes and streams and then, in the first or second year of their life, they are carried along by rivers or mountain streams to the ocean.

Paragraph 3

One of the most feared creatures in the jungles of Africa and South America is the army ant, but it is also one of the most helpless looking.

_____ **Q.** Still, this ant is a killer, a flesh eater, and for this reason lions, tigers, leopards, and even men have learned to run as quickly as possible from an approaching horde of army ants.

_____ **R.** After the new generation is grown and the ants are on the move, they travel through the jungle in huge masses that can number as many as twenty million ants, marching in a close-packed line that can stretch for miles.

_____ **S.** As the ants move along, columns branch out from the main group of ants, and often an animal or man stumbles unawares into one of the branching columns and is quickly consumed.

_____ **T.** This insect is barely a quarter of an inch long, lives out in the open, is so vulnerable that bright sunlight can kill it in a few minutes, and is totally blind.

_____ **U.** A colony of army ants moves through the jungle only two weeks out of every month since the remaining two weeks are spent in one spot, raising a new generation of young ants.

Paragraph 4

Richard Spruce, the son of a poor English schoolmaster, found his life's work and his fame in the jungles of South America.

_____ **Q.** After that, he pressed the specimen between pages of blotting paper and sent it back to England to be part of the great plant collection at Kew Gardens in London; in all, Spruce sent 30,000 specimens home.

_____ **R.** Spruce grew up in a part of England that was filled with plants growing wild and free, and his love of plants developed early in his life.

_____ **S.** While in South America, Spruce explored more of the Amazon jungle than any other European before him, and he mapped the Amazon River and studied its plant life.

_____ **T.** Every time Spruce came upon a new plant, he took a specimen of it and labeled it with both the scientific name and the name that the Amazon Indians gave it and described what the whole plant looked like and what it was used for.

_____ **U.** When he read a book describing the strange and different animals of the Amazon jungle in Brazil, he decided to go there to study the area's plant life.

Paragraph 5

Many people have the misconception that chess is so complicated that it can only be played by older people.

_____ **Q.** He figured out the rules by himself and within a few years was considered the best player in the world.

_____ **R.** José Raoul Capablanca of Cuba, for instance, started to play chess when he was only nine after watching his father and his uncle play the game.

_____ **S.** But the truth is that most of the world's great chess players showed signs of their greatness when they were in their teens and usually reached the peak of their prowess when they were in their twenties.

_____ **T.** This fact prompted chess experts to conclude that a 13-year-old boy who is learning the game today may be as good as the recognized experts in a year and better than any of them in two years.

_____ **U.** A second example, Bobby Fischer, who was the strongest player in America and possibly the strongest in the world, also revealed his special genius at an early age, winning the United States chess championship at the age of 13.

Logical Reasoning

QUESTIONS 11–20

DIRECTIONS: Read the scenarios below, then choose the **best** answer to each question. You must base your response **only on the information given.**

Read the scenarios with careful attention to the precise meaning of the words used. For example, "The red shirt hanging **between** the yellow and blue shirts" does not necessarily mean "The red shirt is **between and next to** the yellow and blue shirts"; one or more other articles of clothing may separate the red shirt from the yellow shirt or from the blue shirt. This precaution also applies to location words such as **above, below, before, after, ahead of,** and **behind.**

11. Five pictures of horses are hung next to each other on a wall. They are numbered consecutively from left to right.

1) One horse is a thoroughbred.
2) A pinto is in the second position.
3) There are exactly three horses between the quarter horse and the palomino.
4) There is exactly one horse to the right of the thoroughbred.

In which position is the Arabian?

A. second
B. third
C. fourth
D. fifth
E. Cannot be determined from the information given.

12. Billy will not go to the movies if Teddy does. Teddy will go to the movies if Janet does not forget her backpack.

Based only on the information above, which of the following **must** be true?

F. If Janet forgets her backpack, Billy will go to the movies.
G. If Teddy does not go to the movies, Billy will go to the movies.
H. If Janet does not forget her backpack, Teddy will go to the movies.
J. Teddy will not go to the movies if Janet does not forget her backpack.
K. If Janet goes to the movies, Teddy will also go to the movies.

13. At a flooring store, there are five different kinds of flooring samples piled one on top of the other. The flooring samples consist of linoleum, laminate wood, ceramic tile, carpeting, and marble.

1) The linoleum sample is higher in the stack than the marble sample is.
2) Exactly two flooring samples are between the marble sample and the ceramic tile sample.
3) The laminate wood sample is third from the top.

If the bottom sample is marble, which sample is on the top?

A. carpeting
B. linoleum
C. ceramic tile
D. Either carpeting or linoleum, but it is not possible to determine which one.
E. Either carpeting or ceramic tile, but it is not possible to determine which one.

14. Most of the people attending the lecture on architecture are students. However, some of them are not studying architecture.

F. Most of the students who are attending the lecture on architecture are studying architecture.
G. Some of the people who are attending the lecture on architecture are advanced students.
H. Being a student of architecture is necessary to attend the lecture on architecture.
J. Most people attending the lecture on architecture are not studying architecture.
K. People who are not studying architecture can attend the lecture on architecture.

Questions 15 and 16 refer to the following information.

In the code below, (1) each letter always represents the same word, (2) each word is represented by only one letter, and (3) in any given sentence, the letters may or may not be presented in the same order as the words.

M	O	R	T	W	means
"George	wants	to	write	Diane."	
M	N	X	T	R	means
"Helen	wants	to	write	Leonardo."	
Q	M	X	R	T	means
"Carla	wants	to	write	Helen."	
M	Z	R	O	Y	means
"Beth	wants	to	call	George."	

15. Which letter represents the word "Carla"?

A. R
B. M
C. Q
D. X
E. Cannot be determined from the information given.

16. Which word is represented by the letter Z?

F. write
G. Helen
H. Beth
J. call
K. Cannot be determined from the information given.

17. Five children are standing in a line according to their heights, from tallest to shortest, left to right.

1) Anna is taller than Diane, but shorter than Nathan.

2) Diane is shorter than Phil, but taller than Tom.
3) Phil is shorter than Nathan.

In which position is Phil?

A. first
B. second
C. third
D. fourth
E. Cannot be determined from the information given.

18. People who are on the Hyannis High soccer team are expected to practice five afternoons a week. Dan practices only four afternoons a week.

Based only on the information above, which of the following **must** be true?

F. Dan is not a member of the Hyannis High soccer team.
G. If Dan had more time, he would practice five afternoons.
H. If all members of the soccer team practiced only four afternoons a week, the Hyannis High soccer team would not continue to exist.
J. Dan is on another sports team, so he cannot practice all five afternoons.
K. If Dan is a member of the Hyannis High soccer team, he does not do what is expected of him.

19. Four people were driving to a baseball game. They arrived at the game one at a time. Their cars were all different makes. One was

a Dodge, another was a Ford, and the other two were a Chevrolet and a Chrysler.

1) Paul arrived right after Anna.
2) Anna was driving the Dodge.
3) Dan was not driving the Chrysler.
4) The Dodge got to the baseball game before the Chevrolet.
5) Vicky, who was driving the Ford, arrived after Paul.

Based only on the information above, which of the following **must** be true?

A. The Dodge arrived before the Chrysler.
B. Dan arrived third.
C. Paul was in the Chevrolet.
D. Vicky arrived third.
E. The Chevrolet arrived second.

20. If it is cold, our class picnic will be canceled. If it is hot, our class picnic will be held.

Based only on the information above, which of the following **must** be true?

F. When it is not cold, the picnic will take place.
G. The picnic will not take place if it is windy.
H. The weather may cause the picnic to be canceled.
J. If the picnic is canceled, it must be cold.
K. The picnic is canceled more often than it takes place.

Reading

QUESTIONS 21–50

DIRECTIONS: Read the passages below, then answer the questions that appear after each passage. Select the **best** answer for each question. Reread the passages if necessary. You must base your response **only on information contained in the passage.**

For centuries explorers and traders looked for a way to travel from Europe to Asia without making the long and dangerous trip around the continent of Africa or South America. Before the Suez Canal and the Panama Canal were in existence, that route was the Northwest Passage through the Arctic Ocean. The first recorded attempt to traverse the Northwest Passage was in 1497. A that time John Cabot became the first European to discover the mainland of North America. Cabot could go no farther, however. The Arctic was completely frozen and impassable. Consequently, he returned to Italy. Over the years others tried and failed. Some disappeared, never to be heard from again. In 1854 the British explorer John Rae arrived. He mapped the Canadian Arctic coastline and learned how to survive the brutal winter with the help of the Cree Indians. But he too found the waters locked with thick ice.

Finally, in 1903, Norwegian Roald Amundsen departed Norway in a small fishing trawler, the *Gjoa*. It was only 70 feet long and had a crew of just six. His expedition did successfully navigate the Northwest Passage. The trip from the Atlantic to the Pacific took three years. The ship was stuck frozen in two-foot-thick ice for two winters. But everyone survived. He too learned survival skills from the Inuit Eskimos including how to hunt for food. Then in 1906 he sailed through the Bering Strait and on to Nome, Alaska. If he had taken a larger ship, he would never have made it through because the water in some parts was only three feet deep!

The Northwest Passage on the eastern side starts with the Davis Strait between Greenland and Canada. It continues into Baffin Bay and then through the Canadian Arctic Archipelago, a collection of more than 36,000 islands, most of them small and uninhabited. There are seven possible routes through the Archipelago, but some of them are too shallow for anything but a small ship. From there one passes into the Beaufort Sea, the Chukchi Sea, through the Bering Strait, and into the Pacific Ocean.

Today, because of global warming, the Northwest Passage is becoming an option for shipping. At the beginning of

55 the twenty-first century, it was recorded that parts of the route were ice free for a short time in the summer months. For 36 days in August and September 2007, the entire route was ice free and navigable.

60 Continuing record sea ice melting may indicate that the route will be used by commercial carriers, instead of going through the Panama Canal. It is about 2,500 miles less in distance, which would save both time and money. As a result, the Canadian government

65 is experiencing new challenges from other governments about international travel through the Northwest Passage.

21. Which of the following best tells what this passage is about?

A. how the Canadian government is dealing with use of the Northwest Passage
B. what waterways the Northwest Passage runs through
C. how the Northwest Passage was discovered and its future potential
D. how explorers discovered the North American mainland
E. why the Northwest Passage will be useful in the future

22. The passage suggests that the search for the Northwest Passage

F. caused tension between the various explorers.
G. was undertaken by the Canadian government.
H. was paid for by European royalty.
J. was only undertaken after the Suez Canal was built.
K. caused the death of many explorers.

23. Which of these statements is true about Roald Amundsen?

A. He became a rich man because of the discovery.
B. He realized that a big ship would never get through the Northwest Passage.

C. He had great determination in the face of significant hardships.

D. He was the only explorer to learn how to deal with the cold from natives.

E. He was a better sailor than the other explorers.

24. Which of the following best describes what is suggested by the statement that "Continuing record sea ice melting may indicate that the route will be used by commercial carriers . . ." (lines 60–62)?

F. Global warming is impacting which routes commercial carriers may take.

G. Fewer small ships will transverse the Northwest Passage because of global warming.

H. Commercial carriers are forced to carry heavier cargoes because of global warming.

J. Global warming will make the Northwest Passage more difficult to use.

K. Global warming will cause more commercial carriers to use smaller ships.

25. Which of these is most likely what the Canadian government will do in the future?

A. The government will declare the Northwest Passage an international zone.

B. The government will allow only European countries to use the route without interference.

C. The government will develop rules about traveling through the Northwest Passage.

D. The government will create a series of canals in the Northwest Passage.

E. The government will close the Northwest Passage.

26. What is **not** mentioned as a difficulty in traversing the Northwest Passage?

F. ice floes

G. cold weather

H. shallow waters

J. hostile natives

K. finding food

The Netherlands has spent much effort managing and controlling the sea and the many rivers that run through it using a system of barriers. Dike construction goes back to the tenth century in this European country along the North Sea. One-quarter of the country is at or below sea level, and the country is very densely populated. There are over 3,000 polders, low-lying tracts of land reclaimed from fens and marshes that are protected by dikes. Every Dutch child knows the fable of Hans Brinker, the boy who put his finger in a hole in the dike and saved his town from flooding.

In January 1953, a cataclysmic windstorm accompanied by an abnormally high spring tide pushed an 18-foot-high storm surge toward the province of Zeeland, a popular summer resort area in southwestern Holland. Dikes were destroyed, and nearly 600 square miles of land were flooded. Over 1,800 people died. The government immediately formed a commission to study how to increase safety in the delta areas. The problem was how to improve safety but still leave the economically important seaways to the Dutch ports open for shipping.

After years of studies and testimony by engineers, fishermen, farmers, and environmentalists, a plan was agreed upon. It was called Delta Works (*Deltawerken* in Dutch). Work began on the ambitious plan at once. By 1958 the first phase of Delta Works was completed. A storm surge barrier was built across the river Hollandse IJssel, a branch of the Rhine delta. The barrier is

almost 300 feet wide and over 40 feet high. Over the next four decades, a system of storm surge barriers, sluices, dams, and dikes was constructed all across the Netherlands. They manage everything from the North Sea to the runoff from the spring snow melt in the Alps that cascades down the Rhine and its tributaries. They total more than 6,000 miles in length with 300 structures. Many of the Delta Works have visitor centers where one can learn about the history of the area and the construction of the dams. There are many miles of walking and bicycle paths as well.

Delta Works is an undertaking that many consider to be one of the Seven Wonders of the Modern World. They rate it in the same category as the Panama Canal and the Empire State Building. The project is far from over. It is constantly ongoing. Managing saltwater levels and freshwater levels, and balancing the needs of fishing and agriculture, are continually being reassessed. Planners are currently researching rising sea levels because of global warming. Delta Works is an example of what can happen when people work together to ensure that something like the Great Flood of 1953 will never occur again.

27. Which of the following best tells what this passage is about?

A. why Delta Works is known as one of the Seven Wonders of the Modern World

B. why the snow runoff from the Alps affects the Netherlands

C. how the Netherlands is the only country that has a plan to manage sea and river water

D. how the Netherlands was forced to find a way to deal with destructive storm surges

E. how the Netherlands developed and built its sea and river water management program

28. The author implies that without Delta Works

F. the Netherlands would have been absorbed by another European country.

G. the lives of the people of the Netherlands would be unchanged.

H. the Netherlands would have been less popular as a tourist destination.

J. the Netherlands might have lost land to the water.

K. many of the people of the Netherlands would have moved to other countries.

29. Why did the author include the information about Hans Brinker at the beginning of the article?

A. to explain how leaks are stopped in dikes

B. to show that everyone can help when flooding is imminent

C. to show that the Netherlands has many stories written about it

D. to suggest that the Dutch are very imaginative when there are problems

E. to emphasize that the Netherlands has always had a problem with flooding

30. Which best describes the way in which the plan for Delta Works was developed?

F. It was a cooperative effort by many different people.

G. It was the work of advanced engineers.

H. It was modeled after a similar program in Germany.

J. It came about somewhat by accident.

K. It was conceived by the people living near the dikes.

31. Which of the following is **not** true of Delta Works?

A. It is an engineering masterpiece.
B. It was built because of the Great Flood of 1953.
C. Many people are interested in seeing it.
D. It is completely finished.
E. Some parts of the system were built before others.

32. With which of the statements would the author most agree?

F. More countries should follow the example of the Netherlands in solving their problems.
G. Delta Works will soon be the most popular stop for people traveling to Europe.
H. More time should have been taken in developing the plan to contain storm surges.
J. Delta Works did not have to be as large a project as it was.
K. The people of the Netherlands did not want to have Delta Works constructed.

The traditional Mehndi Celebration that takes place as part of an Indian wedding has roots that go back over 9,000 years. This was when the henna plant was first used for a variety of purposes in Africa, the Middle East, and India. The origin for using henna was its cooling and astringent effect when applied to the skin. But the custom was also tied into the cultural belief that henna would bring love and good fortune.

Henna, or *mehndi* in Hindi, was brought to India in the twelfth century by the Mongols who traded for centuries with Arabs. It grows best in desert-like conditions; hence, when the desert people of Rajasthan, Punjab, and Gujarat states became aware of the cooling aspect of the plant, they dipped their hands and feet in a paste made from the plant's crushed leaves. While the plant did cool, it also left a red stain. After a while, some women grew tired of

having bright red palms all the time. They found that one large dot in the center of the palm still provided a cooling effect. It was also pleasing to the eye. Other, smaller dots were placed around the center dot. This gradually gave way to the idea of creating artistic designs on the palms.

Over time this process evolved into the Mehndi Celebration for the bride. Indian weddings are elaborate affairs, with many festivities before, during, and after the wedding. A Mehndi Celebration is held at the bride's house before the wedding. Her palms and feet are painted, usually by a henna professional. The stronger the paste, the longer the designs will last. This is desirable since the designs are a symbol of a long-lasting marriage. Popular images of mehndi art are conch shells, flowers, and peacocks. Hidden in the detailed designs of the bridal henna design is the bride's future husband's name. The groom must find his name in the mehndi to show his future wife that he has sharp eyes and an alert mind. While henna has long been used to dye hair in the United States, it is only in recent years that henna tattoo body painting has become popular as a painless, fashionable, and nonpermanent tattoo alternative. Henna body art kits and traditional mehndi stencils are even available for do-it-yourselfers. Henna art studios are popping up across the United States offering traditional Indian bridal designs as well as decorative body painting. Some artists will come to parties or events and decorate everyone there. The tradition of henna body painting has definitely evolved with the times.

33. Which of the following best tells what this passage is about?

A. what needs to be done to apply henna
B. how henna designs are made
C. how henna is used to dye hair
D. which people use henna
E. the use of henna to create body designs

34. What reason is **not** mentioned for using henna?

F. enables a change of hair color
G. ease of using
H. not a permanent design

J. symbol of love
K. attractive designs

35. According to the passage, which of the following statements about the Mehndi Celebration is probably true?

A. The participants of the celebration are all women.
B. The celebration lasts for several days.
C. Many Indian brides do not participate in such a celebration.
D. Indian weddings in the United States do not have this tradition.
E. The celebration is costly, and only wealthy Indians have it.

36. The information about a bride's mehndi containing the name of her future husband was included in order to illustrate

F. how the designs are attractive to the future husband.
G. why the designs take so long to do.
H. how beautiful the designs are.
J. what the future husband thinks of mehndi.
K. how complex the designs can be.

37. Which of the following is the most likely reason that Indian brides choose to have mehndi designs on their palms?

A. They are attractive to look at.
B. They cool the skin.
C. They leave a red stain.
D. They are easy to get rid of.
E. They promise good luck.

38. Why does the author say that the "tradition of henna body painting has definitely evolved with the times"?

F. to make readers aware that henna body painting is used more now than in the past

G. to show that people use mehndi for wedding celebrations

H. to show that henna uses have changed over the years

J. to make light of people who use henna

K. to suggest that today's henna is nothing like the henna that was used years ago

Learning how to swim is very important for people of all ages. It's never too late to learn how to swim. Accidents can happen around water of any kind, from a swimming pool to a lake or river to the ocean, even a water park. Knowing how to react if you fall into the water can save your life, especially if you are young. In fact, according to the Centers for Disease Control (CDC), drowning is the second most likely cause of death in children under the age of 14. Learning how to swim is something you can't do online or learn by yourself. You need a teacher, either with a group or privately. YMCAs, community centers, and gyms all offer swimming lessons.

Certain groups of people, like African Americans and Hispanics, statistically face greater dangers near water. A survey conducted by USA Swimmers, an organization that governs competitive swimming in the United States at all age levels, polled nearly 1,800 people in 16 cities. They found that 60 percent of black children under 14 cannot swim. That is twice the rate for white children. Most of the black children who could not swim were inner city residents. Black children drown at three times the rate of white children. Several conclusions emerged. The children didn't learn to swim because their parents didn't learn to swim and were afraid of the water. Also, many parents and grandparents grew up when pools were segregated, and few pools were available to African Americans. USA Swimmers is expanding its learn-to-swim programs, especially in inner cities, to

reach more African American children.

The best place to learn how to swim is in a swimming pool. It is possible to start out where a person can stand with his or her head above water. All professional teachers of swimming will tell you that the first step is to get rid of the fear of water. Whether a child or an adult, the individual needs to get used to being in the water. Beginners need to realize that when swimming, the body is more buoyant, the muscles loosen up, and the spine becomes more relaxed. That is why water is used extensively for rehabilitation from accident or injury.

Beginners will learn to develop confidence in the water as well as the motor skills necessary for swimming. They will learn about safety and what to do if somebody accidentally falls into the water. Discipline is necessary. A beginning swimmer needs to obey rules, do what the lifeguard says to do, and most important, never swim alone. Ultimately, the student will discover, as USA Swimmers states on its website, that “swimming is a technical and specialized activity involving extensive skill development.”

39. Which of the following best tells what this passage is about?

A. the reasons for people drowning
B. how people learn to swim in a pool
C. the reasons why people should learn to swim
D. the kind of motor skills that are used when swimming
E. how USA Swimmers helps children learn to swim

40. The best reason why the author included the information that drowning is the second most likely cause of death in children under the age of 14 was to

F. remind the reader that many people drown every year.
G. stress to the reader how important knowing how to swim is.

H. suggest that swimming can help improve a child's life experience.

J. show how difficult it is for children to get swimming classes.

K. emphasize how many children need to learn to swim.

41. Which reason is **not** given for learning to swim?

A. to feel confident in the water

B. to know what to do if you fall into the water

C. to improve new motor skills

D. to feel at home in the water

E. to be able to become a life guard

42. Why should beginning swimmers swim with someone else?

F. It will help them improve their swimming.

G. They will learn to compete with others.

H. They will enjoy the company.

J. They will learn to swim faster.

K. It will keep them safer.

43. The author includes information about African American children to demonstrate

A. the difficulty of making this group aware of the water's dangers.

B. his knowledge of the African American population's history.

C. why African Americans are good athletes.

D. how cultural disparities have influenced children in terms of learning to swim.

E. how African Americans feel about learning to swim.

44. Why do you suppose that USA Swimmers is expanding its programs in inner cities?

F. to create jobs for inner city people
G. to explore the new possibilities that come with expansion
H. to lower the cost of swimming classes in cities
J. to show that swimming is something that everyone can do
K. to reach more African American and Hispanic children

The great African American explorer Matthew Henson was born in Maryland in 1866 to parents who were sharecroppers. When he was four, the family moved to Washington, DC, to find work, but tragically, when he was 12, both his parents died. Hard work and determination were traits that showed up early in Henson. They were most evident when he walked to Baltimore to get a job as a cabin boy.

The captain of the merchant ship *Katie Hines* was struck by this boy's persistence and hired him. Soon enough Henson learned to be a good seaman. The captain took an interest in him and taught him geography, math, and history. Henson also learned how to navigate and he traveled with the captain to China, Africa, and the Middle East. After the captain suddenly died, Matthew returned to Washington and found a job working for a furrier.

It was at the furrier's that he met Robert E. Peary, a naval officer. Peary, who traded with Eskimos in Greenland, brought furs with him to sell. Peary realized that Henson had a great sense of adventure and offered him a job as his assistant. The two made several trips to Greenland. Henson learned to speak Inuit, and the Eskimos taught him how survive the harsh climate. Peary told Henson about his dream to be the first person to reach the North Pole, and

Henson eagerly accepted his invitation to join him. Between 1898 and 1908 the Peary made numerous attempts to reach the Pole, each time to turn back in failure.

In 1908 Peary's expedition, including Henson, left on the USS *Roosevelt* from New York for Cape Columbia, the northernmost point of Canada. On March 1, 1909, the team pointed their dog sleds north and set out on the 475-mile journey. Crossing the ice-covered Arctic Ocean was treacherous because the shifting ice would split open without warning. Most of the expedition members turned back. Some were angered that Henson, a black person, had better skills than they did. Finally, on April 6, Peary, Henson, and four Eskimos reached their goal: 90 degrees north.

On their return, Peary was given many honors. Because of the prevailing prejudice against blacks at that time, Henson was ignored. But in 1937 Henson was made an honorary member of the famous Explorers Club of New York. Henson died in 1955 and was buried in New York. In 1987 there was a movement to have the remains of both Henson and his wife moved to lie next to Robert Peary in Arlington National Cemetery. President Reagan granted permission, and on the seventy-ninth anniversary of the discovery of the North Pole, Henson was laid to rest near his fellow explorer. On Henson's tomb there is a quote from his autobiography: "The lure of the Arctic is tugging at my heart. To me the trail is calling. The old trail. The trail that is always new."

45. Which of the following best tells what this passage is about?

A. the relationship between Matthew Henson and Robert Peary
B. why Matthew Henson became a cabin boy
C. how Matthew Henson learned Inuit
D. how Matthew Henson made his mark on history
E. how the North Pole was discovered

46. How did the experience of being a cabin boy affect Matthew Henson?

F. He was protected from dangers.
G. He started to read a lot.
H. He learned many new things.
J. He met many people who helped him.
K. He found what he wanted to do in life.

47. Which of the following was **not** something that Matthew Henson did?

A. traveled all around the globe
B. learned to trade for furs
C. learned to speak Inuit
D. traveled to the North Pole
E. wrote an autobiography

48. Why did some members of the expedition not like Matthew Henson?

F. They were trained explorers.
G. They did not like him telling them what to do.
H. They were jealous of Peary's friendship with him.
J. They thought he was too strict.
K. They were prejudiced against black people.

49. Why do you suppose Matthew Henson was made an honorary member of the famous Explorers Club of New York?

A. to give him the respect he deserved for reaching the North Pole
B. to show that he had the same skills as other explorers

C. to inspire other African Americans to become explorers

D. to show that the group was not prejudiced against black people

E. to help him to find work

50. What can be inferred about Matthew Henson from the quotation from his autobiography?

F. He enjoyed writing.

G. He thought he would die in the Arctic.

H. He was a kind and thoughtful man.

J. He lacked determination.

K. He was a born explorer.

PART 2
MATHEMATICS 75 Minutes ▪ 50 Questions

General Instructions

Find the solution to each problem, then choose the best answer from the options given. Mark the letter of your answer choice on the answer sheet. You may use your test booklet or paper provided by the proctor as scrap paper for your calculations. However, DO NOT make calculations or put any stray marks on your answer sheet.

Important Notes

(1) No formulas or definitions of mathematical terms and symbols are provided.

(2) Diagrams are not necessarily drawn to scale. Be careful not to make assumptions about relationships in diagrams.

(3) Diagrams can be assumed to be in one plane unless otherwise stated.

(4) Graphs are drawn to scale. Therefore, you may assume relationships according to appearance. This means lines that look parallel can be assumed to be parallel, angles that look like right angles can be assumed to be right angles, and so forth.

(5) All fractions must be reduced to lowest terms.

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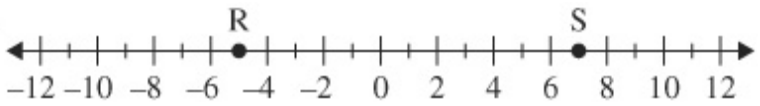
51. What is the greatest common factor of 135 and 210?

A. 3
B. 5
C. 9
D. 15
E. 25

52.



If $\overline{RS}$ is divided into 3 equal parts by points P and Q (not shown), what position will point Q fall on? (Assume $P < Q$)

F. –2
G. –1
H. 1
J. 3
K. 5

53. Convert $\frac{8}{25}$ to decimal form.

A. 0.0032
B. 0.032
C. 0.32
D. 3.20
E. 32.0

54. Tony bought a computer, a cell phone, and a television. The computer cost 2.5 times as much as the television. The television cost 5 times as much as the cell phone. If Tony spent a total of $925, how much did the cell phone cost?

F. $37
G. $50
H. $68
J. $74
K. $80

55. The table below shows the number of hours of volunteer work completed by 20 students. What is the mean number of hours?

HOURS OF VOLUNTEER WORK COMPLETED

Hours	Number of Students
16	7
12	6
20	4
10	3

A. 10.7
B. 13.2
C. 14.7
D. 16.5

E. 19.5

56. A 9-sided polygon has 1 side of length x centimeters, 4 sides each of length $1.25x$, and 4 sides each of length $1.5x$. If the perimeter of the polygon is 156 centimeters, what is the value of x?

F. 11
G. 12
H. 13
J. 14
K. 15

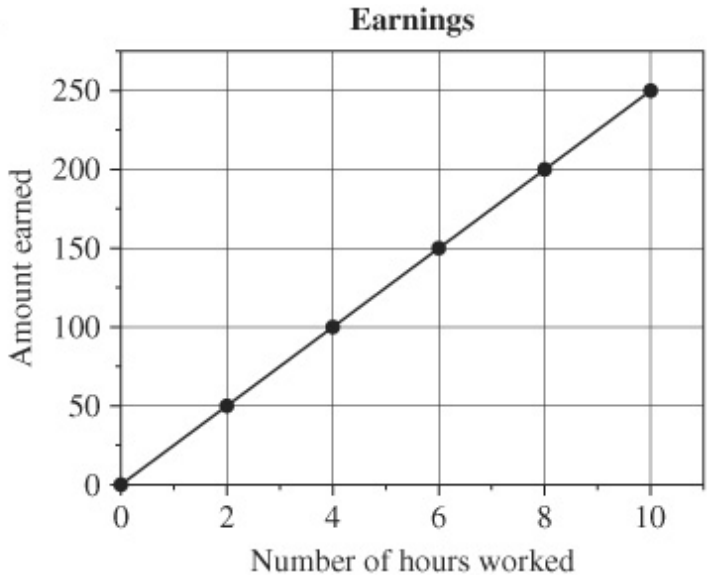
57. $6x(2y-9) =$

A. $8xy-9$
B. $8xy-3x$
C. $12xy-9$
D. $12y-54x$
E. $12xy-54x$

58. $(\sqrt{9})(\sqrt{49}) =$

F. 21
G. 24
H. 27
J. 28
K. 36

59.



The graph shows the relationship between the number of hours Megan works and the amount she earns. How much does Megan earn per hour?

A. $10
B. $15
C. $20
D. $25
E. $30

60. What is the least integer greater than $\frac{43}{7}$?

F. 4
G. 5
H. 6
J. 7
K. 8

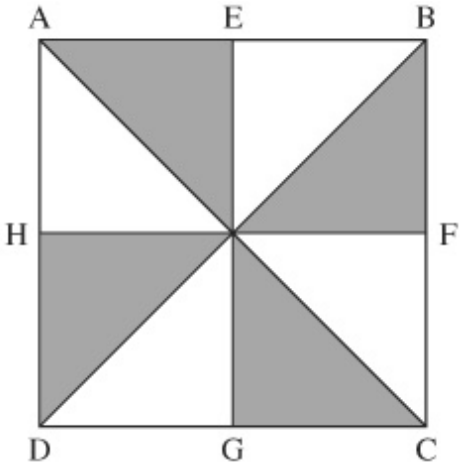
61. What is the value of $|6x| - |-2y|$ if $x = -3$ and $y = 5$?

A. –28
B. –8
C. 2
D. 8
E. 28

62. At 5:00 a.m., the temperature was 15 degrees below zero Fahrenheit. Then the temperature rose 5 degrees per hour for 8 hours. What was the temperature at 12:00 p.m?

F. 15°
G. 20°
H. 25°
J. 30°
K. 35°

63.



ABCD is a square. Line segments $\overline{EG}$ and $\overline{FH}$ and diagonals $\overline{AC}$ and $\overline{BD}$ pass through the same point. The total area of the shaded triangles is 18 square centimeters. What is the side length of square ABCD?

A. 6
B. 9
C. 12
D. 24
E. 36

64. Which of the following is equivalent to the inequality $21 + x < -7$?

F. $x < -28$
G. $x > -28$
H. $x > -14$
J. $x < 14$
K. $x < 28$

65. Brooke worked $3\frac{1}{2}$ hours every day for 5 days. She earned $5.10 per hour. How much did Brooke earn in all?

A. $17.85
B. $40.80
C. $43.35
D. $81.60
E. $89.25

66. What is the difference between 15% of 7 and 7% of 7?

F. –0.385
G. –3.85
H. 0.56
J. 1.54
K. 5.95

67. Twenty-one out of 70 customers at the restaurant ordered dessert. What percentage of the customers did **not** order

dessert?

A. 21%
B. 30%
C. 49%
D. 65%
E. 70%

68. Andy is driving in a straight line on the freeway heading east. Morgan is driving on the same freeway heading west. At 1:00 p.m., they are exactly 60 miles apart. They pass each other at 1:30 p.m. If Andy is driving at 55 miles per hour, at what speed is Morgan driving?

F. 45 miles per hour
G. 50 miles per hour
H. 55 miles per hour
J. 60 miles per hour
K. 65 miles per hour

69. A circle has a circumference of C inches and an area of A square inches. If $C = \frac{2}{5}A$, what is the diameter of the circle?

A. 0.4 inches
B. 0.8 inches
C. 2.5 inches
D. 5 inches
E. 10 inches

70. One-third the difference of two numbers is 12. If the larger number is 9, what is the sum of the two numbers?

F. –27
G. –18
H. 36
J. 45
K. 54

71. For what negative value of *x* does $\frac{5}{8} = \frac{40}{x^2}$?

A. –8
B. –7
C. –6
D. –5
E. –4

72. The perimeter of a rectangle is 40 inches. If the width is 9 inches, what is the area of the rectangle?

F. 18 sq in.
G. 27 sq in.
H. 99 sq in.
J. 198 sq in.
K. 279 sq in.

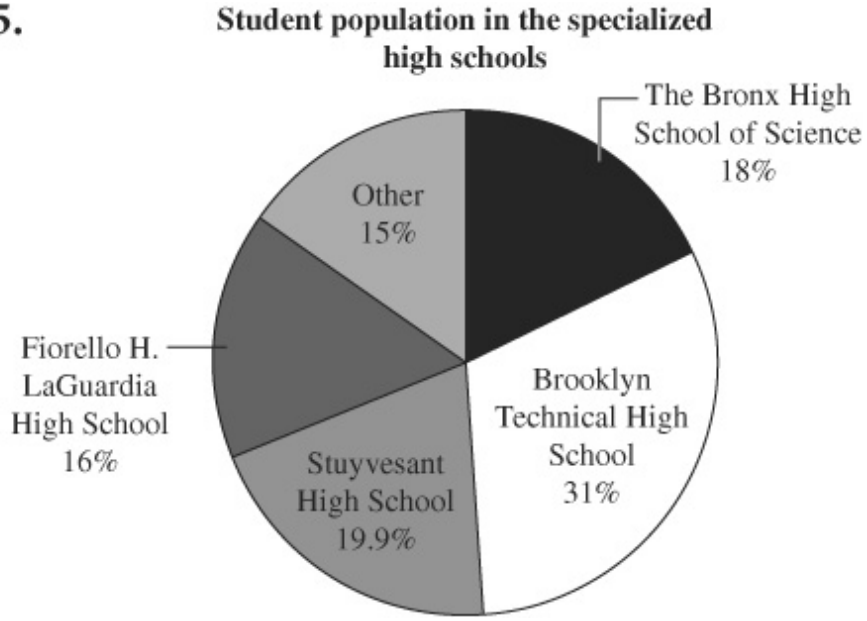
73. The sporting goods store sells 6 different brands of tennis rackets and 4 different brands of tennis balls. If Amanda buys one brand of tennis racket and one brand of tennis balls, how many different combinations of brands can she buy?

A. 10
B. 12
C. 16
D. 24
E. 36

74. If $\ddot{x} = \frac{x}{4}$, what is the value of $\ddot{5}(\ddot{8})$?

F. 1.25
G. 2.5
H. 3.75
J. 5.0
K. 10.0

75.


Student population in the specialized high schools
The Bronx High School of Science 18%
Brooklyn Technical High School 31%
Stuyvesant High School 19.9%
Fiorello H. LaGuardia High School 16%
Other 15%

There are 16,405 students enrolled in specialized high schools. The graph shows the approximate percentage of students enrolled in the different schools. Approximately how many students are enrolled at Stuyvesant High School?

A. 2,526
B. 3,002
C. 3,265
D. 4,101
E. 5,086

76. Fernando is now *x* years old. Erika is 7 years older than Fernando. In terms of *x*, how old was Erika 4 years ago?

F. $7x$
G. $7x-4$
H. $x+11$
J. $x+3$
K. $x-4$

77. If $x=23$ and $y=15$, what is the value of $\frac{x+y}{x-4}$?

A. 2.00
B. 3.75
C. 6.00
D. 8.00
E. 9.50

78. What is the least common multiple of *X* and *Y*?

$X=3\times3\times5$

$Y=2\times2\times3\times5$

F. 15

G. 30
H. 65
J. 120
K. 180

79. In a scale drawing, 1 inch represents 25 feet. How many square inches on the diagram represent 1 square foot?

A. 0.0016
B. 0.009
C. 0.04
D. 0.48
E. 0.625

80. Katy has a bag of yellow and red balls. There are a total of 30 balls in the bag. The probability of drawing a yellow ball is $\frac{1}{6}$. How many red balls should be removed from the bag in order to increase the probability to $\frac{1}{3}$?

F. 5
G. 9
H. 10
J. 12
K. 15

81. Cody has a box of cereal. On Monday, he ate $\frac{1}{7}$ of the cereal. On Tuesday, he ate $\frac{1}{6}$ of the remaining cereal. What fraction of the cereal is left in the box?

A. $\frac{5}{6}$

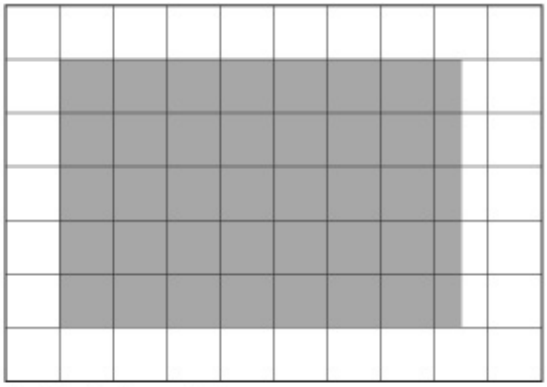
B. $\frac{2}{7}$

C. $\frac{5}{7}$

D. $\frac{2}{13}$

E. $\frac{12}{13}$

82.



Scale: 1 in. = 4 ft

The grid above is made up of 1-inch squares. About how many square yards are in the shaded region?

F. $12\frac{2}{3}$

G. $16\frac{2}{3}$

H. $22\frac{1}{3}$

J. $37\frac{1}{2}$

K. $66\frac{2}{3}$

83. A pyramid has a rectangular base. Its volume is 75 cubic centimeters and its height is 5 centimeters. If the length of the base is 9 centimeters, what is the width?

A. 5 cm

B. 15 cm
C. 25 cm
D. 35 cm
E. 45 cm

84. What number is halfway between $\frac{1}{2}$ and 0.85?

F. 0.525
G. 0.575
H. 0.625
J. 0.675
K. 0.725

85. Julianne is half as old as Benita. If Benita will be 24 years old in 8 years, how old was Julianne 3 years ago?

A. 3
B. 5
C. 7
D. 9
E. 11

86. There are approximately 13,000 registered voters in Montour County. About 46% of the voters are registered as Republicans. About 38% of the voters are registered as Democrats. About how many more registered voters are Republican than Democrat?

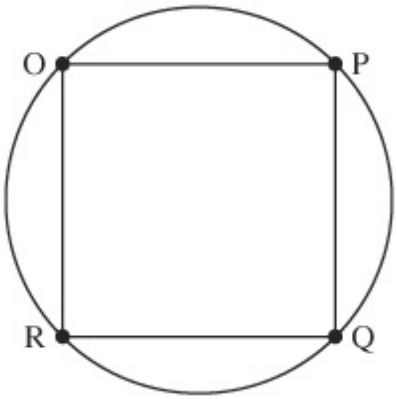
F. 800
G. 900
H. 1,000

J. 2,000
K. 3,000

87. Tameka has 220 yards of fabric. She cuts the fabric into equal sized pieces that are each 12 yards long. How many yards of fabric does she have left over?

A. 1
B. 2
C. 3
D. 4
E. 5

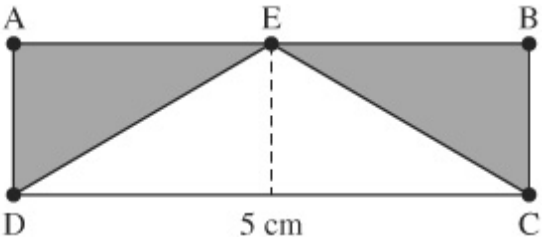
88.



In the figure above, O, P, Q, R are points on a circle and OPQR is a square. If the area of the circle is 36π square centimeters, what is the measure of the diagonal of the square?

F. 6 cm
G. 8 cm
H. 10 cm
J. 12 cm
K. 18 cm

89.



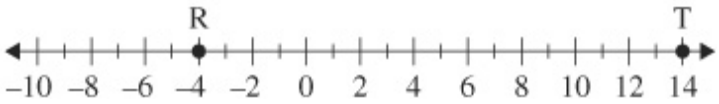
In the diagram above, ABCD is a rectangle and E is a point on $\overline{AB}$. The area of triangle CDE is 25 square centimeters, and its height is 5 centimeters. What is the area of the shaded region?

A. 10 sq cm
B. 15 sq cm
C. 25 sq cm
D. 40 sq cm
E. 50 sq cm

90. If $x = 7$ and $(3x - 1)(y - 2x) = 60$, what is the value of *y*?

F. 11
G. 17
H. 21
J. 30
K. 44

91.



Point S (not shown) is located on line segment $\overline{RT}$ so that $\overline{RS}$ is $\frac{1}{3}$ the length of $\overline{ST}$. What is the location of point S?

A. –2

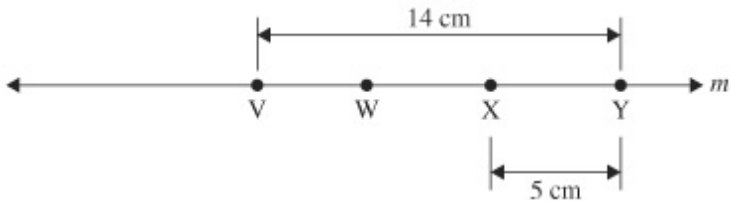
B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. 2

92.



On line *m*, if $VW = WX$, what is the length of WY?

F. 7.0

G. 7.5

H. 9.0

J. 9.5

K. 10.0

93. Julio bought a book for *x* dollars. The electronic version of the book costs *y* dollars. The electronic version costs one dollar less than one-fourth the cost of Julio's book. What is the value of *y* in terms of *x*?

A. $\frac{1}{4} - x$

B. $\frac{1}{4}x - 1$

C. $1 - \frac{1}{4}x$

D. $1 - 4x$

E. $4x - 1$

94. Tia has a box containing 15 blue marbles, 12 green marbles, 7 red marbles, and 1 white marble. Tia removed 8 marbles, 6 of which were blue. If she removes one more marble at random, what is the probability that it will be blue?

F. $\frac{1}{3}$

G. $\frac{2}{5}$

H. $\frac{1}{7}$

J. $\frac{2}{9}$

K. $\frac{5}{9}$

95. $\left(\frac{2}{6}+\frac{1}{7}\right)\div\frac{2}{5}=$

A. $\frac{6}{65}$

B. $\frac{4}{21}$

C. $\frac{15}{26}$

D. $\frac{25}{21}$

E. $\frac{28}{5}$

96. Lance is 39 years old. Ted is 22 years old. Rosalinda is 45 years old. How old is Erik if the mean age of the four friends is 33.5?

F. 19

G. 28

H. 33

J. 48

K. 52

97. One side of a square lies on the *x*-axis of a coordinate system. The coordinates of one corner of the square are (2, 0) and the coordinates of another corner are (6, 0). What could be the coordinates of the third corner?

A. (–4, 4)
B. (–2, –4)
C. (0, 4)
D. (2, –4)
E. (4, 0)

98. Allan can drive 32 miles on one gallon of gas. How many miles can he drive if he only has $\frac{7}{8}$ of a gallon?

F. 21
G. 23
H. 25
J. 27
K. 28

99. Melissa uses $\frac{3}{5}$ as much water as Neil to water her plants. If Melissa use $4\frac{1}{3}$ gallons of water, how many gallons does Neil use?

A. $1\frac{3}{5}$
B. $2\frac{3}{5}$
C. $4\frac{4}{9}$
D. $6\frac{2}{3}$
E. $7\frac{2}{9}$

100. Mr. Hirsch teaches 5 classes. The mean number of students per class is 17.4. If he teaches one more class, the mean

increases to 18.5. How many students are in the sixth class?

F. 19
G. 21
H. 24
J. 38
K. 31

THIS IS THE END OF THE TEST. IF TIME REMAINS, YOU MAY CHECK YOUR ANSWERS TO PART 2 AND PART 1. BE SURE THAT THERE ARE NO STRAY MARKS, PARTIALLY FILLED ANSWER CIRCLES, OR INCOMPLETE ERASURES ON YOUR ANSWER SHEET.

Test 2 Answer Key

Paragraph 1
T R Q U S

Paragraph 2
U R Q S T

Paragraph 3
T Q U R S

Paragraph 4
R U S T Q

Paragraph 5
S R Q U T

11. B
12. H
13. D
14. K
15. C
16. K
17. E
18. K
19. A
20. H
21. C
22. K
23. C
24. F
25. C
26. J
27. E
28. J
29. E
30. F
31. D
32. F
33. E
34. G
35. A
36. K

37. E
38. H
39. C
40. G
41. E
42. K
43. D
44. K
45. D
46. H
47. B
48. K
49. A
50. K
51. D
52. J
53. C
54. G
55. C
56. H
57. E
58. F
59. D
60. J
61. D
62. G

63. A

64. F

65. E

66. H

67. E

68. K

69. E

70. G

71. A

72. H

73. D

74. G

75. C

76. J

77. A

78. K

79. A

80. K

81. C

82. K

83. A

84. J

85. B

86. H

87. D

88. J

89. C

90. G

91. D

92. J

93. B

94. F

95. D

96. G

97. D

98. K

99. E

100. H

Explanation of Answers—Verbal Section

Scrambled Paragraphs

Paragraph 1: T R Q U S

T follows the given sentence that introduces the subject of the first zoo because it gives more information about why the king created the zoo. R is a natural continuation of the subject of the first zoos; the words “Shortly thereafter” help you identify the sequence. Q talks about the difficulty of taking care of the animals, and U talks about how little early zookeepers knew about their animals. S is a natural conclusion to the information in U, so it obviously comes after it.

Paragraph 2: U R Q S T

Sentence U follows the given sentence because it talks about the beginning of the salmon’s life cycle. Logically R follows next since it talks about what happens while the salmon are in the ocean. Q comes next because it follows the sequential order of the salmon’s development. S follows since it tells of the salmon’s journey

upstream. T is the next since it is the conclusion of the life cycle of the salmon.

Paragraph 3: T Q U R S

The given sentence tells about the army ant being fearful but also helpless looking. T is the logical next sentence since it tells what the insect looks like. “Still” helps you figure out that Q would follow next because it tells in what way the army ant is dangerous even though it looks helpless. U would need to come next since it talks about army ants moving through the jungle for only two weeks and raising another generation of ants in the remaining two weeks. It has to go before R, which starts with “After the new generation is grown . . . ,” so it is a logical next step to S, which is a good conclusion since it tells what may happen if someone or something “stumbles” on a group of army ants.

Paragraph 4: R U S T Q

The given sentence talks about what Richard Spruce did to win fame. Chronologically R comes next because it talks about his childhood. U follows next because it continues the narrative about Richard Spruce. The beginning of S, “While in South America,” is the clue that it comes next because it talks about Richard Spruce being in South America and what he did. T explains in greater detail what Richard Spruce did, and Q follows next as a natural conclusion to what he accomplished.

Paragraph 5: S R Q U T

Sentence S come after the given sentence since it explains what the “misconception” about chess is. R gives an example of the information in S. Q is a continuation of the information in S so it is next. The beginning of U, “A second example…” is a clue that it should come after Q and S. T follows logically since it tells what the chess experts concluded about what Bobby Fischer did in U.

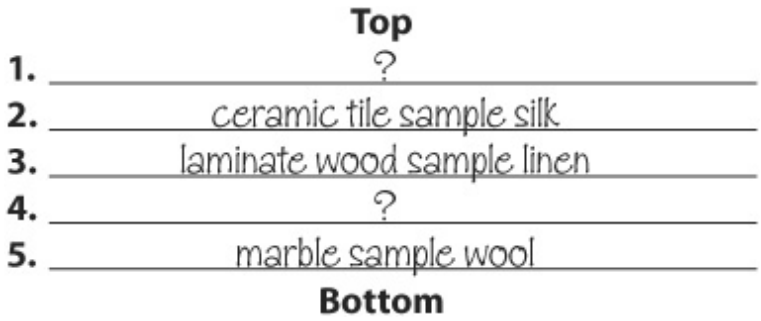
Logical Reasoning

11. **(B)** Use a diagram to help solve this question. Write “left” and “right” on your scrap paper and the numbers 1 through 5,

beginning on the left with 1. Then look for specific locations of each horse. Statement two says that the pinto is in the second position; write “pinto” above 2. Statement four tells you that there is exactly one horse to the right of the thoroughbred, so the thoroughbred must be in the fourth position. Statement three says that there are exactly three horses between the quarter horse and the palomino, so the quarter horse and palomino must be in the first and last positions, even though you cannot tell which one goes in which position. The question asks you in what position the Arabian is. Since you know that position 1 and 5 are taken and that the pinto is in the second position and the thoroughbred in the fourth position, you are left with the third position.

12. (H) This question contains two conditional statements. It is important that you read each statement carefully. The first sentence may be restated as, “If Teddy goes to the movies, Billy will not go to the movies.” This is simply a restatement of the two clauses in the first condition. Make sure to observe that the sentence only tells us what will happen *if* Teddy goes to the movies. *It says nothing about what will happen if Teddy does* ***not*** *go to the movies.* Therefore, it is incorrect to conclude that Billy will go to the movies if Teddy does not go. We simply do not know what will happen. Likewise, the second sentence can be rearranged as, “If Janet does **not** forget her backpack, Teddy will go to the movies.” But the sentence does not tell us what will happen if Janet *does* forget her backpack. Option H is correct; if Janet does not forget her backpack, Teddy will go to the movies. Option F is incorrect because the two conditions do not address what Billy will do if Janet forgets her backpack. Option G is wrong because the first condition does not say what Billy will do if Teddy does not go to the movies. Option J contradicts the second statement, and Option K can be ruled out because no information is given about Janet going to the movies.

13. (D) Use a diagram to help answer this question. Write “top” and “bottom” on your scrap paper. Then record what the question says about each sample. The question says that the bottom sample is marble, and statement three says that the laminate wood sample is third from the top. Statement two tells you that exactly two samples are between the marble sample and ceramic tile sample, which means the ceramic tile sample is in position 2.



That leaves the carpeting and linoleum samples for the remaining open positions, which are positions 1 and 4. According to statement one, the linoleum sample is higher in the stack than the marble. This condition will be fulfilled whether the linoleum sample is in position 1 or 4, so we cannot determine which position it is in. There is no information at all that can be used to determine the location of the carpeting. Thus, the correct answer is Option D: “Either carpeting or linoleum, but it is not possible to determine which one.”

14. (K) Option F is not a valid conclusion because while most people attending the lecture are students, only some of them are studying architecture. Option G could be appealing, but there is nothing in the statement about how advanced the students are. This could be true, but we cannot assume it is. Option H is a value judgment that is not supported by the information given. Option J contradicts statement that “some” students are not studying architecture. There is no basis for concluding that “most” people attending the lecture are not studying architecture. Option K is a valid conclusion. People who are not studying architecture can go to the lecture on architecture.

15. and 16. Read the directions carefully. The letters in a sentence may or may not appear in the same order as the words in that sentence. For example, in the first sentence the first letter (M) may or may not represent the first word (George). Remember that you need not find out what every letter represents in the code.

15. (C) “Carla” appears only once in the code, in the third sentence. The first, second, and third sentences are identical except for the names. Thus the letters that are found in all three sentences must represent the words “wants to write.” The letters common to those three sentences are M, R, and T. The name Helen appears in both sentence two and three, so it must be X. The other letter in each sentence would be the name of the other person in each sentence.

16. (K) The letter Z appears only in the fourth sentence. The difference between the fourth sentence and the other three is the word “call” and the name Beth. You can figure out that M and R must represent “wants to” since these letters are found in all four sentences. The two letters that are only in the fourth sentence are Z and Y, but you cannot figure out whether Z represents “Beth” or “call.” So the answer cannot be determined from the information given.

17. (E) It can be very helpful to draw a diagram for this kind of problem. Let an initial stand for each name. Write “tallest” on the left-hand side of your scrap paper. Then diagram the first statement.

Tallest: N A D

After reading the second statement, you can put Tom into the diagram, but not Phil. All we know about Phil is that he goes somewhere to the right of Nathan; we don’t know how his height compares with that of Anna.

After reading the second statement,

Tallest: N A D T

After reading the third statement, we are still not sure about Phil's position. He is shorter than Nathan, but he could fit on either side of Anna. Therefore, Phil is either second or third, but his exact position "cannot be determined from the information given."

18. (K) We need to check each option against the facts to figure out which one is valid. We know nothing about Dan except how many afternoons he practices for the Hyannis High soccer team. So Option F, although it *might* be true, cannot be verified. Options G and H also *might* be true, but we don't know that from the information that is given. The same is true for Option J. Option K is a valid conclusion, since we are told that team members are expected to practice five afternoons a week, more than Dan does.

19. (A) This question asks you to match each driver with his or her car and place the cars in the order of arriving at the baseball game. The first thing to do is to look for statements that provide information about the person driving each car. Statement two says that Anna was driving the Dodge, statement five says that Vicky was driving the Ford, and statement three says that Dan was not driving the Chrysler. Therefore, the person driving the Chrysler must be Paul, which means that Dan was driving the Chevrolet. You can put the four cars and the people driving them in order. Based on statement one, draw a diagram showing the order in which the cars arrived at the game.

First

Anna (Dodge)

Paul (Chrysler)

Last

Based on statements four and five, Dan (Chevrolet) and Vicky (Ford) arrived after Paul. Either of the following orders could be correct, based on the information provided.

First	**First**
Anna	Anna
Paul	Paul
Dan	Vicky
Vicky	Dan
Last	**Last**

In either order, the Dodge, which Anna drove, arrived before the Chrysler, driven by Paul. So the correct answer is A.

20. (H) The problem says that the class picnic will be canceled if it is cold, but that it will take place if it is hot. We *do not* know that these are the only two circumstances that influence the picnic taking place. Therefore, you can rule out Option F. Option G is wrong because there is nothing in the statements about what happens when it is windy. Option J is wrong. There is no reason to believe that its being cold is the only reason the picnic might be canceled. Option K is a false conclusion. There is no way to tell whether it is cold more times than it is hot. Option H is correct. The weather may cause the picnic to be canceled.

Reading

21. (C) This question asks you to determine the general theme of the passage. Option A is mentioned in passing; it is a detail. Option B is discussed in detail, but it is not the theme of the passage. Options D and E are both details that are mentioned, but not the focus of the entire passage. The passage describes how the Northwest Passage was discovered and its future potential. Thus, the best answer is Option C.

22. (K) This question asks you to make an inference based on the information in the passage. Option F may be true, but there is not enough information in the passage to draw this conclusion. Option G does not agree with the information in the passage about the nationality of the explorers. Option H might be true, but there is nothing to suggest this was so. A close reading of the text shows that Option J is incorrect. The search started before

the Suez Canal was built, not afterward. A close reading of the text will show that Option K is the correct answer since the sentence “Some disappeared, never to be heard from again” (lines 16–17) strongly suggests this was the case.

23. (C) The question asks specifically about Roald Amundsen. In order to answer the question, you must look at the information in the passage and use it to infer the answer. Option A may be true, but there is nothing to support this idea in the passage. Option B could be true or false, but the passage does not discuss this possibility. Option C is supported by the passage, which states, “The ship was stuck frozen in two-foot-thick ice for two winters. But everyone survived” (lines 30–32). Only someone with great determination would endure this. Option D is contrary to the information in the passage. Option E does not appear to be true. There is nothing to suggest that he was a better sailor than other explorers.

24. (F) The statement “Continuing record sea ice melting may indicate that the route will be used by commercial carriers . . .” implies that global warming is the reason the ice is melting, therefore allowing commercial carriers to traverse the Northwest Passage, so there is support for Option F. It does not imply that Option G is correct; there is no suggestion that fewer small ships will use the Northwest Passage. There is also no evidence for Option H, J, or K.

25. (C) This is an inference question. It is necessary to look for information that supports one of the options. Neither Option A nor B is supported by the passage. There is no evidence of the Canadian government creating canals in the Northwest Passage, so Option D is not correct. It is also doubtful that Option E would happen. The best answer is Option C.

26. (J) To answer this question, you have to go through the passage and try to find the one topic that is not discussed. Options F and G are discussed in the paragraph. So is Option H. The passage says that the Eskimos taught Roald Amundsen survival skills,

"including how to hunt for food" (line 34). This suggests that finding food was a difficulty as well. That leaves Option J. There is no mention of hostile natives.

27. (E) This question asks you to determine the general theme of the passage. Option A is not discussed in the passage. Option B is just a detail, not a main theme. Option C is not mentioned in the passage at all, and nothing in the article says that the Netherlands was forced to find a solution (Option D), although the country was certainly motivated by the storm surge. Option E is the best answer.

28. (J) This is an inference question. You need to make an inference based on the information in the passage. Option F is not supported by the passage. There is nothing to suggest this. Option G is probably not true; without Delta Works the lives of some of the people of the Netherlands would have been affected. Option H might be true, but there is nothing to directly support this idea. Option J is supported by the passage, which talks about "low-lying tracts of land reclaimed from fens and marshes that are protected by dikes" (lines 10–12), and it can be inferred that without Delta Works much of this land might revert to the sea. Option K is not supported by the passage either. It might have happened, but there is nothing to suggest it.

29. (E) To answer this question, you need to read the passage carefully to see which option is correct. Option A might appeal, except the rest of the passage shows that fixing dikes is much more complicated than that. Option B really has no bearing on the passage, so it cannot be the correct answer. While it is true that the Netherlands has had many stories written about it, this is not the reason the author included the story of Hans Brinker at the beginning of the article. Nor is Option D the reason, since the story of a boy sticking his finger in a hole is not really a story of imagination, but a story about a town imperiled by a flood. Thus, Option E is the correct answer. This can be inferred from the passage.

30. (F) To answer this question, you need to reread the section of the passage that talks about how Delta Works was developed. If you do this, you will find that only Option F is an accurate description of how many people contributed to the plan. Option G might appeal, but there is nothing in the passage to indicate that only engineers worked on the plan. There is nothing to suggest that Option H or J is correct. Option K contradicts the information in the passage.

31. (D) To answer this question, you need to review the passage and find what is true of Delta Works. Option A is not likely because the author calls Delta Works "one of the Seven Wonders of the Modern World" (lines 53–54). Option B is something that is directly stated in the passage, so it can be eliminated. Options C and E are also correct according to the passage. However, Option D is not true; the project is ongoing. So Option D is the correct answer.

32. (F) To make an inference, you need to weigh all the information in a passage and come to the most logical conclusion. Options G and H don't seem to be likely. Nothing that the author says would indicate that Option J is correct, and Option K does not seem to be the correct choice either. Option F seems to be the only logical reason. Therefore, it is the correct answer.

33. (E) This question asks you to determine what the main idea or general theme of the passage is. Option A is mentioned in the passage, but it is only a detail, and not a general theme. Options B and C are details in the passage, but not the main focus of it. Option D might be an option, but most of the passage is about the use of henna to create body designs. Therefore, the correct answer is Option E.

34. (G) Check back through the passage to answer this question. Review the reasons that are mentioned for using henna. Although the word *dye* is not mentioned in Option F, this means the same thing. Only Option G is not given as a reason for using henna.

35. (A) This question requires you to take the information in the passage and make an inference. Based on the information in the passage, which only talks about women putting henna designs on their palms before weddings, Option A seems to be a logical conclusion. There is no suggestion that Option B is correct, although it could be true. Option C does not seem likely; there is nothing to suggest this is so. Option D is probably not true either. Option E could be true, but there is nothing to suggest this.

36. (K) You have to figure out why the author used this information in the passage. Option F might be a possibility, although this is not mentioned in the passage. Option G is not correct. There is no reference to how long the designs take to be done. Option H is probably true, but that is not why the information about the name of the bride's future husband is included. There is no suggestion that Option J is correct. Option K is the best answer.

37. (E) You need to use your best judgment to answer this question. While all of the options are true, Option E seems to be the best answer. Option A is tempting. A bride wants to be attractive. But you need to weigh all the answers before choosing one. Options B, C, and D are possible, but not as likely as E.

38. (H) To answer this question, you have to examine each option carefully. Option F is appealing, but that is not stated in the passage. Option G might be true, but this is not what the phrase states. Option H seems to be correct. This seems to be a good reason. Option J and Option K do not seem likely. Option H is the best answer.

39. (C) This question asks you to determine what the main idea or general theme is of the passage. Option A is mentioned in the passage, but it is only a detail, not a general theme. Option B is also mentioned but is only a detail, not what the passage is mostly about. Options D and E are only discussed in passing. Most of the article is about why people should learn to swim. Therefore, the correct answer is Option C.

40. (G) The statistic offers a strong motivation for the author's premise that learning to swim is important. The other options are incorrect.

41. (E) You will need to review the passage to answer this question. Scan through the passage to find what reasons were mentioned for learning to swim. Options A, B, and C are definitely mentioned, and Option D could be inferred from what is said in the passage. Hence Option E is the correct answer.

42. (K) A clue to the answer is the sentence "A beginning swimmer needs to obey rules . . ." Rules are meant to keep new swimmers safe; not swimming alone is one of the rules.

43. (D) You will need to analyze the author's intent from what the passage says. It talks about how many parents or grandparents of African American children were not allowed to swim in pools reserved for use only by white people; there were not many pools where blacks could swim. Showing how cultural disparities have influenced children in terms of swimming is the reason for including this information. Option A is not the reason, nor is Option B: you must keep in mind the general subject and purpose of the passage, which is water safety. Option C has no bearing on the importance of learning to swim. Option E is not correct; it is not even mentioned.

44. (K) While some of the options may occur as a result of USA Swimmers expanding its program in inner cities, such as Options F and G, the most important reason is Option K. Neither Option H nor Option J would probably be a reason.

45. (D) This question asks you to determine the main idea or general theme of the passage. Option A is definitely a possibility, but while this is important, it is not the main focus of the passage. Options B and C are details in the passage, not the main focus of it. Option E is also a topic, but it is not the main theme. The passage describes how Matthew Henson made his mark on history, so Option D is the correct answer.

46. (H) Use the information in the passage to help find the answer to this question. The passage says that Matthew Henson traveled all over the world as a cabin boy; from that information it could be inferred that there were many dangers. Option G could be a possibility; perhaps he did read a lot. But that is not the best answer. Option H is something that the passage supports. Therefore it is the correct answer. Option J is probably true, but it was not the main effect. Option K is not supported by the passage since he ended up being an explorer who made his way to the North Pole.

47. (B) A careful review of the passage will help you answer this question. Option A is something that Matthew Henson did, as are Options C and D. Option E was not stated directly, but there is a quote from his autobiography in the passage, so this is something he did as well. There is no mention in the passage that he learned to trade furs, so Option B is correct.

48. (K) The passage talks about how many of the expedition did not like Matthew Henson because he had better skills; but the main point is that he was a black man. Later in the passage it talks about how prejudiced people were against African Americans at that time, so it is possible to infer that Option K was the reason they did not like him. Option F is probably true of these members of the expedition, but that was not the underlying reason for their dislike of him. There is nothing in the passage to suggest Option G, H, or J is correct. Therefore option K is the best answer.

49. (A) You have to use the information and your own prior knowledge to answer this question. Option A is appealing. The fact that Matthew Henson helped Peary get to the North Pole was originally ignored; having him become an honorary member of the Explorers Club was one way to rectify what had happened earlier. While he did have the same, or better, skills as other explorers, this was not the reason for the honor, so Option B is not correct. While the honor might inspire other African Americans, this was not the main reason he received this honor, so Option C is not correct either. Option D is not the reason even

if it is true. There is no mention of Henson needing work, so Option E is not the answer. The best answer is Option A.

50. (K) You need to use the information in the passage to make an inference about Matthew Henson. Options F and G do not seem to be the correct choices; he may have enjoyed writing, but that is not what the quote suggests, nor does it suggest he thought he would die in the Arctic. Option H might be true, but this is not what is suggested by the quote. Option J is clearly incorrect. Option K is what is suggested by the quotation from his autobiography.

Explanation of Answers—Mathematics Section

51. (D) Factor each number into its prime factors:

$$135 = 3 \cdot 3 \cdot 3 \cdot 5$$
$$210 = 2 \cdot 3 \cdot 5 \cdot 7$$

The common prime factors are 3 and 5:

$$135 = \boxed{3} \cdot 3 \cdot 3 \cdot \boxed{5}$$
$$210 = 2 \cdot \boxed{3} \cdot \boxed{5} \cdot 7$$

The greatest common factor (GCF) is the product of the common prime factors:

$$\text{GCF} = 3 \times 5 = 15$$

52. (J) The formula for the distance between two points *a* and *b* on a number line is $b - a$. Use the formula to find the length of $\overline{\text{RS}}$:

$$\begin{aligned} RS &= S - R \\ &= 7 - (-5) \\ &= 7 + 5 \\ &= 12 \end{aligned}$$

$\overline{RS}$ is divided into 3 equal parts, so each part is $12 \div 3 = 4$ units long. Start at point R and count 4 units to the right to point P. Then count another 4 units to point Q.

53. (C) Divide 25 into 8:

$$\begin{array}{r} 0.32 \\ 25\overline{)8.00} \\ -750 \\ \hline 50 \\ -50 \\ \hline 0 \end{array}$$

54. (G) Let x represent the cost of the cell phone. The television cost 5 times as much as the cell phone, so the cost of the television is $5x$. The computer cost 2.5 times as much as the television, so the cost of the computer is $2.5(5x)$. Together, they cost \$925, so $2.5(5x) + 5x + x = \$925$. Solve for x:

$$\begin{aligned} 2.5(5x) + 5x + x &= \$925 \\ 12.5x + 6x &= \$925 \\ 18.5x &= \$925 \\ x &= \$50 \end{aligned}$$

55. (C) The mean is the total number of hours divided by the number of students. The table shows that 7 students completed 16 hours, 6 students completed 12 hours, 4 students completed 20 hours, and 3 students completed 10 hours. The mean number of hours completed is:

$$\frac{(7\times16)+(6\times12)+(4\times20)+(3\times10)}{20}=\frac{112+72+80+30}{20}$$
$$=\frac{294}{20}$$
$$=14.7$$

56. (H) The sum of all nine sides of the polygon is 156, so $x+4(1.25x)+4(1.5x)=156$. Solve for *x*:

$$x+4(1.25x)+4(1.5x)=156$$
$$x+5x+6x=156$$
$$12x=156$$
$$x=13$$

57. (E) Use the distributive property to write $6x(2y-9)$ as an equivalent expression.

$$6x(2y-9)=6x(2y)-6x(9)$$
$$=12xy-54x$$

58. (F) $(\sqrt{9})(\sqrt{49})=(3)(7)=21$

59. (D) According to the graph, Megan earns \$50 if she works 2 hours, \$100 if she works 4 hours, and so on. The graph is a straight line, so the rate of change is constant.

The amount Megan earns per hour is:

$$\frac{\$50}{2}=\$25$$

60. (J) $\frac{43}{7}=6\frac{1}{7}$, so the least integer that is greater than $6\frac{1}{7}$ is 7.

61. (D) Substitute the values of *x* and *y* into the expression:

$$\begin{aligned}|6x|-|-2y| &= |6(-3)|-|-2(5)| \\ &= |-18|-|-10| \\ &= 18-10 \\ &= 8\end{aligned}$$

62. (G) At 5:00 a.m., the temperature was –15°. At 12:00 p.m., the temperature had risen 5 degrees per hour for 7 hours, for a total rise of $5° \times 7 = 35°$. The temperature at 12:00 p.m. was $-15° + 35° = 20°$.

63. (A) Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at the center of the square. Therefore, they divide square ABCD into four large triangles of the same size. Line segments $\overline{EG}$ and $\overline{FH}$ also pass through the center of the square, so they divide each of the large triangles into 2 smaller triangles of the same size. Four of the smaller triangles are shaded, and their total area is 18 sq cm. Therefore, the total area of all 8 triangles is $18 \times 2 = 36$ sq cm. This is also the area of the square. If the area of the square is 36 sq cm, then the side length is $\sqrt{36} = 6$ cm.

64. (F)

$$\begin{aligned}21 + x &< -7 \\ 21 - 21 + x &< -7 - 21 \\ x &< -28\end{aligned}$$

65. (E) Brooke worked a total of $3\frac{1}{2} \times 5 = 17.5$ hours. Therefore, she earned $17.5 \times \$5.10 = \$89.25$.

66. (H)

$$\begin{aligned}15\% \text{ of } 7 &= (0.15)(7) = 1.05 \\ 7\% \text{ of } 7 &= (0.07)(7) = 0.49 \\ 1.05 - 0.49 &= 0.56\end{aligned}$$

67. (E) The number of customers who did not order dessert is $70-21=49$, so the fraction who did not order dessert is $\frac{49}{70}$. Reduce the fraction to lowest terms and then multiply by 100 to find the percentage who did not order dessert:

$$\frac{49}{70}=\frac{7}{10}$$

$$\frac{7}{10}\times 100=70\%$$

68. (K) Let *M* represent Morgan's speed. Use the formula distance = rate × time to find the distance Morgan traveled in 0.5 hours: distance = rate × time = 0.5*M*.

In 0.5 hours, Andy traveled $55\times 0.5=27.5$ miles. Together, they traveled a total distance of 60 miles:

$$27.5+0.5M=60$$

$$0.5M=32.5$$

$$M=65$$

69. (E) The formula for the circumference of a circle is $C=2\pi r$. The formula for the area of a circle is $A=\pi r^2$. Substitute the formulas into the equation $C=\frac{2}{5}A$ to solve for the radius of the circle:

$$C=\frac{2}{5}A$$

$$2\pi r=\frac{2}{5}(\pi r^2)$$

$$\frac{5}{2}(2\pi r)=\pi r^2$$

$$5r=r^2$$

$$5=r$$

The diameter is $5\times 2=10$ inches.

70. (G) Let *x* and *y* represent the two numbers. Translate the verbal description into an algebraic equation:

$$\frac{1}{3}(x-y)=12$$
$$x-y=36$$
$$9-y=36$$
$$-y=27$$
$$y=-27$$

The sum of *x* and *y* is $9+(-27)=-18$.

71. (A) Cross-multiply and then solve for *x*:

$$\frac{5}{8}=\frac{40}{x^2}$$
$$5x^2=8(40)$$
$$x^2=\frac{320}{5}$$
$$x^2=64$$
$$x=8 \text{ or } -8$$

72. (H) Let *x* represent the length of the rectangle:

$$9+9+x+x=40$$
$$18+2x=40$$
$$2x=22$$
$$x=11$$

The width is 9 inches and the length is 11 inches, so the area of the rectangle is $9\times11=99$ square inches.

73. (D) Amanda can choose among 6 brands of tennis rackets and 4 brands of tennis balls. Use the Fundamental Counting Principle

to find the different combinations of brands that she can buy:

$$6 \times 4 = 24$$

74. (G)

$$\dddot{5}(\dddot{8}) = \frac{5}{4}\left(\frac{8}{4}\right) = \frac{40}{16} = \frac{5}{2} = 2.5$$

75. (C) According to the circle graph, approximately 19.9% of the students are enrolled at Stuyvesant High School:

$$16,405 \times 0.199 = 3,264.595$$

3,264.595 rounded to the nearest whole number is 3,265.

76. (J) Erika is 7 years older than Fernando, so she is now $x + 7$. Four years ago, her age was $x + 7 - 4 = x + 3$.

77. (A) Substitute the values of *x* and *y* into the expression:

$$\frac{x+y}{x-4} = \frac{23+15}{23-4} = \frac{38}{19} = 2$$

78. (K) The least common multiple (LCM) of *X* and *Y* is the product of the greatest power of each of its prime factors:

$$X = 3 \times 3 \times 5 = 3^2 \times 5$$

$$Y = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$$

The LCM is $2^2 \times 3^2 \times 5 = 4 \times 9 \times 5 = 180$.

79. (A) If 1 inch is equivalent to 25 feet, then 1 square inch is equivalent to 625 square feet:

$$(1 \text{ in.})^2 = (25 \text{ ft})^2$$

$$1 \text{ sq in.} = 625 \text{ sq ft}$$

Set up a proportion to find how many square inches is equivalent to 1 square foot:

$$\frac{1 \text{ sq in.}}{625 \text{ sq ft}} = \frac{x}{1}$$
$$625x = 1$$
$$x = \frac{1}{625}$$
$$= 0.0016$$

80. (K) Let *x* represent the number of yellow balls in the bag. There are30 balls in the bag, and the probability of randomly drawing a yellow one is $\frac{1}{6}$. Set up a proportion to solve for *x*:

$$\frac{x}{30} = \frac{1}{6}$$
$$6x = 30$$
$$x = 5$$

Let *y* represent the number of red balls to be removed in order to increase the probability to $\frac{1}{3}$. Set up another proportion to solve for *y*:

$$\frac{5}{30 - y} = \frac{1}{3}$$
$$30 - y = 15$$
$$-y = -15$$
$$y = 15$$

81. (C) Cody ate $\frac{1}{7}$ of the cereal on Monday, so the fraction left in the box is $1-\frac{1}{7}=\frac{6}{7}$. On Tuesday, he ate $\frac{1}{6}\times\frac{6}{7}=\frac{1}{7}$ of the cereal. Therefore, Cody has eaten a total of $\frac{1}{7}+\frac{1}{7}=\frac{2}{7}$ of the cereal. The fraction that is left in the box after Tuesday is $1-\frac{2}{7}=\frac{5}{7}$.

82. (K) On the grid, the shaded region has a width of 5 inches and a length of about 7.5 inches. The scale is 1 in. = 4 feet, so the actual width is $5\times4=20$ feet and the actual length is $7.5\times4=30$ feet. One yard is equal to 3 feet, so the width is $\frac{20}{3}$ yd and the length is $\frac{30}{3}$ yd. Therefore, the area of the shaded region is $\frac{20}{3}\times\frac{30}{3}=66\frac{2}{3}$ square yards.

83. (A) The formula for the volume of a pyramid is $V=\frac{1}{3}bh$, where *b* is the area of the base and *h* is the height. Substitute the given values into the formula and solve for *b*:

$$V=\frac{1}{3}bh$$
$$75=\frac{5}{3}b$$
$$\frac{3}{5}(75)=b$$
$$45=b$$

The area of the base is 45 square centimeters. The base has a length of 9 centimeters, so it has a width of $45\div9=5$ centimeters.

84. (J) To find the value that is halfway between $\frac{1}{2}$ and 0.85, add and then divide by 2:

$$\frac{\frac{1}{2}+0.85}{2}=\frac{0.5+0.85}{2}$$
$$=\frac{1.35}{2}$$
$$=0.675$$

85. (B) Benita will be 24 years old in 8 years, so her age now is $24-8=16$. Julianne is half as old as Benita, so Julianne is now $16\div 2=8$ years old. Three years ago, Julianne was $8-3=5$ years old.

86. (H) The number of registered Republicans is about $13{,}000\times 0.46=5{,}980$. The number of registered Democrats is about $13{,}000\times 0.38=4{,}940$. There are about $5{,}980-4{,}940=1{,}040$ more Republicans than Democrats. The number 1,040 rounded to the nearest thousand is 1,000.

87. (D) Divide 220 by 12. The remainder represents the amount of fabric Tameka has left over:

$$220\div 12=18\,\mathrm{R}4$$

88. (J) Since points O, P, Q, and R are points on the circle, the diagonal of the square is equal to the diameter of the circle. Use the formula for the area of a circle, $A=\pi r^2$, to find the radius:

$$A=\pi r^2$$
$$36\pi=\pi r^2$$
$$36=r^2$$
$$6=r$$

Diameter is 2*r*, so the answer is 12.

89. (C) Use the formula for the area of a triangle, $A=\frac{1}{2}bh$, to find the measure of the base:

$$A=\frac{1}{2}bh$$
$$25=\frac{1}{2}b(5)$$
$$50=5b$$
$$10=b$$

The base of the triangle is equal to the length of the rectangle, and the height of the triangle is equal to the width of the rectangle, so the area of the rectangle is $10\times5=50$ sq cm. The area of the shaded region is $50-25=25$ sq cm.

90. (G) Substitute the value of *x* into the expression and solve for *y*:

$$(3x-1)(y-2x)=60$$
$$(21-1)(y-14)=60$$
$$20(y-14)=60$$
$$y-14=3$$
$$y=17$$

91. (D) Let *s* represent the location of point S. The distance between points R and S on the number line is $s-(-4)=s+4$. The distance between points S and T on the number line is 14–*s*. We know that $RS=\frac{1}{3}ST$. So:

$$s+4=\frac{1}{3}(14-s)$$
$$3s+12=14-s$$
$$4s=2$$
$$s=\frac{1}{2}$$

92. (J) It is given that $\mathrm{VY}=14\text{ cm}$ and that $\mathrm{XY}=5\text{ cm}$. We can use this information to find the length of $\overline{\mathrm{VX}}$:

$$\mathrm{VX}=\mathrm{VY}-\mathrm{XY}=14-5=9\text{ cm}$$

If $\mathrm{VW}=\mathrm{WX}$, then they are each $9\div 2=4.5\text{ cm}$. Therefore, $\mathrm{WY}=4.5+5=9.5\text{ cm}$.

93. (B) Translate the verbal description into an algebraic equation: if the electronic version costs one dollar less than one-fourth the cost of Julio's book, then:

$$y=\frac{1}{4}x-1$$

94. (F) Tia had a total of 35 marbles in the box. Before she removed any marbles, the probability of randomly drawing a blue marble was $\frac{15}{35}$. After she removed the 8 marbles, the probability of drawing a blue marble is:

$$\frac{15-6}{35-8}=\frac{9}{27}=\frac{1}{3}$$

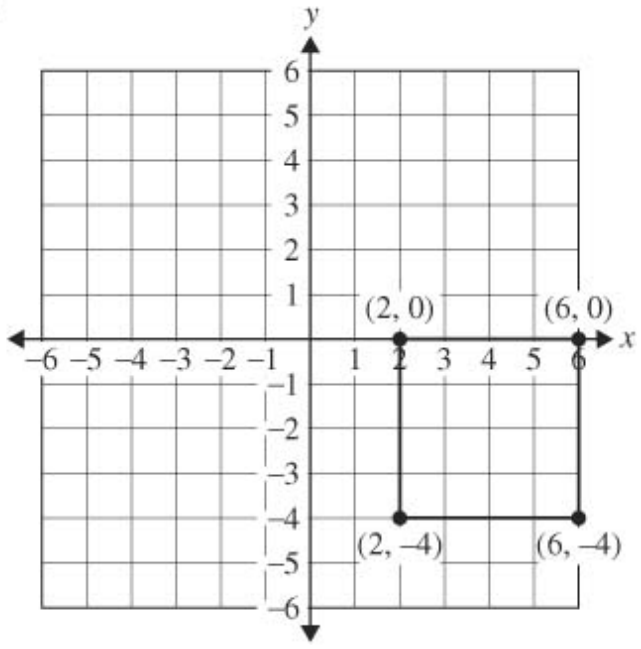
95. (D)

$$\left(\frac{2}{6}+\frac{1}{7}\right)\div\frac{2}{5}=\left(\frac{14}{42}+\frac{6}{42}\right)\div\frac{2}{5}$$
$$=\left(\frac{20}{42}\right)\div\frac{2}{5}$$
$$=\frac{20}{42}\times\frac{5}{2}$$
$$=\frac{100}{84}$$
$$=\frac{25}{21}$$

96. (G) Let x represent Erik's age:

$$\frac{39+22+45+x}{4}=33.5$$
$$\frac{106+x}{4}=33.5$$
$$106+x=134$$
$$x=28$$

97. (D)



98. (K) Let *x* represent the number of miles Allan can drive. Set up a proportion to solve for *x*:

$$\frac{x}{\frac{7}{8}\text{ gallon of gas}} = \frac{32\text{ miles}}{1\text{ gallon of gas}}$$

$$x = 32 \times \frac{7}{8}$$
$$= 28$$

99. (E) Let *x* represent the number of gallons of water Neil uses to water his plants:

$$\frac{3}{5}x = 4\frac{1}{3}$$
$$\frac{3}{5}x = \frac{13}{3}$$
$$x = \frac{13}{3} \div \frac{3}{5}$$
$$= \frac{13}{3} \times \frac{5}{3}$$
$$= \frac{65}{9}$$
$$= 7\frac{2}{9}$$

100. (H) Let *x* represent the total number of students in the 5 classes:

$$\frac{x}{5} = 17.4$$
$$x = 87$$

Let *y* represent the number of students in the sixth class:

$$\frac{87+y}{6}=18.5$$
$$87+y=111$$
$$y=24$$